11 Descriptions

11.1 Russell's Theory of Descriptions

On an earlier handout, we saw that Russell used the notation $(\imath x)Ax$ to mean "the x such that Ax". However, he did not take expressions of this form as genuine terms to which you could instantiate quantifiers, and so on. Nor did he give a direct definition of these expressions outright. Instead, he gave a contextual definition that provides a means for defining any sentence in which they occur. In particular:

$$B((\imath x)Ax) \;\; \textit{abbreviates} \;\; \exists x(\forall y(Ay \leftrightarrow y = x) \;\&\; Bx)$$

The parentheses around "$(\imath x)$" are unnecessary, just as it is unnecessary to write, as some books do, "$(\forall x)$" or "$(\exists x)$" instead of "$\forall x$" or "$\exists x$". We may write instead:

$$B(\imath x\, Ax) \;\; \textit{abbreviates} \;\; \exists x(\forall y(Ay \leftrightarrow y = x) \;\&\; Bx)$$

Or, equivalently:

$$\ldots \; \textit{abbreviates} \; \exists x(Ax \;\&\; \forall y(Ay \rightarrow y = x) \;\&\; Bx)$$

(We here assume that y is the first variable alphabetically not occurring in $B(\imath x\, Ax)$.) Garson uses the notation "$!x\, Ax$" instead of "$\imath x\, Ax$", but this is too weird and idiosyncratic to follow along with.

The component of Russell's definition:

$$Ax \;\&\; \forall y(Ay \rightarrow y = x)$$

can be read as saying that x and x alone is A. Garson likes to abbreviate this as $1Ax$ for "x is the one and only A". Indeed, he gives this definition:

$$1At \;\; \textit{abbreviates} \;\; At \;\&\; \forall y(Ay \rightarrow y = t)$$

In that case, Russell's abbreviation can be stated:

$$B(\imath x\, Ax) \;\; \textit{abbreviates} \;\; \exists x(1Ax \;\&\; Bx)$$

Garson's definition of $1At$, however, is not rigorous. Consider, for example, the expression 1Rab. This could mean either:

$$\mathrm{Rab} \;\&\; \forall \mathrm{x}(\mathrm{Rxb} \rightarrow \mathrm{x} = \mathrm{a})$$

Or:

$$\mathrm{Rab} \;\&\; \forall \mathrm{x}(\mathrm{Rax} \rightarrow \mathrm{x} = \mathrm{b})$$

Garson ignores this difficulty. It is usually obvious when used in the context $\exists x(1Ax \;\&\; Bx)$ that one means to say that x alone is A; however, I shall usually avoid this notation wherever possible.

Speaking of ambiguities, one which Russell himself stressed is that his definition can be applied in different ways when B is not a simple predicate. Consider, "the King of France is not bald," written:

$$\sim\mathrm{B}(\imath \mathrm{x}\, \mathrm{Kx})$$

One can either interpret the $B(\ldots)$ (italicized B) in the definition above as the whole of "$\sim$B(...)" here, giving us:

$$\exists \mathrm{x}(\mathrm{Kx} \;\&\; \forall \mathrm{y}(\mathrm{Ky} \rightarrow \mathrm{y} = \mathrm{x}) \;\&\; \sim\mathrm{Bx})$$

When the contextual definition for the description is applied to the entire statement in which it occurs, Russell says the description has *primary occurrence*. The above can be read as saying that there is one and only one King of France, which is not bald, which is false.

However, it is also possible to interpret the $B(\ldots)$ of the definition as just B(...) in $\sim\mathrm{B}(\imath \mathrm{x}\, \mathrm{Kx})$, and then take the "$\sim$" to apply to the result:

$$\sim \exists \mathrm{x}(\mathrm{Kx} \;\&\; \forall \mathrm{y}(\mathrm{Ky} \rightarrow \mathrm{y} = \mathrm{x}) \;\&\; \mathrm{Bx})$$

Here, the description is taken to have a *secondary occurrence*. This can be read as saying that it's not true that there is one and only one King of France who is bald, which is true. If the statement in which a description occurs is even more complex, there may be three, four or many more different readings of the sentence depending on the ways the contextual definition may be applied.

