12 Scope Distinctions

12.1 De Re/De Dicto

We have already seen the *de re*/*de dicto* distinction a number of times. It can be drawn when modal or intensional operators and quantifiers are used in the same statement.

(1) Something is necessarily good.

can be read as either of these:

(1′) $\Box\,\exists x\, Gx$ *de dicto*

(1″) $\exists x\,\Box\, Gx$ *de re*

In one case, the necessity is attached to a complete sentence (*dicto*); in the other, the necessary goodness is attached to some thing (*re*).

The difference here is a scope distinction, regarding which operator has a greater scope (applies to more of the statment): the modal operator or the quantifier.

The same distinction can be drawn with other intensional operators. Consider:

(2) I want a sloop.

Where "**D**" means *I desire it to be true that* ... and "H*xy*" means *x has y*, this can be either:

(2′) $\mathbf{D}\,\exists x(Sx\ \&\ Hix)$ *de dicto*

(2″) $\exists x(Sx\ \&\ \mathbf{D}\,Hix)$ *de re*

If I just wanted to own a sloop, but didn't have my eye on any particular one, (2′) would be true but (2″) false.

The distinction can also be clearly drawn whenever a description is used in the context of a modal or other intensional operator. Informally, we can distinguish two readings of:

(3) The number of planets is necessarily greater than five.

Namely:

(3′) It is true *of* the number that is the number of planets that *it* is necessarily greater than five. (*de re*; usually regarded as true)

(3″) It is necessarily true that the number of planets (whatever it may be) is greater than five. (*de dicto*; false)

Or with other intensional operators:

(4) Sally thinks that the biggest jerk in town is sexy. *Either*:

(4′) The biggest jerk in town is someone Sally thinks is sexy.

(4″) Sally thinks that whoever the biggest jerk in town is is sexy.

Here, we can easily imagine (4′) as true with (4″) false.

Russell's theory of descriptions provides us with a means of representing these different readings using scope distinctions completely on par with those given for (1′) and (1″) earlier.

($3'_R$) $\exists x(Nx\ \&\ \forall y(Ny \to y = x)\ \&\ \Box\, x > 5)$

($3''_R$) $\Box\,\exists x(Nx\ \&\ \forall y(Ny \to y = x)\ \&\ x > 5)$

($4'_R$) $\exists x(Jx\ \&\ \forall y(Jy \to y = x)\ \&$ *Sally-thinks-that* : $Sx)$

($4''_R$) *Sally-thinks-that* : $\exists x(Jx\ \&\ \forall y(Jy \to y = x)\ \&\ Sx)$

Although Garson admits that Russell's theory can accommodate these differences, he faults it for not being able to make a similar move when no descriptions are involved (e.g., believing of Clemens that he is an author, i.e., in virtue of beliving that Twain is, and believing that Clemens is an author outright), though it should be noted that on this score, the theory about to be sketched does no better.

The modal description theory $\imath$K formulated earlier can only accommodate the the *de dicto* readings. That is, given the semantics sketched:

($3''_G$) $\Box\,\imath x\, Nx > 5$

would require that, at each possible world in the model, the object that is the number of planets at that world is greater than five. Similarly:

($4''_G$) *Sally-thinks-that* : $S(\imath x\, Jx)$

gives us something analogous to the *de dicto* reading earlier.

Garson advocates expanding the language of the system in order to allow it to express *de re* modal claims for sentences with descriptions as well.

12.2 Lambda abstracts

Garson does by means of lambda abstracts. Lambda abstracts were originally invented by Alonzo Church for a very different purpose, i.e., stating the inference rules of higher-order logic in a simple recursive form. A lambda abstract is written:

$$\lambda x\, Ax$$

and may be read "being an x such that Ax" (or in context, "... is an x such that Ax"). This notation is usually used to represent complex properties. For example, if "Bx" means x is blue, and "Gx" mean x is green, then:

$$\lambda x(Gx \lor Bx)$$

represents the property of being *either* green or blue. The property of being taller than everything (else) might be written:

$$\lambda x\, \forall y(y \neq x \to Txy)$$

Like "∀" and "ı", "λ" is a variable binding operator; the occurrences of the variable "x" are all bound above. Notice, however, that the above is neither a term nor a sentence; it is a predicate. Like (monadic) predicate letters, it is to be followed by terms to form a sentence. Thus, to say that "a" is taller than everything else, we may write:

$$\lambda x\, \forall y(y \neq x \to Txy)(a)$$

which is equivalent to:

$$\forall y(y \neq a \to Tay)$$

We shall normally require that parentheses and commas used to separate predicates and terms not be left out when using predicate terms, in order to differentiate the predicate, "$\lambda x\, S(x, a)$" which stands for the property of bearing relation S to a, from the sentence "$\lambda x\, S(x)(a)$" which predicates the property of being an x such that Sx to a, which would be obscured if we wrote either as "$\lambda x\, Sxa$".

Typically, a language involving lambda abstracts adopts a principle such as the following schema:

(λ) $\lambda x\, Ax(c) \leftrightarrow Ac$

This is sometimes known as the *principle of lambda abstraction*, and an inference from one half of the biconditional is sometimes known as *lambda conversion* (or lambda contraction in the left-to-right, or lambda expansion in the right-to-left direction).

In the present context, the use of lambda abstracts is that it allows us to remove descriptions from the scope of modal operators, repalce them with variables, but still predicate the thing in question of the thing which satisfies the description. In particular, we could render the *de re* readings of (3) and (4) from the previous section as:

($3'_G$) $\lambda y(\Box\, y > 5)(\imath x\, Nx)$

i.e., the number of planets has the property of being something necessarily greater than five.

($4'_G$) $\lambda y(\textit{Sally-thinks-that} : Sy)(\imath x\, Jx)$

i.e., the biggest jerk in town has the property of being someone Sally thinks is nice.

The reason these can be read *de re* is that unlike the earlier examples, the descriptions are not in the scope of the "□", so that their truth or falsity does not depend on the semantic values of these descriptions at *other* worlds; these must be contrasted with the *de dicto* statements given earlier, or even the following, which are equivalent to the *de dicto* readings:

($3''_G\star$) $\Box\, \lambda y(y > 5)(\imath x\, Nx)$

($4''_G\star$) $\textit{Sally-thinks-that} : \lambda y(Sy)(\imath x\, Jx)$

The difference then is not merely whether or not lambda abstracts are involved, but whether the descriptions occur in the scope of "□".

12.3 The System λK

To be more precise, we now expand the system *ıK* by adding lambda abstracts to our syntax and adopting the corresponding rules. Now terms, predicates and sentences must be defined together, so that we have:

1. Constants are terms.
2. "⊥" is a sentence.
3. Predicate letters are predicates.
4. If $t_1, \ldots, t_n$ are terms and P is a predicate then $P(t_1, \ldots, t_n)$ is a sentence.
5. If A is a sentence, then $\Box\, A$ is a sentence.
6. If A and B are sentences, then $(A \to B)$ is a sentence.
7. If At is a sentence containing term t, and x is a variable, then $\imath x\, Ax$ is a term.
8. If At is a sentence containing term t, and x is a variable, then $\lambda x\, Ax$ is a predicate.
9. If At is a sentence containing term t, and x is a variable, then $\forall x\, Ax$ is a sentence.

The system is fairly minimal with regard to its inference rules. For the system λK (—we could define λB, λS5, etc., similarly—) we adopt the principle (λ) to the left as an axiom schema. However, in order to avoid the *de re*:

($3'_G$) $\lambda y(\Box\, y > 5)(\imath x\, Nx)$

collapsing into the *de dicto* statement:

($3''_G$) $\Box\, \imath x\, Nx > 5$

We must restrict the schema (λ) to *constants* only (terms that are designed to be rigid); we do not adopt the more general:

$$\lambda x\, Ax(t) \leftrightarrow At$$

for terms t besides constants.

The identity rule (=Out) is specified so that one may infer $P(u_1,\ldots,u_n,t,v_1,\ldots,v_m)$ from $P(u_1,\ldots,u_n,s,v_1,\ldots,v_m)$ and $s=t$ where P is either a predicate letter or a lambda abstract. (Like every other rule, we cannot apply this rule to parts of lines, or when the predicate is inside the scope of some other operator.)

Treating "8" as a constant, we may go from:

$$\Box\, 8 > 5$$

to:

$$\lambda\mathrm{y}(\Box\,\mathrm{y} > 5)(8)$$

by (λ). From this and:

$$8 = \imath\mathrm{x}\,\mathrm{Nx}$$

one may use the new (=Out) to infer:

$$\lambda\mathrm{y}(\Box\,\mathrm{y} > 5)(\imath\mathrm{x}\,\mathrm{Nx})$$

Since this is the *de re* claim claim, not the *de dicto* claim, this is unproblematic. Because we cannot replace identicals inside the scope of modal operators, or apply (λ) to description terms, there is no way to get the more problematic:

$$\Box\,\imath\mathrm{x}\,\mathrm{Nx} > 5$$

Similarly, if $\imath\mathrm{z}\,\mathrm{Bz}$ is the number of members of the Brady family, from:

$$\lambda\mathrm{y}(\Box\,\mathrm{y} > 5)(\imath\mathrm{x}\,\mathrm{Nx})$$

and

$$\imath\mathrm{x}\,\mathrm{Nx} = \imath\mathrm{z}\,\mathrm{Bz}$$

one may infer the *de re* claim:

$$\lambda\mathrm{y}(\Box\,\mathrm{y} > 5)(\imath\mathrm{z}\,\mathrm{Bz})$$

Because this is *de re*, this is unobjectionable. We may not proceed to:

$$\Box\,\imath\mathrm{z}\,\mathrm{Bz} > 5$$

12.4 Semantics for λK

To do formal semantics for the above, we need to modify the notion of a $\imath$K model $\langle \mathbf{W}, \mathbf{R}, \mathbb{D}, \mathbf{a}\rangle$ so that the assignment function assigns semantic values also to lambda abstracts; like predicate letters, these are assigned extensions at each world. In particular, we define a λK model such that the following holds:

$\mathbf{a_w}(\lambda x\, Ax)$ = the set of objects $\mathbf{d}$ in $\mathbf{D}$ such that $\mathbf{a_w}(A\mathbf{d}) = \mathrm{T}$ (where $A\mathbf{d}$ is the approprite hybrid).

The semantics for sentences in which lambda abstracts are used are the same as for predicates generall in objectual semantics, so that we take (where each of $\mathbf{o}_1 \ldots \mathbf{o}_n$ is either an object or term):

$$\mathbf{a_w}(P(\mathbf{o}_1,\ldots,\mathbf{o}_n)) = \begin{cases} \mathrm{T} & \text{if } \langle \mathbf{a_w}(\mathbf{o}_1),\ldots,\mathbf{a_w}(\mathbf{o}_n)\rangle \\ & \quad \text{is a member of } \mathbf{a_w}(P) \\ \mathrm{F} & \text{otherwise} \end{cases}$$

to apply both when P is a simple predicate letter and when it is a lambda abstract, in which case, the above can be read:

$$\mathbf{a_w}(\lambda x\, Ax(\mathbf{o})) = \begin{cases} \mathrm{T} & \text{if } \mathbf{a_w}(\mathbf{o}) \text{ is a member of} \\ & \quad \mathbf{a_w}(\lambda x\, Ax) \\ \mathrm{F} & \text{otherwise} \end{cases}$$

Putting these together, in effect, we get that $\mathbf{a_w}(\lambda x\, Ax(t)) = \mathrm{T}$ iff $\mathbf{a_w}(t)$ is an object $\mathbf{d}$ such that, for the hybrid, $\mathbf{a_w}(A\mathbf{d}) = \mathrm{T}$.

The deductive system sketched in the previous section is sound and complete with regard to this semantics, in the sense that we have:

$$A_1,\ldots,A_n \vdash_{\lambda\mathrm{K}} B \text{ if and only if } A_1,\ldots,A_n \vDash_{\lambda\mathrm{K}} B$$

How then do the semantics of *de re* and *de dicto* readings differ from another? Consider, then any *de dicto* claim of the form:

$$\Box\, P(\imath x\, Ax)$$

For this, we have $\mathbf{a_w}(\Box\, P(\imath x\, Ax)) = \mathrm{T}$ iff $\mathbf{a_v}(P(\imath x\, Ax)) = \mathrm{T}$ for all world $\mathbf{v}$ such that $\mathbf{wRv}$. This is in turn the case if $\mathbf{a_v}(\imath x\, Ax)$ is a member of $\mathbf{a_v}(P)$ for each such world.

On the other hand, consider:

$$\lambda y(\Box\, Py)(\imath x\, Ax)$$

This *de re* claim will be true in case $\mathbf{a_w}(\imath x\, Ax)$ stands for an object $\mathbf{d}$, and that object $\mathbf{d}$ is a member of $\mathbf{a_w}(\lambda y(\Box\, Py))$, which it will be just in case it is a member of $\mathbf{a_v}(P)$ for every world $\mathbf{v}$ such that $\mathbf{wRv}$.

In other words, the difference is that the *de re* claim depends on whether $\mathbf{a_w}(\imath x\, Ax)$ is a member of $\mathbf{a_v}(P)$ for the worlds $\mathbf{v}$ accessible to $\mathbf{w}$, whereas, for the *de dicto* statement, what matters is whether $\mathbf{a_v}(\imath x\, Ax)$ is a member of $\mathbf{a_v}(P)$. Assuming the description term is non-rigid, these may come apart. For the *de re* claim, what matters is whether or not the actual thing that is the A is P in every possible world; in the *de dicto* claim, it depends on different things in different world, depending on which things (if any) are the A in those worlds.

Notice that if all terms were rigid, the distinction would completely collapse. Since constants are required to be rigid in λK (recall that λK is built on $\imath$K, which is in turn built on rK), the *de re/de dicto* distinction doesn't apply to them. (This is also the reason why lambda conversion is unobjectionable or valid for constants.)

12.5 Quantifying In and Essences

Recall Quine's complaint that moving from something like:

$$\Box\, 8 > 5$$

to:

$$\exists x \Box\, x > 5$$

is illegitimate because "8" does not really have a referential use in the first example; since if it referred to the object itself, it ought to be true regardless of how this object is described.

And indeed, inferences of this form can be seen as problematic in general, since moving from the *de dicto* claim:

$$\Box(\text{The shortest spy is a spy.})$$

To the *de re* claim:

$$\exists x \Box(x \text{ is a spy})$$

does not seem valid. Indeed, in general, *de re* modalities involve quantifying in to a modal context, which Quine and others find illegitimate.

Garson's work seems to provide a partial response to these worries; the truth-values of quantified statements is defined in terms of hybrids, and in general, it is not possible in his systems to make any inferences similar to the shortest spy example, and it is possible in the first example only if "8" is treated as a constant (and one for something that exists).

This general line of approach coheres somewhat with a line suggested by David Kaplan years ago in which *de re* modalities can be seen as true when certain *de dicto* claims are true, not just any *de dicto* claims that involve a description naming an existing object, but only those making use of some sort of privileged name more closely associated with the object itself. (As "8" is with the number; it rigidly designates that selfsame number in every world.)

Nevertheless, in even assinging truth-values to hybrids like "$\Box\, A\mathbf{d}$", where $\mathbf{d}$ is the object itself, divorced from the way in which it is referred to or named, Garson's approach arguably commits him to essentialism; the idea that some properties an object has, it (—the thing itself—) has necessarily (its essential properties or essence), and others it doesn't have necessarily (so called "accidents"), or at least if he wants his logic to allow for interesting results.

The same might be said merely in accommodating *de re* modality at all. Even admitting that the thing which is the number of planets is necessarily greater than 5 seems to suggest that the number 8 has certain properties (e.g., its purely mathematical ones), essentially, and others (like numbering the planets), accidentally. Those who accept a view of necessity that reduces it to analyticity may object, since necessary truths must be true in virtue of meaning, but meaning attaches to the label for the thing, or the means of representing it, not the thing itself.

Quine has argued against the coherence of essences by citing the following argument:

(1) Mathematicians are necessarily rational.
(2) Mathematicians are not necessarily two legged.
(3) Cyclists are not necessarily rational.
(4) Cyclists are necessarily two legged.
(5) John is a cyclist and John is a mathematician (and John exists).

Translated as follows, these generate contradictions (that John both is and isn't necessarily rational, and necessarily two legged ...):

(1⋆) $\forall x(Mx \rightarrow \Box\, Rx)$
(2⋆) $\forall x(Mx \rightarrow \sim\Box\, Tx)$
(3⋆) $\forall x(Cx \rightarrow \sim\Box\, Rx)$
(4⋆) $\forall x(Cx \rightarrow \Box\, Tx)$
(5⋆) $(Cj \,\&\, Mj) \,\&\, Ej$

Garson's response is to deny that (2⋆) and (3⋆) are the correct translations of the above. (2) he suggests ought to be rather, instead, either $\sim\forall x(Mx \rightarrow \Box\, Tx)$ or $\sim\Box\,\forall x(Mx \rightarrow Tx)$, neither of which conflicts with (4⋆). Personally, I think the point would only be plausible if made even stronger: (1⋆) should be interpreted as the weaker $\Box\,\forall x(Mx \rightarrow Rx)$.

However, Garson does not defend any particular conception of what essential properties anything has, but instead only wants to argue that the idea behind essentialism is not logically coherent. If it is lacking, it is lacking for *philosophical* reasons, whereas modal logic itself can remain neutral on the issue.