Phil 511: Modal Logic

Spring 2008 — Take Home Final

Due in my box in 352 Bartlett on Friday, May 23.

The exam consists of three parts: A, B and C. Complete all three parts, and five questions in total.

Part A — Derivations and Trees

Choose *any three* of the following five problems.

(For the derivation problems, all derived rules and theorem schemata proven in the book, handouts or homework are fair game, so long as they apply to the system in question, or one on which it's built.)

(1) Construct derivations to show:

$\forall x\,\Box(Ax \rightarrow Bx) \ \vdash_{\text{qK}} \Box(\forall x\,Ax \rightarrow \forall x\,Bx)$

$\vdash_{\text{qT}} \exists x\,\Diamond(\Box\,Ax \rightarrow \forall y\,\Box\,Ay)$

$\forall x(\Box\,Ax \lor \Box\,{\sim}Ax) \ \vdash_{\text{qS4}} \forall x\,\Box(\Box\,Ax \leftrightarrow Ax)$

(2) Construct derivations to show:

$\Diamond\,\forall x\,Ax \,\&\, \Box\,\exists x\,Bx \ \vdash_{\text{fK}} \Diamond\,\exists x(Ax \,\&\, Bx)$

$\Box\,\forall x(Ax \lor Bx) \ \vdash_{\text{fT}} \Diamond\,\forall x\,Ax \lor \exists x\,\Diamond\,Bx$

$\vdash_{\text{fS5}} \forall x\,\Box\,\text{E}x \rightarrow (\exists x\,\Diamond\,\Diamond\,Ax \rightarrow \Box\,\Diamond\,\exists x\,Ax)$

(3) Construct derivations to show:

$\exists x\,\forall y\,\Diamond\,x = y \vdash_{\text{rK}} \exists x\,\forall y\,\Box\,x = y$

$\forall x(Ax \rightarrow x = \imath y\,\Diamond\,{\sim}Ay) \vdash_{\imath\text{T}} {\sim}\exists x\,\Box\,Ax$

$\vdash_{\lambda\text{K}} \lambda x\,\Box\,Ax(c) \leftrightarrow \Box\,\lambda x\,Ax(c)$, where c is a constant.

(4) Use *trees* to determine whether or not the following arguments are qK valid, *as well as* whether or not they are tK (=fK) valid.

$\Diamond\,\exists\text{x}\,\text{Fx} \ \ / \therefore \exists\text{x}\,\Diamond\,\text{Fx}$

$\Box\,\exists\text{x}\,\text{Fx} \ \ / \therefore \exists\text{x}\,\Box\,\text{Fx}$

$\forall\text{x}(\text{Fx} \rightarrow \Box\,\text{Gx}) \,\&\, \Box\,\forall\text{x}(\text{Gx} \rightarrow \text{Hx}) \ \ / \therefore \forall\text{x}(\text{Fx} \rightarrow \Box\,\text{Hx})$

$/ \therefore \forall\text{x}(\Box\,\text{Fx} \lor \Box\,{\sim}\text{Fx}) \lor \forall\text{x}(\Diamond\,\text{Fx} \,\&\, \Diamond\,{\sim}\text{Fx})$

$\exists\text{x}\,\forall\text{y}\,\Box\,\text{Rxy} \ \ / \therefore \forall\text{y}\,\Box\,\exists\text{x}\,\text{Rxy}$

$\forall\text{x}(\Diamond\,\text{Fx} \,\&\, \Diamond\,{\sim}\text{Fx}) \ \ / \therefore \forall\text{x}(\Box\,\text{Fx} \leftrightarrow (\text{Gx} \,\&\, {\sim}\text{Gx}))$

(5) Use trees to determine whether or not the following arguments have the sort of validity indicated:

$\vDash_{\text{qS5}} \Box\,\forall\text{x}\,\text{Fx} \leftrightarrow \Diamond\,\forall\text{x}\,\Box\,\text{Fx}$??

$\Box\,\exists\text{x}\,\text{Fx} \vDash_{\text{tT (=fT)}} \exists\text{x}\,\forall\text{y}(\Diamond\,\text{Fy} \rightarrow \text{y} = \text{x}) \rightarrow \Box\,\forall\text{x}\,\forall\text{y}[(\text{Fx} \,\&\, \text{Fy}) \rightarrow \text{x} = \text{y}]$??

$\exists\text{x}\,\forall\text{y}\,\Diamond\,\text{x} = \text{y} \vDash_{\text{rK}} \forall\text{x}\,\forall\text{y}\,\text{x} = \text{y}$??

(and finally "The New Ontological Argument for God":) $\Diamond\,\text{Eg}, \Box(\text{Eg} \rightarrow \Box\,\text{Eg}) \vDash_{\text{fB}} \text{Eg}$??

Part B — Semantics and Metatheory

Choose *any one* of the following four problems.

(6) *In your own words*, explain the difference between substitutional, objectual and intensional semantics for quantified modal logic. What advantages or disadvantages do you see with each approach? Does one of these approaches seem best to you? If so, why? Explain your answer.

(7) *In your own words*, but informally, explain both why it is that if an argument $A_1, \ldots, A_n \ / \therefore B$ is tK valid, then the tree for it closes, and why it is that if its tree closes, then there is an fK-derivation for it. (You may take for granted the parts of these proofs that carry over from propositional modal logic.) Illustrate with an example.

(8) *In your own words*, explain the proof of the soundness of system fK relative to iK models, that is, that if $A_1, \ldots, A_n \ \vdash_{\text{fK}} B$ then $A_1, \ldots, A_n \ \vDash_{\text{iK}} B$. (You may take for granted the parts of this proof carried over from propositional modal logic.)

(9) *In your own words*, explain the proof that if an argument is valid according to intensional semantics then it is also valid according to substitutional semantics, i.e., that if $A_1, \ldots, A_n \ \vDash_{\text{iK}} B$ then $A_1, \ldots, A_n \ \vDash_{\text{tK}} B$.

Part C — Philosophical Issues and More

Choose *any one* of the following four problems. (Ideally, your answer should be not more than 2–3 pages in length.)

(10) Compare and contrast in some detail the treatment of descriptions and the de re/de dicto distinctions from Garson's systems $\imath$K and λK with the approach inherent in Russell's theory of descriptions. Illustrate with examples. Does one seem like a better approach than the other to you? Why or why not?

(11) Compare and contrast Lewis's Counterpart Theory to the other theories of quantified modal logic studied in this course, and briefly sketch Lewis's means for translating the latter into the former. Does it seem to you that Lewis's approach offers any particular advantages or disadvantages?

(12) Explain why Kripke thinks that names are rigid designators and what does it lead him to conclude about the necessity, or lack thereof, of identity statements? Discuss these matters in light of Garson's system rK, and explain how it is that having rigid constants helps solve certain technical issues in objectual semantics. (See, e.g., page 261 and sec. 13.6 of the textbook.) Do you agree? Why or why not?

(13) Explain Kripke's defense of the notion of identity of an object across worlds, and explain his diagnosis of why other theorists were misled into thinking there was a problem there. Do you agree? Explain your answer.

Standard disclaimer: *This is not a group exericse. You are expecteed to work on your own, though you may ask me for help.*