At times, Russell used scope markers to disambiguate. The scope markers were simply the description itself, placed within [...], placed right before the beginning of the context to which the contextual definition would be applied. Hence, to get the primary occurrence reading, we write:

$$[\imath \mathrm{x}\, \mathrm{Kx}]\sim\mathrm{B}(\imath \mathrm{x}\, \mathrm{Kx})$$

And for the secondary reading:

$$\sim[\imath \mathrm{x}\, \mathrm{Kx}]\mathrm{B}(\imath \mathrm{x}\, \mathrm{Kx})$$

These scope ambiguities, while annoying, are sometimes seen as a strength of Russell's analysis, since in many cases it does seem as if ordinary language is similarly ambiguous. We shall see (indeed, we have already seen to some extent) that this is definitely true when modal operators are involved.

11.2 Motivations for Russell's Theory

In his famous 1905 paper "On Denoting," Russell used certain puzzles to attempt to motivate his theory of descriptions. They are these:

Contingency of Existence Claims

Russell accepted classical quantifier rules. As we have seen, on such a theory"

$$\mathrm{E}c$$

is a theorem of pure logic for every genuine (constant) name c. This seems to rule out existence claims being interesting, logically contingent or indeed, sometimes false. On Russell's theory, however, a claim of the form:

$$\mathrm{E}(\imath x Ax)$$

interpreted as:

$$\exists x(Ax \mathbin{\&} \forall y(Ay \rightarrow y = x) \mathbin{\&} \mathrm{E}x)$$

(which, bearing in the mind the definition of E, is equivalent to simply:)

$$\exists x(Ax \mathbin{\&} \forall y(Ay \rightarrow y = x))$$

is by no means a logical truth (nor a logical falsity except for certain exceptional cases of Ax—such as $x \neq x$).

Meaningfulness of Sentences with Empty Descriptions

The sentence "The King of France is bald" is not nonsense; it has a meaning. Russell thinks it is "clearly false". According to Russell's analysis it does come out as false.

Other theories of descriptions may not yield this result, or might give the statement the wrong meaning. (For Frege, the ordinary language sentence, "the King of France is bald" is neither true nor false, and the replacement for it in his logical language would be true if the empty class is bald, which seems to give it the wrong truth conditions,)

Of course, the suggestion that sentences with empty descriptions should be seen as having truth values has been questioned (e.g., by Strawson among others). Indeed, the example of "the King of France is bald" is not as plain as Russell suggestions. Many would say that this sentence *presupposes* that there is a King of France; it doesn't actually assert it.

But what of "My mother is dating the King of France," or "the King of France did my laundry this morning". While intuitions may still vary, these seem more clearly false to me.

In any case, these sentences must have some meaning, whether we ascribe them truth values or not. They're not gibberish.

Actually, our Free Logic models aren't as clear cut on these issues as you might think. Really, they just push the problem into the metalanguage; while free logic allows models to have varying domains in different worlds, and to give terms semantic values that don't exist at a given world, this only leads us to the question of what the entities in **D** not in **Dw** are supposed to be anyway, on our "intended model" where **w** is the actual world. The bare abstract possibility in our model structure doesn't help explain what those additional things could be. Russell's theory obviates the need for this assumption altogether.

Apparent Exceptions to the Law of Excluded Middle

Russell points out that if you took a tally of the things that are bald, and another tally of the things that are not bald, you wouldn't find the King of France on either list. A naive conclusion would be that neither "the King of France is bald" nor "the King of France is not bald" would then be true. "Hegelians, who love a synthesis, will probably conclude that he wears a wig," Russell quipped.

