UNIT 3: TRANSLATIONS IN IDENTITY LOGIC

1. Translation Forms In Identity Logic; Simplified Versions

1. a is the only F
$\forall x\{Fx \leftrightarrow x=a\}$
equ: $Fa \ \& \ {\sim}\exists x(x \neq a \ \& \ Fx)$

a and b are the only F's
$\forall x\{Fx \leftrightarrow [x=a \lor x=b]\}$
equ: $Fa \ \& \ Fb \ \& \ {\sim}\exists x(x \neq a \ \& \ x \neq b \ \& \ Fx)$

a,b,c are the only F's
$\forall x\{Fx \leftrightarrow [x=a \lor x=b \lor x=c]\}$
equ: $Fa \ \& \ Fb \ Fc \ \& \ {\sim}\exists x(x \neq a \ \& \ x \neq b \ \& \ x \neq c \ \& \ Fx)$

2. there are at least two F's
$\exists x \exists y\{x \neq y \ \& \ Fx \ \& \ Fy\}$

there are at least three F's
$\exists x \exists y \exists z\{x \neq y \ \& \ x \neq z \ \& \ y \neq z \ \& \ Fx \ \& \ Fy \ \& \ Fz\}$

there are at least four F's
$\exists w \exists x \exists y \exists z\{w \neq x \ \& \ w \neq y \ \& \ w \neq z \ \& \ x \neq y \ \& \ x \neq z \ \& \ y \neq z \ \& \ Fw \ \& \ Fx \ \& \ Fy \ \& \ Fz\}$

3. there is at most one F
$\exists x \forall y\{Fy \rightarrow y=x\}$
equ: ${\sim}\exists x \exists y\{x \neq y \ \& \ Fx \ \& \ Fy\}$
equ: $\forall x \forall y\{[Fx \ \& \ Fy] \rightarrow x=y\}$

there are at most two F's
$\exists x \exists y \forall z\{Fz \rightarrow [z=x \lor z=y]\}$
equ: ${\sim}\exists x \exists y \exists z\{x \neq y \ \& \ x \neq z \ \& \ y \neq z \ \& \ Fx \ \& \ Fy \ \& \ Fz\}$
equ: $\forall x \forall y \forall z\{[Fx \ \& \ Fy \ \& \ Fz] \rightarrow [x=y \lor x=z \lor y=z]\}$

there are at most three F's
$\exists x \exists y \exists z \forall w\{Fw \rightarrow [w=x \lor w=y \lor w=z]\}$
equ: ${\sim}\exists w \exists x \exists y \exists z\{w \neq x \ \& \ w \neq y \ \& \ w \neq z \ \& \ x \neq y \ \& \ x \neq z \ \& \ y \neq z \ \& \ Fw \ \& \ Fx \ \& \ Fy \ \& \ Fz\}$
equ: $\forall w \forall x \forall y \forall z\{[Fw \ \& \ Fx \ \& \ Fy \ \& \ Fz] \rightarrow [w=x \lor w=y \lor w=z \lor x=y \lor x=z \lor y=z]\}$

4. there is exactly one F
$\exists x \forall y\{Fy \leftrightarrow y=x\}$

there are exactly two F's
$\exists x \exists y\{x \neq y \ \& \ \forall z\{Fz \leftrightarrow [z=x \lor z=y]\}\}$

there are exactly three F's
$\exists x \exists y \exists z\{x \neq y \ \& \ x \neq z \ \& \ y \neq z \ \& \ \forall w\{Fw \leftrightarrow [w=x \lor w=y \lor w=z]\}$

5. there is exactly one F, which is G
$\exists x\{\forall y\{Fy \leftrightarrow y=x\} \ \& \ Gx\}$

there is exactly one F that is G; there is exactly one thing that is both F and G
$\exists x \forall y\{[Fy \ \& \ Gy] \leftrightarrow y=x\}$

there is exactly one thing, which is both F and G
$\exists x\{\forall y[y=x] \ \& \ [Fx \ \& \ Gx]$

