

Phil 511: Modal Logic

Spring 2008 — Take Home Midterm

Due at the start of class on Thursday, April 3.

The exam consists of three parts: A, B and C. Complete all three parts, and five questions in total.

Part A — Derivations

Choose *any two* of the following four problems.

(1) Without using *any* derived rules or theorems — i.e., using only the primitive inference rules of PL and definitions, show the following:

$$\begin{aligned} A \leftrightarrow B &\vdash_{\text{PL}} (A \& B) \vee (\sim A \& \sim B) \\ \sim(A \leftrightarrow B) &\vdash_{\text{PL}} (A \& \sim B) \vee (\sim A \& B) \end{aligned}$$

(2) Construct derivations to show the following. All derived rules and theorems in the book or handouts are fair game:

$$\begin{aligned} \diamond \Box A \& \diamond \Box B &\vdash_{\text{K}} \diamond \Box (A \& B) \\ \Box A \rightarrow \Box B &\vdash_{\text{K}} \Box C \rightarrow \Box (A \rightarrow B) \\ \vdash_{\text{K}} (\Box A \rightarrow \Box B) &\rightarrow \Box (A \rightarrow B) \end{aligned}$$

(3) Construct derivations to show the following. All derived rules and theorems in the book or handouts are fair game:

$$\begin{aligned} \diamond(A \rightarrow \Box \Box B) &\vdash_{\text{T}} \Box A \rightarrow \diamond \Box B \\ \Box \Box A \& \diamond \Box B &\vdash_{\text{S4}} \Box \Box (\Box \Box A \& \Box \Box B) \\ \vdash_{\text{B}} \diamond(\Box A \rightarrow \Box B) &\rightarrow \Box (A \rightarrow B) \end{aligned}$$

(4) Construct derivations to show the following. All derived rules and theorems in the book or handouts are fair game:

$$\begin{aligned} \Box \Box A \rightarrow \diamond \Box A &\vdash_{\text{S5}} A \leftrightarrow \Box A \\ \Box(A \leftrightarrow \Box B) &\vdash_{\text{S5}} \Box A \vee \Box \sim A \\ \vdash_{\text{S5}} (A \rightarrow \Box B) &\vee (B \rightarrow \Box A) \end{aligned}$$

Part B — Trees

Choose *any two* of the following three questions.

(5) Construct semantic trees to test the K-validity of the following statements or arguments; for those that are invalid, describe a counterexample model. (You need not prove that it is a counterexample.)

$$\begin{aligned} \Box \diamond p, \diamond \Box(p \rightarrow q) &\vDash_{\text{K}} \diamond \Box q \text{ ??} \\ \vDash_{\text{K}} \diamond(p \& \diamond q) \rightarrow (\Box p \& \Box q) \text{ ??} \\ \diamond(p \rightarrow p) &\vDash_{\text{K}} \sim \Box(\Box p \& \Box \sim p) \text{ ??} \\ \diamond p \leftrightarrow \Box q &\vDash_{\text{K}} \diamond(p \leftrightarrow \Box q) \text{ ??} \\ \Box \diamond p \& \Box \Box q &\vDash_{\text{K}} \Box \Box(p \& q) \text{ ??} \\ \Box(\Box p \vee q) &\vDash_{\text{K}} \Box(\Box \sim p \rightarrow q) \& \Box \Box(p \vee q) \text{ ??} \end{aligned}$$

(6) Construct semantic trees to test for the indicated sort of validity for the following statements or arguments; for those that are invalid, describe a counterexample model. (You need not prove that it is a counterexample.)

$$\begin{aligned} \vDash_{\text{M}} \diamond(\Box p \& \sim q) \vee \Box(p \rightarrow \Box q) \text{ ??} \\ \diamond \Box p, \diamond \Box q &\vDash_{\text{KB}} \Box \Box(p \& q) \text{ ??} \\ \vDash_{\text{K4}} \Box q \vee \Box(\Box(p \rightarrow q) \rightarrow \sim p) \text{ ??} \\ \vDash_{\text{S4}} \Box(\Box p \rightarrow \Box q) \vee \Box(\Box q \rightarrow \Box p) \text{ ??} \\ \diamond(p \rightarrow \Box q) &\vDash_{\text{K5}} \Box p \rightarrow \Box q \text{ ??} \\ \Box(\Box p \rightarrow p) &\vDash_{\text{S5}} \Box(\Box \sim p \rightarrow \sim p) \text{ ??} \end{aligned}$$

(7) Construct semantic trees to show that the following arguments or statements have the sort of validity indicated, and *then* convert the tree into a derivation in the associated deductive system. The derivation must match the tree, though you may rewrite the tree skipping tree rules that were not needed to close it before conversion.

$$\begin{aligned} \vDash_{\text{B}} \Box \Box \Box p \rightarrow \Box p \\ \Box \Box p \& \Box \Box q &\vDash_{\text{K4}} \Box \Box (\Box \Box p \& \Box \Box q) \\ \diamond \Box p, \diamond q &\vDash_{\text{S5}} \Box(p \& \Box q) \end{aligned}$$

Part C — Metatheory

Choose *any one* of the following three questions.

(8) *In your own words,** explain the proof that if the S5-tree for $A_1, A_2, \dots, A_n, \sim B$ closes, then $A_1, A_2, \dots, A_n \vDash_{\text{S5}} B$, and illustrate with your own example.

(9) *In your own words,** explain the proof that if $A_1, A_2, \dots, A_n \vDash_{\text{S5}} B$ then $A_1, A_2, \dots, A_n \vDash_{\text{S5}} B$, and illustrate with your own example.

(10) *In your own words,** explain the proof that if $A_1, A_2, \dots, A_n \vDash_{\text{S5}} B$ then the S5-tree for $A_1, A_2, \dots, A_n, \sim B$ closes, and illustrate with your own example.

* I realize that this may be a strange sounding request, and a difficult one to follow, but the important thing is that you do not copy the proof either directly from the book or directly from a handout. Of course, you can consult these things, but you should “word”, “structure” and “explain” the proof in your own way. The better job you do convincing me that you understand the proof on your own terms, the better your grade will be.

This is not a group exercise. You are expected to work on your own, though you may ask me for help.