On Russell's view, both can be false if the description is interpreted with primary scope:

$$\exists \mathrm{x}(\mathrm{Kx} \mathbin{\&} \forall \mathrm{y}(\mathrm{Ky} \rightarrow \mathrm{y} = \mathrm{x}) \mathbin{\&} \mathrm{Bx})$$

$$\exists \mathrm{x}(\mathrm{Kx} \mathbin{\&} \forall \mathrm{y}(\mathrm{Ky} \rightarrow \mathrm{y} = \mathrm{x}) \mathbin{\&} \sim\mathrm{Bx})$$

But by the same token, these secondary-scope statements are true, and these are the true negations of the above:

$$\sim \exists \mathrm{x}(\mathrm{Kx} \mathbin{\&} \forall \mathrm{y}(\mathrm{Ky} \rightarrow \mathrm{y} = \mathrm{x}) \mathbin{\&} \mathrm{Bx})$$

$$\sim \exists \mathrm{x}(\mathrm{Kx} \mathbin{\&} \forall \mathrm{y}(\mathrm{Ky} \rightarrow \mathrm{y} = \mathrm{x}) \mathbin{\&} \sim\mathrm{Bx})$$

Hence, there is no violation of the law of excluded middle.

Violations of Law of Contradiction

Naively, one might be prone to think that "the round square is round" is true, and so is "the round square is square" (and in virtue of the latter that "the round square is not round" must be true as well). However, when interpreted à la Russell, as:

$$\exists x((Rx \,\&\, Sx) \,\&\, \forall y((Ry \,\&\, Sy) \rightarrow y = x) \,\&\, Rx)$$

$$\exists x((Rx \,\&\, Sx) \,\&\, \forall y((Ry \,\&\, Sy) \rightarrow y = x) \,\&\, Sx)$$

Both turn out false. It's neither true that the round square is round, nor that the round square is square (not round).

Belief Puzzles

Insisting that it would be absurd to accuse the first Gentleman of Europe of having "an interest in the law of identity," Rusell insisted that it was wrong to conclude that George IV wished to know whether Scott was Scott from the premises that George wished to know whehter Scott was the author of Waverly and that Scott is the author of Waverly.

Here Russell appealed to the scope differentiations in his theory to mark the difference between (here using **K** as an intensional operator for George wishing to know whether ...):

$$\mathbf{K}\, \exists x(Ax \,\&\, \forall y(Ay \rightarrow y = x) \,\&\, s = x)$$

and:

$$\exists x(Ax \,\&\, \forall y(Ay \rightarrow y = x) \,\&\, \mathbf{K}s = x)$$

The former is clearly what we mean, but the result that George wished to know whether Scott was Scott follows only from the latter along with the premise that Scott is the author of Waverly. This solution, moreover, does not require us to put any restrictions on the substitutivity of coreferring terms in belief contexts, etc., since the descriptions are not genuine terms.

The first of the above is called a *de dicto* claim, since the intensional operator is applied to the entire statement. It says something about a complete thought: that George wondered about it. The second of the above is called *de re* claim since it seems tomake a claim about the thing itself which is the author of Waverly: that George wanted to know whether *it* (or *he*) is Scott.

Resolving Scope Ambiguities

Scope ambiguities, or statements involving modal operators capable of being given both *de re* and *de dicto* readings, are especially important for modal logic (though Russell himself doesn't discuss such cases). As we've already discussed, "the number of planets is necessarily greater than five" may be read as either:

$$\exists x(Nx \,\&\, \forall y(Ny \rightarrow y = x) \,\&\, \Box\, x > 5)$$

or as:

$$\Box\, \exists x(Nx \,\&\, \forall y(Ny \rightarrow y = x) \,\&\, x > 5)$$

By most peoples' intuitions, it would seem that the former is true, the latter is false.

Nor can it be thought that these ambiguities are entirely a product of Russell's theory. There are intelligible sentences of ordinary modal discourse than can only be made sense of by positing some distinction like this.

Consider, "the inventor of bifocals might not have invented anything."

There is a reading of this that makes it true, i.e., since it is not true of Ben Franklin, *him*, that *he* necessarily invented anything. The *de dicto* reading, that it is possible for there to be someone who invented bifocals without invented anything, seems difficult to swallow, but the de reading remains intact. Russell's theory allows us to differentiate:

$$\exists x(Ixb \,\&\, \forall y(Iyb \rightarrow y = x) \,\&\, \Diamond \sim \exists z\, Ixz)$$

from:

$$\Diamond\, \exists x(Ixb \,\&\, \forall y(Iyb \rightarrow y = x) \,\&\, \sim \exists z\, Ixz)$$

Russell's theory leaves us free to say that the former is true while the latter is false.