2. Translation Forms In Identity Logic; General Versions

1. is/are the only

a is the only v such that $\mathbb{F}[v]$	$\forall v\{\mathbb{F}[v] \leftrightarrow v=a\}$
a and b are the only v such that $\mathbb{F}[v]$	$\forall v\{\mathbb{F}[v] \leftrightarrow (v=a \lor v=b\}$
a,b,c are the only v such that $\mathbb{F}[v]$	$\forall v\{\mathbb{F}[v] \leftrightarrow (v=a \lor v=b \lor v=c)\}$
e.g.,	
a is the only x such that Fx	$\forall x\{Fx \leftrightarrow x=a\}$

2. at least 1, at least 2, etc.

there is at least 1 v such that $\mathbb{F}[v]$	$\exists v\mathbb{F}[v]$
there are at least 2 v such that $\mathbb{F}[v]$	$\exists v_1\exists v_2\{v_1{\neq}v_2 \;\&\; \mathbb{F}[v_1] \;\&\; \mathbb{F}[v_2]\}$
there are at least 3 v such that $\mathbb{F}[v]$	$\exists v_1\exists v_2\exists v_3\{v_1{\neq}v_2 \;\&\; v_1{\neq}v_3 \;\&\; v_2{\neq}v_3 \;\&$
	$\mathbb{F}[v_1] \;\&\; \mathbb{F}[v_2] \;\&\; \mathbb{F}[v_3]\}$
e.g.,	
there are at least 2 x such that Fx	$\exists x_1\exists x_2\{x_1 \neq x_2 \;\&\; Fx_1 \;\&\; Fx_2\}$

3. at most 1, at most 2, etc.

there is at most 1 v such that $\mathbb{F}[v]$	$\exists v_1\forall v\{\mathbb{F}[v] \rightarrow v=v_1\}$
there are at most 2 v such that $\mathbb{F}[v]$	$\exists v_1\exists v_2\forall v\{\mathbb{F}[v] \rightarrow (v=v_1 \lor v=v_2)\}$
e.g.,	
there is at most 1 x such that Fx	$\exists x_1\forall x\{Fx \rightarrow x=x_1\}$

4. exactly 1, exactly 2, etc.

there is exactly 1 v such that $\mathbb{F}[v]$	$\exists v_1\forall v\{\mathbb{F}[v] \leftrightarrow v=v_1\}$
there are exactly 2 v such that $\mathbb{F}[v]$	$\exists v_1\exists v_2\{v_1{\neq}v_2 \;\&\; \forall v\{\mathbb{F}[v] \leftrightarrow (v=v_1 \lor v=v_2)\}\}$
e.g.,	
there is exactly 1 x such that Fx	$\exists x_1\forall x\{Fx \leftrightarrow x=x_1\}$

5. there is exactly one F, which is G
there is exactly one F, and it is G $\exists x\ \{\ \forall y\{Fy \leftrightarrow y=x\} \;\&\; Gx\ \}$

there is exactly one F that is G
there is exactly one thing that is both F and G $\exists x\forall y\ \{\ [Fy \;\&\; Gy] \leftrightarrow y=x\ \}$

3. EXERCISES

Directions: Using the suggested abbreviations, translate each of the following statements into the language of Identity Logic. Write down your lexicon, using the directions for the Unit 1 exercises. In case of syntactically ambiguous sentences, symbolize both (all) readings.

1. EXERCISE SET A

1. The SQUARE of TWO is FOUR.
2. The SQUARE of TWO is not FIVE.
3. TWO is the square ROOT of ONE PLUS THREE.
4. THREE is not the square ROOT of TWO PLUS FOUR.
5. CAIN was not his BROTHER'S KEEPER.
6. JAY'S and KAY's FAVORITE songs are the same.
7. JAY'S MIDDLE name is the same as the FIRST name of the PRESIDENT of the US.
8. JAY'S FAVORITE song is not the FAVORITE song of the HEAD of CHRYSLER Corporation.
9. JAY and KAY have the same LAST name, which is ‘SMITH’.
10. John and Jack are the same, but ‘John’ and ‘Jack’ are not.
    [Make up you own lexicon.]

2. EXERCISE SET B

11. Everyone has someone who is his/her MOTHER.

12. There is no one who is everyone's MOTHER.

13. Every MOTHER is someone's MOTHER.

14. POE is the AUTHOR of every interesting GHOST story.

15. No one PRESENT is the HEAD of IBM.

16. The HEAD of IBM is the SPOUSE of someone PRESENT.

17. The SPOUSE of JONES is the MOTHER of everyone PRESENT.

18. Every MARRIED person is someone's SPOUSE.

19. A person is MARRIED if and only if he/she is someone's SPOUSE.

20. One of the people PRESENT is the KILLER.

21. Every PROFESSOR is the FAVORITE of some STUDENT [or other].

22. No PROFESSOR is the FAVORITE of every STUDENT.

23. Every POLITICIAN has a RELATIVE whose MIDDLE name is the same as the politician's MIDDLE name.

24. The LAST name of the WIFE of every MALE-chauvinist is the same as her HUSBAND'S.

25. The FAVORITE song of the HEAD of a NATION is also the FAVORITE song of every CITIZEN of that nation.

26. At least one NATURAL number is (equal to) its own SQUARE.

27. For any pair of NUMBERS, if the former equals the latter, then the SQUARE of the former equals the SQUARE of the latter.

28. The SUM of any NUMBER and its NEGATIVE is ZERO.

29. The square ROOT of TWO is not the QUOTIENT of any [pair of] WHOLE numbers.

30. A number is RATIONAL if and only if it is the QUOTIENT of a pair of WHOLE numbers.

31. A number is not PRIME if it is the PRODUCT of any two numbers other than 1.

32. The SQUARE of the LENGTH of the HYPOTENUSE of any RIGHT triangle is equal to the SUM of the SQUARES of the LENGTHS of its BASE and VERTICAL sides.

33. FRESHMEN all have the same FAVORITE song.

3. EXERCISE SET C

34. There is at least one FRESHMAN in the class.

35. There is at most one FRESHMAN in the class.

36. There is exactly one FRESHMAN in the class.

37. There are at least two FRESHMEN in the class.

38. There are at most two FRESHMEN in the class.

39. There are exactly two FRESHMEN in the class.

40. There are at least three FRESHMEN in the class.

41. There are at most three FRESHMEN in the class.

42. There are exactly three FRESHMEN in the class.

43. There are at least two but no more than three FRESHMEN.

44. There are no more than two FRESHMEN.

45. JAY is the only FRESHMAN in the class.

46. JAY and KAY are the only FRESHMEN in the class.

47. JAY, KAY, and CHRIS are the only FRESHMEN in the class.

48. Everyone PASSED except JAY.

49. Everyone PASSED except JAY and KAY.

50. Everyone PASSED except JAY, KAY, and CHRIS.

51. No one PASSED except JAY.

52. No one PASSED except JAY and KAY.

53. No one PASSED except JAY, KAY, and CHRIS.

54. JAY RESPECTS only himself.

55. KAY RESPECTS everyone but herself.

56. JAY RESPECTS everyone but/except his FATHER.

57. KAY RESPECTS no one but/except her MOTHER.

58. Everyone HONORS his(her) FATHER unless he is the PRESIDENT of LIBYA.

59. No one HONORS his(her) FATHER if he is the PRESIDENT of LIBYA.

60. Every person RESPECTS only his/her MOTHER and FATHER.

4. EXERCISE SET D

61. Everyone has at least two PARENTS.

62. Every AFFLUENT person OWNS at least two CARS.

63. No POOR person OWNS more than one CAR.

64. No CRIMINAL has more than two FRIENDS.

65. No more than one BOSTONIAN will WIN the LOTTERY.

66. JONES KNOWS exactly two of SMITH's RELATIVES.

67. There is at most one PERSON who KNOWS every SONG.

68. No PERSON KNOWS more than two SONGS.

69. Every PERSON KNOWS at most one SONG.

70. There is at most one REPUBLICAN who is RESPECTED by every DEMOCRAT.

71. Every POLITICIAN has exactly two FRIENDS.

72. No HOUSE has exactly one ROACH LIVING in it.

73. There is exactly one PERSON who KNOWS the PRESIDENT of every COUNTRY.

74. There are at least two STUDENTS who are RESPECTED by every PROFESSOR.

75. There is at most one PROFESSOR who is RESPECTED by every STUDENT.

76. Every STUDENT RESPECTS at least two PROFESSORS.

77. There is exactly one STUDENT who RESPECTS every PROFESSOR.

78. Every STUDENT RESPECTS exactly one PROFESSOR.

79. There are at least two STUDENTS who RESPECT every PROFESSOR.

80. LINCOLN is the only PRESIDENT who is RESPECTED by everyone.

81. ADAMS and BROWN are the only PROFESSORS who are RESPECTED by every STUDENT.

82. ARISTOTLE and HEGEL are the only PHILOSOPHERS who are SMARTER than all of their DISCIPLES.

83. ADAMS is the only FRESHMAN who is SMARTER than the PROFESSOR of every COURSE in which he/she is ENROLLED.

84. The PRESIDENT of the US is the only AMERICAN who KNOWS the PRESIDENT of every COUNTRY.

5. EXERCISE SET E

85. Anyone who RESPECTS no one but his/her MOTHER is unHEALTHY.
86. No one who RESPECTS everyone but his/her MOTHER is HEALTHY.
87. No one RESPECTS anyone who RESPECTS only him(her)self.
88. No one RESPECTS anyone who RESPECTS everyone but him(her)self.
89. No MAN RESPECTS any WOMAN who RESPECTS everyone but him.
90. Other than JAY and KAY, no HUMAN UNDERSTANDS every PROBLEM in this exercise set.
91. Except for 1, there is no POSITIVE number that is equal to its own SQUARE.
92. Except for 0 and 1, no number is (equal to) its own SQUARE.
93. Except for the HEAD of CHRYSLER Corp., no PERSON is RESPECTED by every EMPLOYEE of his/her COMPANY.
94. JAY is the only FRESHMAN who KNOWS at least two PROFESSORS.
95. JAY is the only STUDENT RESPECTED by more than one PROFESSOR.
96. KAY is the only STUDENT who RESPECTS no PROFESSOR but JONES.
97. For every POLITICIAN, there are at least two VOTERS who RESPECT him/her.
98. For every POLITICIAN, there is at least one VOTER who RESPECTS no POLITICIAN but him/her.
99. For every POLITICIAN, there is at least one VOTER who RESPECTS every POLITICIAN but him/her.
100. Anyone who RESPECTS exactly one person is RESPECTED by at least one person.
101. Anyone who RESPECTS no one but him(her)self is RESPECTED by at most one person.
102. No one RESPECTS anyone who RESPECTS at most one person.
103. There is exactly one FRESHMAN who KNOWS every PROFESSOR.
104. There is exactly one FRESHMAN, and he(she) KNOWS every PROFESSOR.
105. There is exactly one STUDENT who KNOWS exactly one PROFESSOR.
106. There is exactly one STUDENT, and he(she) KNOWS exactly one PROFESSOR.

6. EXERCISE SET F

107. JONES is the TALLEST person in the CLASS.
[T[α,β] : α is taller than β]

108. JONES is FASTER than every other RUNNER.

109. If an ARGUMENT is VALID, but its CONCLUSION is SENTENCE #100, then at least one of its PREMISES is FALSE.

100. The truth VALUE of a CONDITIONAL is F if the truth VALUE of its ANTECEDENT is T and the truth VALUE of its CONSEQUENT is F.

111. The truth VALUE of a CONJUNCTION is T if and only if the truth VALUE of every one of its CONJUNCTS is T.

112. The truth VALUE of a DISJUNCTION is F if and only if the truth VALUE of both of its DISJUNCTS is F.

113. Every FORMULA whose FIRST SYMBOL is a TILDE is the NEGATION of some FORMULA.

7. EXERCISE SET G

114. Everything is identical to something.

115. If anything is identical to everything, then everything is identical to everything.

116. Everything is identical to itself.

117. For any two things, if the former is identical to the latter, then the latter is identical to the former.

118. Any two things that are identical to a third thing are identical to each other.

119. Nothing is identical to everything, unless there is exactly one thing.

121. If there is exactly one thing, then everything is identical to it.

4. ANSWERS TO UNIT 3 EXERCISES

1. $s(2)=4$
2. $s(2)\neq 5$
3. $2=r(p(1,3))$; $2=p(r(1),3)$
4. $3\neq r(p(2,4))$; $3\neq p(r(2),4)$
5. $c\neq k(b(c))$
6. $f(j)=f(k)$
7. $m(j)=f(p(u))$
8. $f(j)\neq f(h(c))$
9. $l(j)=l(k)\ \&\ l(k)=s$
10. $a=b\ \&\ c\neq d$
    a : John c : 'John' b : Jack d : 'Jack'
11. $\forall x\exists y[y=m(x)]$
12. $\sim\exists x\forall y[x=m(y)]$
13. $\forall x\{Mx\rightarrow\exists y[x=m(y)]\}$
14. $\forall x\{Gx\rightarrow p=a(x)\}$
15. $\sim\exists x\{Px\ \&\ x=h(i)\}$
16. $\exists x\{Px\ \&\ h(i)=s(x)\}$
17. $\forall x\{Px\rightarrow s(j)=m(x)\}$
18. $\forall x\{Mx\rightarrow\exists y[x=s(y)]\}$
19. $\forall x\{Mx\leftrightarrow\exists y[x=s(y)]\}$
20. $\exists x\{Px\ \&\ x=k\}$
21. $\forall x\{Px\rightarrow\exists y[Sy\ \&\ x=f(y)]\}$
22. $\sim\exists x\{Px\ \&\ \forall y[Sy\rightarrow x=f(y)]\}$
23. $\forall x\{Px\rightarrow\exists y\{Ryx\ \&\ m(y)=m(x)\}\}$
24. $\forall x\{Mx\rightarrow l(w(x))=l(h(w(x)))\}$
25. $\forall x\{Nx\rightarrow\forall y\{Cyx\rightarrow f(h(x))=f(y)\}\}$
26. $\exists x\{Nx\ \&\ x=s(x)\}$
27. $\forall x\forall y\{[Nx\ \&\ Ny]\rightarrow\{x=y\rightarrow s(x)=s(y)\}\}$
28. $\forall x\{Nx\rightarrow s(x,n(x))=0\}$
29. $\forall x\forall y\{[Wx\ \&\ Wy]\rightarrow r(2)\neq q(x,y)\}$
30. $\forall x\{Rx\leftrightarrow\exists y\exists z\{Wy\ \&\ Wz\ \&\ x=q(y,z)\}\}$
31. $\forall x\{\exists y\exists z\{y\neq 1\ \&\ z\neq 1\ \&\ x=p(y,z)\}\rightarrow\sim Px\}$
    $\forall x\forall y\forall z\{\{y\neq 1\ \&\ z\neq 1\ \&\ x=p(y,z)\}\rightarrow\sim Px\}$
32. $\forall x\{Rx\rightarrow q(l(h(x)))=s(q(l(b(x))),q(l(v(x))))\}$
    $q(\alpha)$: the square of α
33. $\forall x\forall y\{[Fx\ \&\ Fy]\rightarrow f(x)=f(y)\}$

34. $\exists xFx$

35. $\exists x\forall y\{Fy \rightarrow y=x\}$

36. $\exists x\forall y\{Fy \leftrightarrow y=x\}$

37. $\exists x\exists y\{x\neq y \,\&\, Fx \,\&\, Fy\}$

38. $\exists x\exists y\forall z\{Fz \rightarrow \{z=x \vee z=y\}\}$

39. $\exists x\exists y\{x\neq y \,\&\, \forall z\{Fz \leftrightarrow \{z=x \vee z=y\}\}\}$

40. $\exists x\exists y\exists z\{x\neq y \,\&\, x\neq z \,\&\, y\neq z \,\&\, Fx \,\&\, Fy \,\&\, Fz\}$

41. $\exists x\exists y\exists z\forall w\{Fw \rightarrow [w=x \vee w=y \vee w=z]\}$

