4 Systems for Other Intensional Operators

We briefly turn to consideration of logical systems that count as modal logic only in one of the broader senses, i.e., that do not count as *alethic* modal logic.

Many of these systems are philosophically controversial.

4.1 Deontic Logic: System D

In deontic logic, our sentential operators stand for the moral concepts of *obligation* and *permissibility*. While it would not be a far stretch to understand '$\Box$' as representing the *moral* 'must' and '$\Diamond$' as the moral 'may', to avoid confusion we'll use '**O**' (for "it is obligatory that ...", or "it ought to be that ...") instead of '$\Box$', and add defined signs for "it is permitted that ..." and "it is forbidden that ...":

(Def **P**) $\mathbf{P}A$ abbreviates $\sim\mathbf{O}\sim A$

(Def **F**) $\mathbf{F}A$ abbreviates $\mathbf{O}\sim A$

The correct formulation of the inference rules and axioms is extremely controversial; indeed, some question whether it can be done at all.

The most common formulation, system D, contains deontic analogues of the rules of system K, (**O**-In) and (**O**-Out), as well as the following axiom schema:

(D) $\mathbf{O}A \rightarrow \mathbf{P}A$

I.e., whatever is obligatory is permissible.

Some other popular formulations add additional principles governing iterated deontic operators.

(OO) $\mathbf{O}A \leftrightarrow \mathbf{OO}A$

(OP) $\mathbf{P}A \leftrightarrow \mathbf{OP}A$

(OM) $\mathbf{O}(\mathbf{O}A \rightarrow A)$

(OO) is the deontic equivalent to a biconditional form of (4) of S4, and (OP) is the deontic equivalent to a biconditional form of (5) of S5. However, since S4 and S5 also include (M), which is stronger than (OM)+(D), these do not result in systems as strong as S4 and S5.

Puzzle 1 *All theorems (including all tautologies) are obligatory and their opposites forbidden.*

For example, we have $\vdash_{\mathrm{D}} \mathbf{F}\bot$, and $\vdash_{\mathrm{D}} \mathbf{O}(p \rightarrow p)$. However, it seems odd that one would be obligated to bring about something that couldn't be false, or forbidden to bring about a contradiction.

Puzzle 2 *If something is permitted, then either it or anything else is permitted.*

The proof of the following is easy, and follows from the rules of K alone:

$\vdash_{\mathrm{D}} \mathbf{P}A \rightarrow \mathbf{P}(A \vee B)$

But consider the following instance: *if it is permissible that you hug your grandmother, then it is permissible that you either hug or spit on your grandmother*. A reponse might be that $\mathbf{P}(h \vee s)$ might not be the correct rendering of the English sentence "it is permissible that you either hug or spit on your grandmother", which seems to imply $\mathbf{P}h$ & $\mathbf{P}s$. Indeed, even the English sentence, "either you can hug or you can spit on your grandmother" seems to mean something closer to $\mathbf{P}h$ & $\mathbf{P}s$ than the more literal $\mathbf{P}h \vee \mathbf{P}s$, given the truth-conditions involved.

Similarly, we have:

$\vdash_{\mathrm{D}} \mathbf{O}A \rightarrow \mathbf{O}(A \vee B)$

Suppuse I ought to hug grandma. Does this mean I ought to hug or spit on grandma? If we accept the result that $\mathbf{O}(h \vee s)$, it turns out that if I were to spit on Grandma, I'd be fulfilling one of my obligations, since rendering s true is sufficient to render $h \vee s$ true.

Puzzle 3 *One cannot have conflicting obligations.*

We have the following (again, following from rules of K alone):

(NonCon) $\vdash_{\mathrm{D}} \sim(\mathbf{O}A$ & $\mathbf{O}\sim A)$

Yet, conflicting obligations do not seem impossible. If I've promised to meet a friend for lunch at 1pm, it seems I'm obligated to go to lunch, but if at the same time, my child needs to go to the hospital because of an emergency, I'm obligated not to go to lunch. One response might be to insist that '**O**' only represents our "all things considered" obligations, not our *prima facie* obligations that can be overridden by other considerations. However, even this might show that standard deontic logic is lacking insofar as it does not have the means for representing how different *prima facie* obligations go together to determine or constrain one's final obligations.

Puzzle 4 (Chisholm's paradox) *D cannot adquately capture conditional obligations.*

Consider:

1. It ought to be that Sadie goes to the meeting.
2. It ought to be that if Sadie goes to the meeting, she tells them she's coming.
3. If Sadie doesn't go to the meeting, she ought not tell them she's coming.
4. Sadie doesn't go to the meeting.

Rendering these as:

1. $\mathbf{O}g$
2. $\mathbf{O}(g \rightarrow t)$
3. $\sim g \rightarrow \mathbf{O}\sim t$
4. $\sim g$

We get a contradiction! By the (Dist) schema (from K), (1) and (2) yield $\mathbf{O}t$, but (3) and (4) yield $\mathbf{O}\sim t$, which together contradict (NonCon) above.

If we render (2) instead as $g \rightarrow \mathbf{O}t$, it trivially follows from (4), so does not seem quite what we want. If we render (3) as $\mathbf{O}(\sim g \rightarrow \sim t)$, it follows under the rules K from (1) so doesn't quite seem right either. Indeed, we have the following rather odd theorem schema in D:

$\vdash_{D}$ $\mathbf{O}A \rightarrow \mathbf{O}(\sim A \rightarrow B)$, *whatever B might be!*

These and other puzzles (see, e.g., the Good Samaritan paradox) leave the correct formulation and/or application of deontic logic in some doubt.

4.2 Tense Logic (AKA Temporal Logic): System Kt

Here, rather than adding a single pair of modal operators obeying the modal square of opposition, we have two: one pair dealing with the future, another with the past. Let $\mathbf{G}A$ mean *A will always be the case*, and let $\mathbf{H}A$ means *A has always been the case*. We can then define operators for being true at some time in the future, and being true at some time in the past:

(Def **F**) $\mathbf{F}A$ abbreviates $\sim\mathbf{G}\sim A$
(Def **P**) $\mathbf{P}A$ abbreviates $\sim\mathbf{H}\sim A$

In the standard formulation, we have rules corresponding to the rules K for both "strong" operators, (**G**-In) and (**G**-Out), as well as (**H**-In) and (**H**-Out). These insure that all tautologies, as well as axioms and theorems of the system, have always been, and always will be, true. We also have:

(GP) $A \rightarrow \mathbf{GP}A$

(HF) $A \rightarrow \mathbf{HF}A$

Whatever is the case now, in the future, always will have been true at some time in the past. Whatever is the case now always has been something that was going to be true in the future. (This is not taken to imply determinism.) These give us the duals of the above, and temporal versions of (4) and (4′):

$\vdash_{Kt}$ $\mathbf{PG}A \rightarrow A$
$\vdash_{Kt}$ $\mathbf{FH}A \rightarrow A$
$\vdash_{Kt}$ $\mathbf{G}A \rightarrow \mathbf{GG}A$
$\vdash_{Kt}$ $\mathbf{H}A \rightarrow \mathbf{HH}A$
$\vdash_{Kt}$ $\mathbf{FF}A \rightarrow \mathbf{F}A$
$\vdash_{Kt}$ $\mathbf{PP}A \rightarrow \mathbf{P}A$

The main logical principles of tense logic are not nearly as controversial as other logics with intensional operators, though their correct philosophical interpretation is controversial.

4.3 Doxastic and Epistemic Logic

Doxastic logic is *the logic of belief*. Epistemic logic is *the logic of knowledge*. These are interrelated and often, though not always, treated together. The correct principles for doxastic and epistemic logic are, like deontic logic, extremely controversial.

Here, we add the primitive operators, '$\mathbf{B}_m$', '$\mathbf{B}_n$', etc., and '$\mathbf{K}_m$', '$\mathbf{K}_n$'. The subscripts track different believers or knowers, so that $\mathbf{K}_m p$ might mean that Manuel knows that p, whereas $\mathbf{K}_n p$ might mean that Nancy knows that p. These subscripts are often dropped if one only needs to consider a single believer or knower in a given context. I shall drop them in my discussion below.

(Def **C**) $\mathbf{C}A$ abbreviates $\sim\mathbf{B}\sim A$
(Def **P**) $\mathbf{P}A$ abbreviates $\sim\mathbf{K}\sim A$

According to one popular formulation (owing to Jaakko Hintikka), the logic for '**K**' can be built upon system K, and hence contains the schema:

(Dist-**K**) $\mathbf{K}(A \rightarrow B) \rightarrow (\mathbf{K}A \rightarrow \mathbf{K}B)$

It also adds the following:

(M-**K**) $\mathbf{K}A \rightarrow A$
(KK) $\mathbf{K}A \rightarrow \mathbf{KK}A$

A moment's reflection reveals that this is precisely modal system S4 simply with '**K**' in place of '$\Box$'!

However, this way of formulating epistemic logic is open to certain doubts.

Doubt 1 *By the epistemic necessitation rule, it entails that we know every tautology, as well as the principles of deontic logic themselves.*

Aren't some purely logical truths so complex that we are not in a position to know them? And when it comes to the principles of epistemic logic itself, many people don't even believe the above axioms: so how could they know them?

Doubt 2 *The (Dist) schema entails that knowledge is closed under implication, i.e., that whenever we know a conditional and know its antecedent, we know its consequent.*

This seems to suggest that we actually make every inference we're in a position to make. But sometimes, it would seem, we do not always realize that our knowledge entails something else. Moreover, it seems to conflict with common intuitions about skeptical scenarios. Consider, e.g.:

1. I know that I'm seeing a book.
2. I know that if I'm seeing a book, I'm not a brain in a vat.
3. I don't know that I'm not a brain in a vat.

These three sentences cannot all be true in S4 epistemic logic.

Doubt 3 *It claims that whatever we know is something we know that we know.*

The so-called (KK)-principle or "principle of positive introspection" seems most suitable for a very strong, idealized notion of knowledge, such as that found in Descartes's *Meditations*, where to have knowledge of something is for it to beyond rational doubt. One might think that any reason I'd have for doubting that I knew something would be reason for doubting the thing itself.

However, on more ordinary conceptions of knowledge, it seems tempting to think that one could know something without even having the concept of knowledge, much less explicitly having the belief that you knew it.

On so-called 'externalist' theories of knowledge, (KK) is even more objectionable, since knowledge there often amounts to something like true belief formed or caused in a reliable way. To know that you knew something would require knowledge about what caused your beliefs, which is uncommon.

Nevertheless, these puzzles do not seem as threatening if '**K**p' is reinterpreted to mean that one *is in the position to know* to know p, not that one actually does know that p.

Doubts about whether or not the logic of '**B**' can be treated as an extension of K (so that all tautologies are believed, and belief is closed under implication,) tend to be even more worrisome than for epistemic logic. Perhaps these doubts can be overcome. If so, one natural formulation of doxastic logic would in essence be KD4, accepting:

(D-**B**) $\mathbf{B}A \rightarrow \mathbf{C}A$
(BB) $\mathbf{B}A \rightarrow \mathbf{BB}A$

Note that this leads to the result that no one has incompatible beliefs:

(NonCon-**B**) $\sim(\mathbf{B}A \ \& \ \mathbf{B}{\sim}A)$

This is by no means obviously correct.

As with epistemic logic, some of the puzzles may seem less puzzling if '**B**p' is reinterpreted to mean that you *ought* to believe p or that your beliefs make you *committed* to the truth of p.

Here are some other principles of epistemic/doxastic logic that are sometimes accepted, but often disputed:

- $\mathbf{BB}A \rightarrow \mathbf{B}A$
- $\mathbf{K}A \rightarrow \mathbf{B}A$
- $\mathbf{B}A \rightarrow \mathbf{KB}A$
- ${\sim}\mathbf{K}A \rightarrow \mathbf{K}{\sim}\mathbf{K}A$
- $\mathbf{PK}A \rightarrow \mathbf{KP}A$
- $\mathbf{BK}A \rightarrow \mathbf{B}A$
- ${\sim}\mathbf{B}\bot$

Many of these controversies would be better discussed in the context of an epistemology or philosophy of mind course. Nevertheless, it should be clear why these topics are thought to be closely related to modal logic. If a given philosophical position on the nature of belief or knowledge implies that these notions obey a logic essentially the same as a modal system such as K or S4, the results we already have for modal logic are relevant to them.