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1. Introduction

This text presents an elementary introduction to the metatheory of first-order logic. It presents some of the most important results in metalogic, including the completeness, soundness, and decidability for classical sentential logic and classical predicate logic.

2. Basic Concepts of MetaLogic

A logic specifies what arguments in a given language are valid; this can be done in different ways. For example in intro logic, sentential logic is presented both in terms of truth tables and in terms of derivations.

On the one hand, we define a sentential argument $P_1, P_2, \dots, P_n / C$ to be valid if the truth table is such that there is no line in which the premises are all true but the conclusion is false.

On the other hand, we define an argument to be valid if the conclusion can be derived from the premises using the supplied derivation system.

The natural question that arises is whether the two methods of presentation are mutually consistent – do they specify precisely the same arguments as being valid? Are the D-valid arguments precisely the same as the T-valid arguments? This question divides into two separate questions: is every D-valid argument T-valid; is every T-valid argument D-valid. These notions are given special names in logic, given by the following definition.

A deductive system D is sound wrt to a logic L iff every D-valid argument is L-valid;

A deductive system D is complete wrt to a logic L iff every L-valid argument is D-valid.
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A related question is known as compactness, the terminology being borrowed from topology. In principle, an argument can have any number of premises, including infinitely many. The following is a natural argument form in number theory, which has infinitely many premises.

$$A[0], A[1], A[2], A[3], \dots / \forall x(N[x] \rightarrow A[x])$$

A logical system is compact, in one sense of the word, if the following is true:

if Γ/α is valid, then there is a finite subset Γ^* of Γ , such that Γ^*/α is valid.

Deductive systems are by their very nature finitary; insofar as one can derive α from Γ , one uses only finitely many premises in the derivation; a derivation is by its very nature a finite list of formulas. D-validity is always compact.

But given that D-validity is always compact, completeness and soundness automatically entails compactness. A logic can be adequately axiomatized only if it is compact

3. General Outline

The course is divided into four (unequal) sections.

1. Basic set theory; a very brief introduction:
 2. General metalogical concepts:
 3. Metatheory of classical sentential logic.
 4. Metatheory of classical predicate logic.
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1. Basic set theory; a very brief introduction:
membership, extensionality
operations on sets
ordered pairs, functions, relations
n-tuples, sequences
natural numbers, cardinality, mathematical induction.
 2. General metalogical concepts:

formal syntax, formal semantics, and proof theory.
Completeness, soundness, compactness.
 3. Metatheory of classical sentential logic.
 - a. formal syntax for CSL;
a mathematically precise presentation of syntax of intro logic
 - b. formal semantics for CSL.
a mathematically precise presentation of truth tables;
alternative semantic presentations of classical logic;
 - c. proof theory for CSL
axiomatic systems
natural deduction systems
 - d. soundness and completeness results for CSL.
 - e. failure of absoluteness for traditional presentations of CSL

4. Metatheory of classical predicate logic.

After doing CSL and its variants, we turn to CPL. One thing that was missing in intro logic and intermediate logic was a semantic presentation of validity in predicate logic and so forth. Validity was only presented deductively. Accordingly, we must now make amends, and provide a semantics for FOL, from which a semantic definition of validity can be extracted. This is considerably more complicated than the semantics for CSL.

Having provided a semantic account of validity, what is left is to compare this with the deductive account, both in terms of axiomatic systems and in terms of natural deduction systems.