We can invent similar statements in temporal logics. Take, e.g., "the president won't always be the president". This makes the most sense if we interpret it as saying of the person who is the president that *he* (*de re*) won't be president at some future time. Unless we were skeptical of the prospects of our form of government, there would be little reason to think (*de dicto*) that at some point the person who is president then won't be president then. Or in other words, the first of these seems right, the second seems wrong:

$$\exists x(Px \,\&\, \forall y(Py \rightarrow y = x) \,\&\, \mathbf{F}{\sim}\, \exists y(Py \,\&\, \forall z(Pz \rightarrow z = y) \,\&\, x = y))$$

$$\mathbf{F}{\sim}\, \exists x(Px \,\&\, \forall y(Py \rightarrow y = x) \,\&\, \exists y(Py \,\&\, \forall z(Pz \rightarrow z = y) \,\&\, x = y))$$

Even if we don't accept Russell's theory, we must somehow distinguish *de re* and *de dicto* readings of statements with modal and other intensional operators. Of the nice features of Russell's theory is that it automatically accomodates such differences.

11.3 Problems with Russell's Theory

Despite its many advantages, problems have been seen with Russell's theory.

Perhaps the largest, for our purposes, is that the existence and uniqueness clauses are thought too strong to be part of the actual content expressed by a sentence with a description.

- This makes the theory impossible to apply in cases where there is more than thing x such that Ax; yet, in ordinary language we often speak of "the table" or "the book" without actually asserting that these are the only tables or books there are.
- It makes every sentence containing a (wide scope or primary occurrence) description that is not satisfied false, including such statements as "the crisis was averted" or "the omega particle was never found," to use two of Garson's examples.
- Possibilist readings of quantification don't help out too much, since it would seem to entail that there is exactly one possible crisis that must have been averted.
- Even in the case of "the King of France is bald," many would say it is a presupposition that there is one and only King of France, and if there isn't, the sentence just isn't true or false.

Another set of problems involve the very puzzles Russell used to motivate his theory. In many cases, it seems that the problems apply even in sentences where no descriptions are involved, making it seem as if the theory of descriptions' solution doesn't get to the heart of the matter. For example:

- Existence claims can seem to be contingent and interesting even when formulated not using descriptions (e.g., "Romulus existed," or "Vulcan does not exist", etc.)
- We are left with the problem of explaining how it is that statements with meaningful names continues to be meaningful, with or without the theory of descriptions.
- Belief puzzles can arise with names alone too (e.g., wishing to know whether Twain is Clemens, or Hesperus is Phosphorus).
- The *de re*/*de dicto* distinction can seem to apply even when no direct descriptions are involved; i.e., someone might believe, of Twain, that he smokes, without believing that Twain smokes, etc.
- It is probably for reasons such as the above that Russell came to think that ordinary proper names should be treated as "disguised definite descriptions"; a full assessment of this move on Russell's part will have to be left for another context.
- Even then, there are other belief-like contexts that need to be treated, but Russell's theory is inapplicable since there is no complete sentence involved. (E.g., "the ancient civilization worshipped the sun god." Here we can't interpret this as meaning that one and only one sun god was worshipped, at least not if we want it to turn out true. But there is no other sentential context in which to unpack the contextual definition of the description.

There have also been criticisms of Russell's theory for more straightforward philosophy of language concerns that cannot be fully discussed here:

- It does not give the right truth conditions for so called "referential" uses of definitive descriptions (e.g., Donnellan's famous "the man drinking the martini . . . " example.)
- It has trouble dealing with anaphora. ("The author of Waverly was intelligent. *He* was also very secretive . . . ")
- And so on.

In any case, Garson takes the number of problems as sufficient motivation for introducing another means for dealing with descriptions. It should be noted that Free Logic itself is often thought to provide its own rival solutions to many of the difficulties Russell discussed. (I have my doubts.) In any case, it does seem to remove most of the roadblocks to taking descriptions as genuine terms.

11.4 Syntax for Modal Description Theory

Here, we introduce notation for descriptions as primitive, superficially similar to Russell's notation, but without the contextual definition.

Here, sentences and terms can both occur within one another, and so must be defined together in a single recursive specification: