Unstable particles: characterized by mass and lifetime.

\[ \Gamma = \frac{\hbar}{\tau} \quad W \propto \Gamma \]

Heirarchy of Interactions:

- Decay by the strong force: \(10^{-23} \text{ s} \implies \Gamma \approx 100 \text{ MeV}^{-1}\)
- Weak force: \(10^{-8} \text{ s}\)
- Electromagnetic: \(10^{-16} \text{ s}\)

Density of States or Phase Space causes variation:
- Free neutron: \(10 \text{ s}\)
- Nuclear isotopes: \(1000 \text{ s}\) to years
- Free muon: \(10^{-6} \text{ s}\)

All are weak decays!

By studying the lifetime of various allowed decay modes:
- Learn about forces & interactions.

* If a decay mode is allowed, it will happen.
  \[ \mu \rightarrow e^{-} \nu_{e} \mu \text{ (allowed)} \]
  \[ \mu \rightarrow e^{-} \gamma \text{ (not observed)} \]
LECTURE VI

Symmetries

Classical physics: Consider an odd function. We know a few things:

\[ \int_{-3}^{3} f(x) \, dx = 0 \quad \text{and} \quad \int_{-7}^{0} f(x) \, dx - \int_{0}^{7} f(x) \, dx \]

No cosines in Fourier series, odd powers in Taylor series \(\Rightarrow\) symmetry allowed for a lot of guesses

In physics, intuition can suggest symmetries \(\Rightarrow\) exploit symmetries to understand underlying physics

Example: Space is homogeneous \(\Rightarrow\)

\( L \) invariant under translation

\[ L(\mathbf{r}_i, \mathbf{v}_i, t) \rightarrow \mathbf{r}_i \rightarrow \mathbf{r}_i + \mathbf{a} \]

\[ \delta L = \sum_i \left( \frac{\partial L}{\partial \mathbf{r}_i} \right) \delta \mathbf{r}_i = \mathbf{a} \cdot \sum_i \frac{\partial L}{\partial \mathbf{r}_i} = 0 \]

\[ \Rightarrow \sum_i \frac{\partial L}{\partial \mathbf{r}_i} = 0 \Rightarrow \frac{\partial (\sum_i \mathbf{v}_i)}{\partial t} = 0 \Rightarrow \frac{d\mathbf{v}_i}{dt} = 0 \]

\[ \Rightarrow \quad \frac{d}{dt} \left( \sum_i \frac{\partial L}{\partial \mathbf{r}_i} \right) \frac{\partial L}{\partial \mathbf{r}_i} = 0 \quad (E-1 \text{ Eqn}) \]

\[ \Rightarrow \quad p_i = \text{constant} \]
LECTURE VI

Translational invariance \(\iff\) momentum conservation
Time invariance \(\iff\) energy conservation
Rotational invariance \(\iff\) angular momentum

Noether's Theorem (Sweeping implications for dynamics)
If E-L Eqn is invariant under any coordinate transformation, if an integral of motion
\(\Rightarrow\) conserved quantity or observable

Not just space-time symmetries, also internal symmetries i.e. invariance of Hamiltonian or Lagrangian

Example: \(|\psi\rangle \rightarrow \gamma_{\chi}(\vec{r},t)\); if \(\vec{r} \rightarrow \vec{r} + \vec{r}'\)
then \(\gamma_{\chi}(\vec{r},t) \rightarrow U\gamma_{\chi}(\vec{r},t)\)
with \(U = e^{-(\vec{r} \cdot \vec{r}'/\hbar)}\) \(\Rightarrow\) \(U = e^{-(\vec{r} \cdot \vec{r}'/\hbar)}\)

Invariance under Schrödinger Eqn: \(i\hbar \frac{\partial \psi}{\partial t} = H\psi\)
\(\Rightarrow\) \([H,\psi] = 0\)
For translations: \([H,\vec{p}] = 0\)
In general: \([H,F] = 0\)
Which implies invariance under \(e^{i\xi F}\)
LECTURE VI

Example: Electric charge if \( [\mathcal{H}, \mathcal{E}] = 0 \Rightarrow \frac{d\langle \mathcal{E} \rangle}{dt} = 0 \)

\[ \mathcal{E} \psi = \mathcal{E} \psi \] (\( \psi \) is a simultaneous eigenfunction of \( \mathcal{H} \) and \( \mathcal{E} \))

This implies that \( \mathcal{E}' = e^{i\mathcal{E}t} \mathcal{E} \) leaves \( \mathcal{H} \) invariant

\( \Rightarrow \) a "global" gauge transformation

i.e. \( \mathcal{E} \) is a constant.

Profound Implications: Inspect Hamiltonian, predict \( \leq q_i = \leq q_f \)

Since charge is quantized, define charge \# \( q = \text{Ne} \)

If \( a + b \rightarrow c + d + e \)

\( Na + Nb = Ne + Nd + Ne \)

(Additive conservation law)

\( \Rightarrow \) e.g. \( e \rightarrow \nu \bar{\nu} \) is forbidden.

Baryon \# \( A = 1: \) \( P, n, \Lambda \leq^+ \begin{array}{c} -2 \end{array} \)

\( A = -1 \) for antiparticles

\( P \rightarrow e^+ \pi^0 \) (Violates Baryon #)

Hence proton is stable (lucky us!)

Engin \& baryon \# conservation unknown.
$\bar{\Lambda}^0 \rightarrow n \pi^0$
$A=1$  
$A=1$

$\Sigma^+ \rightarrow p \pi^0$
$A=1$  
$A=1$

$\Sigma^- \rightarrow n\pi^-$
$A=1$  
$A=1$

$\pi^- p \rightarrow \Lambda^0 + K^0$
$A=0$  
$A=1$  
$A=1$  
$A=0$

*Some amount of baryon number violation needed to explain preponderance of matter in the universe.*

Lepton number $\gamma^* \rightarrow e^+ e^-$ or $p\bar{p}$ but not $e^+ p$

$N \rightarrow p \ e^- \ \bar{\nu}_e$
$A=1$  
$A=1$
$L=0$  
$L=0$  
$L=1$  
$L=-1$

$\bar{\nu}_e$ observed for the first time in a reactor experiment

$L=1 \rightarrow e^+ n$

Not seen:

$\bar{\nu}_e n \rightarrow e^- p$
$L=-1 \rightarrow +1$

$\nu_e p \rightarrow e^+ n$
$L=-1 \rightarrow +1$

Seen:

$\nu_e n \rightarrow e^- p$
$L=+1 \rightarrow +1$
LECTURE VI

Neutrinos & anti-neutrinos have opposite lepton #
(Defer discussion of massive neutrinos)

We are obliged to introduce several lepton #s. Why?

Consider $\mu^- \to e^- \nu \bar{\nu}$

If there was only one lepton number, then $\mu^- \to e^- \gamma$ would be allowed.

NOT OBSERVED.

$\Rightarrow$ Assign $L_\mu(\mu^-) = 1$, $L_\mu(\mu^+) = -1$, $L_\mu(e^-) = 0$ etc.

$\mu^- \to e^- \nu e \bar{\nu}$

Both $L_e$ & $L_\mu$ are independently conserved.

$\Rightarrow 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0$

$\Rightarrow \pi^- \to \mu^- \bar{\nu}_\mu \quad \pi^+ \to \mu^+ \nu_\mu$

$\mu^- \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0$

$\Rightarrow \nu_e$ & $\nu_\mu$ are distinct particles. How do you prove it? Make $\nu$ beam from $\pi$-decay.

If $\pi^-$ used, then $\bar{\nu}_\mu$ beam, if $\pi^+$ used, then $\nu_\mu$ beam.

$\Rightarrow \bar{\nu}_\mu P \to \mu^+ n \quad \nu_\mu n \to \mu^- p$

OBSERVED!

$\bar{\nu}_\mu P \to e^+ n \quad \nu_\mu n \to e^- p$

NOT OBSERVED!

1962 Nobel Prize
LECTURE VI

Symmetry Operation: leaves system invariant.
Set of all operations with Closure, Identity, Inverse and Associativity (Note: Commutativity not required)
Finite group $\Rightarrow$ discrete symmetry
Infinite group $\Rightarrow$ continuous symmetry.

In physics, groups can be represented by matrices.
E.g. Lorentz transformations, rotations.

$SO(n)$: Group of $n$-D rotations,
$SO(3) \subseteq SU(2)$

$SU(2)$ is an important group in particle physics.
Invariance under $SU(2)$: Angular momentum conservation.
$\Rightarrow$ Next week, introduce formalism.

**Summary on Lepton Number**

$Le = +1: \nu_e, e^-$
$Le = -1: \bar{\nu}_e, e^+$
$L\mu = +1: \nu_\mu, \mu^-$
$L\mu = -1: \bar{\nu}_\mu, \mu^+$
$L\tau = +1: \nu_\tau, \tau^-$
$L\tau = -1: \bar{\nu}_\tau, \tau^+$

Antiparticles in general have mass and spin that are the same but additive quantum numbers that are the opposite sign.
LECTURE VII

Review of Angular Momentum Formalism:

Spin-1/2 system

\[ |\frac{1}{2}, \frac{1}{2} \rangle \equiv |0 \rangle; |\frac{1}{2}, -\frac{1}{2} \rangle \equiv |1 \rangle \]

General state

\[ \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \alpha |0 \rangle + \beta |1 \rangle \]

\[ |\alpha|^2 = \text{probability that } S_z \text{ yields value } \pm \hbar/2 \]

\[ |\alpha|^2 + |\beta|^2 = 1 \]

To measure probability of measuring \( \pm \hbar/2 \) along \( S_x \) or \( S_y \) construct operators in matrix form,

\[ \hat{S}_x = \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \]

\[ \hat{S}_y = \frac{\hbar}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \]

\[ \hat{S}_z = \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \]

\[ \sigma_i = \frac{\hbar}{2} \sigma_i \quad \sigma_i \text{ Pauli spin matrices} \]

\[ \chi_{\pm} \equiv \text{eigenvectors of } \hat{S}_x \equiv \begin{pmatrix} \hbar/2 \\ \pm \hbar/2 \end{pmatrix} \]

\[ \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = a \begin{pmatrix} \hbar/2 \\ \hbar/2 \end{pmatrix} + b \begin{pmatrix} \hbar/2 \\ -\hbar/2 \end{pmatrix} \]

\[ |a|^2 \text{ is the probability of finding } \pm \hbar/2 \text{ along } S_x \]

\[ |a|^2 + |b|^2 = 1 \]

Algorithm:
- Find matrix for observable
- Find eigenvalues & eigenvectors
- Probability given by coefficients
LECTURE VII

Spin-1/2 formalism occurs often in particle physics.
Example: Free fermions, iso-spin, rotations in spinor space (2-D space)

\[ U(\theta) = e^{-i(\theta \cdot \sigma)/2} \]

\[ \left( \begin{array}{c} \alpha' \\ \beta' \end{array} \right) = U(\theta) \left( \begin{array}{c} \alpha \\ \beta \end{array} \right) \]

Similar to \( \left( \begin{array}{c} x' \\ y' \end{array} \right) = R(\theta) \left( \begin{array}{c} x \\ y \end{array} \right) \) rotations of 3-vector \( \mathbf{R} \)

\[ U(\theta) : \text{2-D representation of SU(2)} \]
\[ R(\theta) : \text{3-D representation of SU(2)} \iff SO(3) \]

Addition of Angular Momenta

\[ \vec{L} \quad \text{or} \quad \vec{S} : \quad L^2 \Rightarrow l(l+1) \hbar^2 \]

\[ L_z \quad \text{or} \quad S_z : \quad L_z \Rightarrow m_l \quad (-l \leq m_l \leq l) \]

Particles are in a state of definite \( L \) or \( S \)

\( L \) can be anything but \( S \) is a ground state property

Examples: \( \pi^0 \) has \( S = 0 \), electron has \( S = 1/2 \)

Example: Electron in hydrogen atom:

\[ |l \text{ } m_l \text{ } s \text{ } m_s \rangle \quad l = -1 \quad s = 1/2 \quad F \text{ state.} \]

Often we need to know \( \vec{J} = \vec{L} + \vec{S} \)

What \( J \) states are allowed and with what probability?
LECTURE VII

Generally
\[ \vec{J} = \vec{J}_1 + \vec{J}_2 \quad |j_1 - j_2| < j < |j_1 + j_2| \]
\[ m = m_1 + m_2. \]

Probabilities can be looked up in table of Clebsch-Gordan coefficients.

\[ |j_1 m_1 \rangle |j_2 m_2 \rangle = \sum_{j = |j_1 \pm j_2|}^{j_1 + j_2} C_{m_1 m_2}^{j_1 j_2} |j m \rangle \]

Suppose electron is in \[ |1/2, -1/2 \rangle \]
\[ l \text{ m s ms}. \]

Possible values of \( j \) are \( 3/2 \) and \( 5/2 \).

\[ m = m_1 + m_2 = -1 + 1/2 = -1/2 \]

Look at the \( 2 \times 1/2 \) table:

<table>
<thead>
<tr>
<th></th>
<th>[m_1]</th>
<th>[m_2]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/2</td>
<td>5/2</td>
<td>3/2</td>
</tr>
<tr>
<td>-1/2</td>
<td>-1/2</td>
<td>3/2</td>
</tr>
<tr>
<td>0</td>
<td>-1/2</td>
<td>3/2</td>
</tr>
<tr>
<td>( -1/2 \pm 1/2 )</td>
<td>2/2</td>
<td>-3/2</td>
</tr>
</tbody>
</table>

Example: A \( q \bar{q} \) meson has \( l = 0 \). What \( s \) values are allowed? \( 1/2 + 1/2 = 1 \) or \( 1/2 - 1/2 = 0 \)

\[ \Rightarrow \text{Spin 1 or Spin 0}. \]

Spin 1 mesons: \( S^+, S^-, S^0, K^*, \phi, \omega \) Vectors

Spin 0 mesons: \( \Pi^+, \Pi^-, \Pi^0, K_S, \eta, \eta' \) Scalars
LECTURE VII

Barbons: 3 quark states: If $l = 0$ then
\( \frac{1}{2} + \frac{1}{2} \) and \( \frac{1}{2} - \frac{1}{2} \) combine with \( \frac{1}{2} \) to give \( +\frac{1}{2} \) or \( +\frac{3}{2} \)

If mesons have non-zero $l$, then $l+1$, $l$ and $l-1$ are possible.
If barbons have non-zero $l$, then $l-\frac{3}{2}$, $l-\frac{1}{2}$, $l+\frac{1}{2}$, and $l+\frac{3}{2}$

Concrete Example of Formalism: ISOSPIN

After neutron discovery (1932), Heisenberg noted:

\[ M_n = M_p \] if electromagnetism is ignored.

then $n$ and $p$ are identical, most of nucleon mass comes from strong force binding.

Propose: $P = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $n = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ are two states of the same particle.

General state: $N = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \alpha P + \beta n$

Introduce new quantum number $I$ or $I$spin (analogous to spin angular momentum)

What is the physics? That strong interaction Hamiltonian is invariant under isospin rotations.

(proof: SU(2) I)

Protons & neutrons are in $\mathbf{2} = D$ representation of isospin rotations: $SU(2) I$
LECTURE VII

3-D representation: triplet
\[ \begin{align*}
\Pi^+ & \rightarrow |1,1\rangle \\
\Pi^- & \rightarrow |1,0\rangle \\
\Pi^0 & \rightarrow |1,-1\rangle \\
\end{align*} \]

Quadruplet
\[ \begin{align*}
\Delta^{++} & \rightarrow |\frac{3}{2},\frac{3}{2}\rangle \\
\Delta^+ & \rightarrow |\frac{3}{2},\frac{1}{2}\rangle \\
\Delta^0 & \rightarrow |\frac{3}{2},\frac{1}{2}\rangle \\
\Delta^- & \rightarrow |\frac{3}{2},-\frac{3}{2}\rangle \\
\end{align*} \]

goes beyond classification:

Example: combine two nucleons: P for n
Can form state of total \( I = 1 \) (triplet) or 0 (singlet)

\[ \begin{align*}
T \left\{ \begin{array}{l}
|1,1\rangle = \frac{1}{\sqrt{2}} |\frac{1}{2},\frac{1}{2}\rangle |\frac{1}{2},\frac{1}{2}\rangle = PP \\
|1,0\rangle = \frac{1}{\sqrt{2}} \left[ |\frac{1}{2},\frac{1}{2}\rangle |\frac{1}{2},-\frac{1}{2}\rangle + |\frac{1}{2},-\frac{1}{2}\rangle |\frac{1}{2},\frac{1}{2}\rangle \right] = \frac{1}{\sqrt{2}} (np+pn) \\
|1,-1\rangle = \frac{1}{\sqrt{2}} |\frac{1}{2},-\frac{1}{2}\rangle |\frac{1}{2},-\frac{1}{2}\rangle = nn
\end{array} \right. \\
\end{align*} \]

Singlet
\[ \begin{align*}
|0,0\rangle = & \frac{1}{\sqrt{2}} \left[ |\frac{1}{2},\frac{1}{2}\rangle |\frac{1}{2},-\frac{1}{2}\rangle - |\frac{1}{2},-\frac{1}{2}\rangle |\frac{1}{2},\frac{1}{2}\rangle \right] \\
= & \frac{1}{\sqrt{2}} (pn-np)
\end{align*} \]

In nature, only one bound 2-nucleon state:
Deuteron \( \Rightarrow \) Singlet \( (I=0) \)

Speculation: Hint = \( k \vec{I}_1 \cdot \vec{I}_2 \) since
\[ \langle \vec{I}_1 \cdot \vec{I}_2 \rangle = +\frac{1}{4} \text{ (triplet)} \text{ or } -\frac{3}{4} \text{ (singlet)} \]
Application to Scattering: Example 1.

\[
\begin{align*}
\frac{1}{\sqrt{2}} [ (1,0) + (0,0) ] \rightarrow & \quad \text{p} + n \\
(1,-1) \rightarrow & \quad n + n
\end{align*}
\]

Matrix element proportional to \( \langle I_f | H_{it} | I_i \rangle = M_{fi} \).

If \( I \) is conserved, then \( M_{fi} \propto \delta_{if} \).

\[
\begin{align*}
\text{I} & \quad \implies \quad \left( \frac{M_{0} | \langle I_{1} | H_{ii} | I_{1} \rangle \rangle |^{2}}{M_{0}^{2} | \langle I_{1} | H_{ii} | I_{1} \rangle \rangle |^{2} + \frac{1}{2} | \langle 0 | H_{ii} | 0 \rangle \rangle |^{2}} \right) ^{2} \\
\text{II} & \quad \implies \quad M_{0}^{2} | \langle I_{1} | H_{ii} | I_{1} \rangle \rangle |^{2} \\
\text{III} & \quad \implies \quad M_{0}^{2} | \langle I_{1} | H_{ii} | I_{1} \rangle \rangle |^{2}
\end{align*}
\]

\[
\Rightarrow \quad \begin{array}{c}
\text{I} : \quad \text{II} : \quad \text{III}
\end{array} \quad \Rightarrow \quad \frac{1}{2} : \quad \frac{1}{2} : \quad 1
\]

Example II: \( \pi p \) scattering.

\[
\begin{align*}
\pi^{+} + p & \quad \longrightarrow \quad \pi^{+} + p \\
(1/2,1/2) & \quad \longrightarrow \quad (1/2,1/2)
\end{align*}
\]

\[
\begin{align*}
| 1/2,1/2 \rangle & \quad = \quad | 3/2,3/2 \rangle \\
(1/2,1/2) & \quad \longrightarrow \quad | 3/2,3/2 \rangle
\end{align*}
\]

\[
\begin{align*}
\pi^{-} + p & \quad \longrightarrow \quad \pi^{-} + p \\
(1/2,1/2) & \quad \longrightarrow \quad (1/2,1/2)
\end{align*}
\]

\[
\begin{align*}
(1/2,1/2) & \quad = \quad \frac{1}{\sqrt{3}} | 1/2,1/2 \rangle - \frac{2}{\sqrt{3}} | 1/2,1/2 \rangle \\
(1/2,1/2) & \quad \longrightarrow \quad \frac{1}{\sqrt{3}} | 1/2,1/2 \rangle - \frac{2}{\sqrt{3}} | 1/2,1/2 \rangle
\end{align*}
\]

\[
\begin{align*}
\pi^{0} + p & \quad \longrightarrow \quad \pi^{0} + n \\
(1/2,1/2) & \quad \longrightarrow \quad (1/2,1/2)
\end{align*}
\]

\[
\begin{align*}
| 1/2,1/2 \rangle & \quad = \quad \sqrt{3/3} | 1/2,1/2 \rangle \\
| 1/2,1/2 \rangle & \quad \longrightarrow \quad \sqrt{3/3} | 1/2,1/2 \rangle + \sqrt{3/3} | 1/2,1/2 \rangle
\end{align*}
\]
LECTURE VII

\[ I \Rightarrow \sqrt{1} \propto |\langle \pi^+ p | H | \pi^+ p \rangle|^2 = |\langle \frac{3}{2} | H | \frac{3}{2} \rangle|^2 \]

\[ II \Rightarrow \sqrt{2} \propto |\langle \pi^- p | H | \pi^- p \rangle|^2 = (\frac{1}{3})^2 |\langle \frac{3}{2} | H | \frac{3}{2} \rangle|^2 + (\frac{2}{3})^2 |\langle \frac{1}{2} | H | \frac{1}{2} \rangle|^2 \]

\[ III \Rightarrow \sqrt{2} \propto |\langle \pi^- p | H | \pi^0 n \rangle|^2 = (\frac{1}{3})^2 |\langle \frac{3}{2} | H | \frac{3}{2} \rangle|^2 + (\frac{2}{3})^2 |\langle \frac{1}{2} | H | \frac{1}{2} \rangle|^2 \]

In \( \pi \pi \) scattering, a large resonance (peak) is observed at invariant mass of about 1.1 GeV. The ratio \( \sigma_I : \sigma_{II} : \sigma_{III} \) is \( 9 : 1 : 2 \). (\( \Delta \) resonance)

We deduce that the resonance has definite I

If \( I = \frac{1}{2} \), then \( |\langle \frac{1}{2} | H | \frac{3}{2} \rangle| = 0 \) \( \Rightarrow \)

\[ \sigma_I : \sigma_{II} : \sigma_{III} \sim 1 : \frac{4}{9} : \frac{2}{9} \text{ or } 9 : 4 : 2. \]

If \( I = \frac{3}{2} \), then \( |\langle \frac{3}{2} | H | \frac{1}{2} \rangle| = 0 \) \( \Rightarrow \)

\[ \sigma_I : \sigma_{II} : \sigma_{III} \text{ is } 1 : \frac{1}{9} : \frac{2}{9} \text{ or } 9 : 1 : 2. \]

We conclude that \( \Delta \) resonance has \( I = \frac{3}{2} \).
LECTURE VIII

There are quantum numbers associated with continuous symmetries & discrete symmetries.

**Discrete Symmetries** C, P, T

**Parity Operation** $P = \text{Inversion} = \text{reflection} + 180^\circ\text{ rotation}

$$
\begin{align*}
  & X \rightarrow -X \\
  & Y \rightarrow -Y \\
  & Z \rightarrow Z
\end{align*}
$$

$P \Psi(\vec{r}) = \Psi(-\vec{r})$

But $\vec{L} \rightarrow -\vec{L}$ because $\vec{L} = \vec{r} \times \vec{P}$

$\vec{r}$ & $\vec{P}$ are (polar) vectors

$\vec{L}$ is an example of an axial or pseudo vector.

Since $P^2 \Psi(x) = \Psi(x)$, the group has 2 elements: $P$ & $I$.

If $[H, P] = 0 \Rightarrow H\Psi = E\Psi$ & $P\Psi = \pm\Psi$, $\pm = \pm 1$

If the Hamiltonian is invariant under parity transformations, the $\pm$ is conserved for $\vec{r}$.

Example: $Y^m(\theta, \phi)$ are parity eigenfunctions with $\pm = (-1)^m$.

In a theory where parity is conserved, vectors & axial vectors do not get added.

E.g. $F = q(\vec{E} + \vec{v} \times \vec{B})$.
LECTURE VIII

Implications of Parity Conservation:

Consider: \( a + b \rightarrow c + d \)

\[ \Pi_{0} = \Pi_{a} \Pi_{b} (-1)^{l} \]  \[ \Pi_{f} = \Pi_{c} \Pi_{d} (-1)^{l'} \]

If \( l \) and \( l' \) reflect relative motion

- One can assign intrinsic parities to particles
- Start with \( \Pi_{p} = \Pi_{n} = +1 \)

Consider \( d \Pi_{-} \rightarrow nn \)

\( \rightarrow nn \bar{\gamma} \) √

\( \rightarrow nn \Pi_{0} \times \)

\( \{ nn \} \) state must be antisymmetric = ½ space ½ spin

spin symmetric

\( \downarrow \)

space antisymmetric

\( \downarrow \)

\( l = 1, 3 \ldots \)

\( d = 0, 2 \ldots \)

\( d: L = 0, S = 1 \Rightarrow \Pi_{d} = \Pi_{p} \Pi_{n} = +1 \)

(Assume \( \Pi_{p} = \Pi_{n} \))

\( d \Pi_{-} \) is known to react in the \( S \)-state.

\( = \) Initial state

\( \Pi_{p} \Pi_{n} \Pi_{-} \)

Total angular momentum = 1 \( \Rightarrow l' = 1 \)

\[ \{ nn \} = \Pi_{n}^{2} (-1)^{l} = -1 \]

\( \Rightarrow \) \[ \Pi_{\Pi}^{-} = -1 \]

One can show \( \Pi_{\Pi_{0}} \) and \( \Pi_{\Pi_{+}} = -1 \)
Particles are assigned $S^\pi$ (spin-parity)

\begin{align*}
\text{P} & \quad 0^- \\
\pi \text{ mesons} & \quad 0^+ \\
\eta \text{ mesons} & \quad 1^- \\
\eta' \text{ mesons} & \quad 1^+ \\
\text{pseudo-scalars} & \\
\text{scalars} & \\
\text{vectors} & \\
\text{pseudo-vectors} & 
\end{align*}

As new particles were found, they were assigned $\pi$:

1956: $\Gamma - \Theta$ puzzle

$\theta^+ \rightarrow \pi^+ + \pi^0$ \quad $\tau^+ \rightarrow \pi^+ + \pi^0 + \pi^0$

\begin{align*}
P &= +1 \\
P &= -1
\end{align*}

Same mass but different parities.

Proposal (Lee & Yang): Particles decayed by weak interaction (narrow width, long lifetime).

What if parity is conserved in strong interactions but not in weak interactions.

C.S. Wu et al. \quad {^{60}}\text{Co} \rightarrow {^{60}}\text{Ni} + e^- + \bar{\nu}_e

$\Delta J = 1$

Orient $^{60}\text{Co}$ via an external B field.

If electron flux not symmetric in $\Theta$, then parity violation.
We now know the $\nu_e$ are left-handed and anti-neutrinos are right-handed.

\[ \uparrow e^- \quad \uparrow \bar{\nu}_e \quad \uparrow \nu_e \quad \downarrow e^- \]

$X$ not possible \quad \text{Observed: electrons mostly left-handed.}

**Charge Conjugation**

$C|P\rangle = |\bar{P}\rangle$

Particle to anti-particle: All quantum numbers flip sign except $M$ & $S$

Only particles that are their own anti-particles are eigenstates of $C$.

$C|\pi^0\rangle = \pm |\pi^0\rangle \quad C|\gamma\rangle = -|\gamma\rangle$

(from Maxwell's equations)

$\pi^0 \rightarrow 2\gamma \Rightarrow C|\pi^0\rangle = +|\pi^0\rangle$

Hence $\pi^0 \rightarrow \gamma \gamma \gamma$ is forbidden.

Other examples: $p + \bar{p} \rightarrow \pi^+ + \pi^- + \pi^0$

The $\pi^+$ & $\pi^-$ distributions must be identical if $C$ is conserved in strong interactions.
LECTURE VIII

Charge Conjugation invariance is NOT obeyed by weak interactions.

\[ \nu_e \rightarrow \pi^+ \rightarrow \mu^+ + \nu_\mu \]
\[ \pi^- \rightarrow \pi^- + \nu_\mu \]
\[ \mu^- \rightarrow \pi^- \]

\[ \nu_\mu \rightarrow \pi^+ \rightarrow \mu^+ + \nu_\mu \]
\[ \pi^- \rightarrow \pi^- + \nu_\mu \]
\[ \mu^- \rightarrow \pi^- \]

Consider \( C \rightarrow \pi^- \rightarrow \pi^+ + \nu_\mu \)
\[ \nu_\mu \rightarrow \pi^+ \rightarrow \mu^+ + \nu_\mu \]
\[ \pi^- \rightarrow \pi^- + \nu_\mu \]
\[ \mu^- \rightarrow \pi^- \]

\( \nu_\mu \rightarrow \pi^+ \rightarrow \mu^+ + \nu_\mu \)
\[ \pi^- \rightarrow \pi^- + \nu_\mu \]
\[ \mu^- \rightarrow \pi^- \]

Not observed

CP is conserved

\( \leftrightarrow \) matter-antimatter

However, small amount of CP violation has been observed in special systems (separate topic)

\[ T \psi(t) = \psi^*(-t) \]

\[ \Rightarrow \] All reactions are reversible if \( T \) is conserved.

\[ CP \Rightarrow T \Rightarrow \text{permanent electric dipole moment non-zero for particles.} \]

\[ \text{CPT invariance means all particles & anti-particles have the same mass & lifetime.} \]

All known Quantum Field Theories require

\[ \text{CPT invariance} \]
LECTURE VIII

Resonance Scattering:

\( \pi^+ + p \) forms an intermediate state of definite mass, width, \( I, L, S, \pi^+ \), i.e., definite quantum \( \text{numbers} \).

Some of the new particles were "strange".

\[
\begin{align*}
\pi^{-} + p & \rightarrow k^{+} + \pi^{-}, \\
\pi^{+} & \rightarrow n + k^{+} \rightarrow n + \pi^{-}
\end{align*}
\]

These particles were very long-lived i.e. very narrowwidth.

\[
\begin{align*}
\pi p & \rightarrow k^{+} \pi^{-}, \\
& \rightarrow \pi^{+} \pi^{-}, \pi^{0} \eta
\end{align*}
\]

'Strange' particles produced in pairs.

\( \pi^+ n p \Rightarrow S=0 \quad \text{and} \quad S=+1 \quad \text{and} \quad S=-1. \)

Propose: Strange particles produced in pairs via the strong force: \( S \) is conserved in strong reaction.

Strange particles must decay by an interaction that does not conserve \( S \): Weak interaction.

\[
\begin{align*}
\pi^{-} + p & \rightarrow \Lambda + k^{0} \\
\Rightarrow & \quad c \\
\Rightarrow & \quad (\pi^{+}) \quad (k^{+}) \quad (\bar{K}^{0}) \quad \text{antiparticles!}
\end{align*}
\]

\[
\begin{array}{c|c|c|c|c|c}
S & 0 & 0 & -1 & +1 & -1 \\
I & 1/2 & 1/2 & 0 & -1/2 & -1/2 \\
I_3 & -1/2 & 0 & 1 & 1/2 & 1/2 \\
\end{array}
\]
LECTURE VIII

Categorization of hadrons:

BARYONS

S = 0
\( \eta \), \( \eta' \), \( P \)  \( I = \frac{1}{2} \)

S = 1
\( \Xi^- \), \( \Xi^0 \), \( \Xi^+ \)  \( I = 1 \)

S = -2
\( \Sigma^- \), \( \Sigma^0 \)  \( I = \frac{1}{2} \)

Along diagonal: same charge!

Increasing \( S \): heavier mass

Same \( S \): similar mass

Mesons

S = 1
\( K_0 \), \( K^+ \)

S = 0
\( \Pi^- \), \( \Pi^0 \), \( \Pi^+ \)

S = -1
\( K^- \), \( \bar{K}_0 \)

Along diagonal: same pattern!

\( Q = I_3 + \frac{B}{2} + \frac{S}{2} \)
LECTURE VIII

\[ S = 0 \quad \Delta^- \quad \Delta^0 \quad \Delta^+ \quad \Delta^{++} \quad I = \frac{3}{2}^+ \]

\[ S = -1 \quad \Xi^*^- \quad \Xi^*^0 \quad \Xi^*^+ \quad I = 1 \]

\[ S = -2 \quad \Xi^*^- \rightarrow \Xi^*^0 \quad I = \frac{1}{2} \]

\[ S = -3 \quad I = 0 \]

\[ S_2^- \text{ predicted: (mass & decay mode) by Gell-Mann} \]
LECTURE IX

Quark Model: Gell-Mann proposal

All hadrons (baryons & mesons) are composed of 3 elementary particles called quarks: u, d, s

Why 3 quarks? We won't go through the detailed group theory arguments. Instead we will take a simpler example of 2 quarks u & d.

\[ I = \frac{1}{2} \quad \Rightarrow \]
\[ u = \left| \frac{1}{2} \right\rangle \quad d = \left| \frac{1}{2} - \frac{1}{2} \right\rangle \]
\[ \bar{d} = \left| \frac{1}{2} \frac{1}{2} \right\rangle \quad \bar{u} = \left| \frac{1}{2} - \frac{1}{2} \right\rangle \]

Propose that mesons are made of \( \bar{q} q \) pairs with integral charge

\[ |1,1\rangle = u \bar{d} \quad \Pi^+ \quad S^+ \]
\[ |1,0\rangle = (u \bar{u} - d \bar{d})/\sqrt{2} \quad \Pi^0 \quad S^0 \]
\[ |1,-1\rangle = d \bar{u} \quad \Pi^- \quad S^- \]
\[ |0,0\rangle = (u \bar{u} + d \bar{d})/\sqrt{2} \quad \text{??} \quad \text{W singlet} \]

In group theory language: \[ 2 \otimes 2 = 3 \oplus 1 \]

However: \[ 3 \otimes 3 = 8 \oplus 1 \quad \text{octet} \oplus \text{singlet} \]
LECTURE IX

Octet:

\[ \begin{align*}
K^0 & \quad K^+ \\
\bar{d}s & \quad \bar{u}s \\
\eta & \quad \eta' \\
\pi^- & \quad \pi^0 \\
\pi^+ & \quad \pi^0
\end{align*} \]

Pseudo-scalars

\[ \begin{align*}
\bar{u}s & \quad \bar{d}s \\
K^- & \quad K^0
\end{align*} \]

\[ \eta = \frac{u\bar{u} + d\bar{d} - 2s\bar{s}}{\sqrt{6}} \quad \eta' = \frac{u\bar{u} + d\bar{d} + s\bar{s}}{\sqrt{3}} \]

Part of octet

Singlet

For the vector mesons:

\[ \begin{align*}
\bar{K}^*0 & \quad K^*+ \\
\bar{K}^- & \quad \phi^0 \\
\phi^+ & \quad \bar{K}^*0
\end{align*} \]

\[ \omega = \frac{u\bar{u} + d\bar{d}}{\sqrt{2}} \quad \phi = \frac{s\bar{s}}{\sqrt{3}} \]

Not the singlet rather these two are mixed.

The reason the masses in a multiplet have different masses is because the $s$ quark is much heavier than the $u$ & $d$ quarks.

\[ 3 \times 3 \times 3 = 10 + 8 + 8 + 1 \]

For baryons:

Deeplet & octet are symmetric under exchange.
LECTURE IX

Total wave function \( \Psi = \Psi_\text{(space)} \Psi_\text{(flavor)} \Psi_\text{(spin)} \Psi_\text{(color)} \)

for lightest baryons, \( \Psi_\text{(space)} \) is symmetric.

The observed baryons correspond to \( \Psi_\text{(flavor)} \Psi_\text{(spin)} \) antisymmetric.

\( \Rightarrow \Psi_\text{(color)} \) antisymmetric! QCD gives reason to believe the only colorless baryons can be observed. \( \Rightarrow \) asymptotic freedom (lack of five quarks)

\[ \begin{align*}
\frac{1}{2}^+ & : n \quad u \bar{d}d \\
\Sigma^- & : d \bar{d} s \\
\Xi^- & : d s s \\
\Xi^0 & : u \bar{d} s \\
\Xi^+ & : u \bar{u} s \quad \text{nonet} \quad \text{octet} + \text{singlet} \\
\Lambda^0 & : u \bar{d} s \\
\frac{3}{2}^+ & : \Delta^0 \quad \Delta^+ \quad \Delta^{++} \\
\Xi^* & : s \bar{s} s \\
\Sigma^- & : s \bar{s} s \\
\Sigma^0 & : s \bar{s} s \\
\Sigma^+ & : s \bar{s} s \\
\Sigma^{*+} & : s \bar{s} s \\
\text{Decuplet} & \end{align*} \]

Quark content; not wavefunction.

\[ \text{e.g. } \Delta^0 \equiv \frac{ddu + dud + udd}{\sqrt{3}} \]
LECTURE IX

Part II of the course: Calculating with QED
- Feynman Rules
- Feynman diagrams to calculate M
- Fermi's Golden Rule
- Cross sections & lifetimes

Origin of Feynman Rules?
- Write down eqns of motion for leptons & quarks
- Recognize internal symmetries of the Lagrangian
- Introduce force quanta via local gauge symmetry.

We start with:
- Relativistic field theory
- Dirac eqn for free spin \( \frac{1}{2} \) fermions
- Gauge invariance

Lagrangian in classical mechanics:
\[ L(q_i, \dot{q}_i) \] can define the action \( S = \int_{t_1}^{t_2} dt \, L(q_i, \dot{q}_i) \)

Hamilton's principle: \( SS = 0 \Rightarrow \) Euler-Lagrange eqns
\[ \frac{\partial L}{\partial q_i} = \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) \]

If \( L = \frac{1}{2} m \ddot{x}^2 - V(x) \)

Then: \( \frac{dv}{dx} = m \ddot{x} = f \)
**LECTURE IX**

**Field Theory:**

$L$ is a functional of fields $\phi_i$ and its space derivatives.

$\Rightarrow$ closed system with infinite degrees of freedom.

This is very general. In particle physics:

- The fields represent fundamental particles
- Composite objects are bound state solutions of the theory
- Kinetic energy terms depend on spin
- Potential energy terms contain interactions

Technically, $L$ is a lagrangian density with

$$L = \int dt \int d^3 x \ L(\phi, \partial \phi)$$

**Relativistic Field Theory:** $L$ is a Lorentz scalar.

with $c^2 t^2 - x^2 = \text{const}$ with $c=1$.

$$a_\mu = (a_0, a_i) \quad \mu = 0 \ldots 3 \quad i = 1 \ldots 3$$

$$x_\mu = (t, x^1, x^2, x^3) = t \phi(t, x^i) = (t, \vec{x})$$

$$a_\mu = (a_0, a_i) = g_{\mu \nu} \Phi_\nu = (a_0, -a_i)$$

$$x_\mu = (t, \vec{x}), \quad x_\mu = (t, -\vec{x}) \quad p_\mu = (E, \vec{p}) \quad p_\mu = (E, -\vec{p})$$

$$\partial_\mu = \frac{\partial}{\partial x_\mu} \quad \partial_\mu = \frac{\partial}{\partial x_\mu} = (\frac{\partial}{\partial t}, \nabla)$$

$$\partial_\mu a_\mu = \frac{\partial a_0}{\partial t} + \nabla \cdot \vec{a} \quad \partial_\mu \partial_\mu = \frac{\partial^2}{\partial t^2} - \nabla^2$$

Define

$$S = \int d^4 x \ L(\phi(x_\mu), \partial_\mu \phi(x_\mu))$$

$$\delta S = 0 \Rightarrow$$
LECTURE IX

\[ \frac{\partial L}{\partial \phi} = \partial \mu \left[ \frac{\partial L}{\partial (\partial \mu \phi)} \right] ; \partial \mu \phi = \partial \phi ; \phi (x_\mu) \]

Example. Real Scalar field (far from source)

\[ L = \frac{1}{2} \left[ (\partial \mu \phi \Omega^{\mu \nu}) - m^2 \phi^2 \right] \]

E-L Eqn. \[ \partial \mu \partial^\mu \phi + m^2 \phi = 0 \]
This describes motion of a free, spin-0 particle.

Note: \( E^2 = p^2 + m^2 \) is automatically satisfied
\[ E = i \partial_0 \quad \vec{p} = -i \vec{\nabla} \Rightarrow E^2 - p^2 = -\partial_0^2 + \nabla^2 = -\partial \mu \partial^\mu \]

Thus \( \partial \mu \partial^\mu \phi \) is the kinetic energy term.
No potential energy term \( \Rightarrow \) free particle

Quantum Field Theory: \( \phi \) has definite energy \( \omega \) with momentum \( \vec{k} \)
\[ \phi = \frac{1}{\sqrt{2 \omega}} \left[ e^{i (\vec{k}, \vec{r}, -\omega t)} - e^{-i (\vec{k}, \vec{r}, -\omega t)} \right] \]

Quantum Field Operator:
\[ \phi = \frac{1}{\sqrt{2 \omega}} \left[ a e^{i (\vec{k}, \vec{r}, -\omega t)} + a^+ e^{-i (\vec{k}, \vec{r}, -\omega t)} \right] \]

- \( a^+ \) creates quanta associated with \( \phi \)
- \( a \) destroys quanta associated with \( \phi \)

Q: \( \gamma \) is the quantum of the EM field
E is the quantum of the Dirac field.