42. $\exists x\exists y\exists z\{x\neq y \,\&\, x\neq z \,\&\, y\neq z \,\&\, \forall w\{Fw \leftrightarrow [w=x \vee w=y \vee w=\mathrm{z}]\}\}$

43. $\exists x\exists y\{x\neq y \,\&\, Fx \,\&\, Fy\} \,\&\, \exists x\exists y\exists z\forall w\{Fw \rightarrow [w=x \vee w=y \vee w=\mathrm{z}]\}$

44. $\exists x\exists y\forall z\{Fz \rightarrow [z=x \vee z=y]\}$

45. $\forall x\{Fx \leftrightarrow x=j\}$

46. $\forall x\{Fx \leftrightarrow [x=j \vee x=k]\}$

47. $\forall x\{Fx \leftrightarrow [x=j \vee x=k \vee x=c]\}$

48. $\forall x\{Px \leftrightarrow x\neq j\}$

49. $\forall x\{Px \leftrightarrow \sim[x=j \vee x=k]\}$

50. $\forall x\{Px \leftrightarrow \sim[x=j \vee x=k \vee x=c]\}$

51. $\forall x\{Px \leftrightarrow x=j\}$

52. $\forall x\{Px \leftrightarrow [x=j \vee x=k]\}$

53. $\forall x\{Px \leftrightarrow [x=j \vee x=k \vee x=c]\}$

54. $\forall x\{Rjx \leftrightarrow x=j\}$

55. $\forall x\{\sim Rkx \leftrightarrow x=k\}$; $\forall x\{Rkx \leftrightarrow x\neq k\}$

56. $\forall x\{\sim Rjx \leftrightarrow x=f(j)\}$; $\forall x\{Rjx \leftrightarrow x\neq f(j)\}$

57. $\forall x\{Rkx \leftrightarrow x=m(k)\}$

58. $\forall x\{H[x,f(x)] \leftrightarrow f(x)\neq p(l)\}$

59. $\sim\exists x\{H[x,f(x)] \,\&\, f(x)=p(l)\}$

60. $\forall x\forall y\{Rxy \leftrightarrow [y=m(x) \vee y=f(x)]\}$

61. $\forall x\exists y\exists z\{y\neq z \,\&\, Pyx \,\&\, Pzx\}$
$P[\alpha,\beta]$: α *is a parent of* β

62. $\forall x\{Ax \rightarrow \exists y\exists z\{y\neq z \,\&\, [Cy \,\&\, Oxy] \,\&\, [Cz \,\&\, Oxz]\}\}$

63. $\sim\exists x\{Px \,\&\, \exists y\exists z\{y\neq z \,\&\, [Cy \,\&\, Oxy] \,\&\, [Cz \,\&\, Oxz]\}\}$

64. $\sim\exists w\{Cw \,\&\, \exists x\exists y\exists z\{x\neq y \,\&\, x\neq z \,\&\, y\neq z \,\&\, Fxw \,\&\, Fyw \,\&\, Fzw\}\}$
$F[\alpha,\beta]$: α *is a friend of* β

65. $\exists x\forall y\{[By \,\&\, Wyl] \rightarrow y=x\}$

66. $\exists x\exists y\{x\neq y \,\&\, \forall z\{[Rzs \,\&\, Kjz] \leftrightarrow [z=x \vee z=y]\}\}$
$R[\alpha,\beta]$: α *is a relative of* β

67. $\exists x\forall y\{[Py \,\&\, \forall z[Sz \rightarrow Kyz]] \rightarrow y=x\}$

68. $\sim\exists w\{Pw \,\&\, \exists x\exists y\exists z\{x{\neq}y \,\&\, x{\neq}z \,\&\, y{\neq}z \,\&$
$[Sx \,\&\, Kwx] \,\&\, [Sy \,\&\, Kwy] \,\&\, [Sz \,\&\, Kwz]\}\}$

69. $\forall x\{Px \to \exists y\forall z\{[Sz \,\&\, Kxz] \to z{=}y\}\}$

70. $\exists x\forall y\{[Ry \,\&\, \forall z\{Dz \to Rzy\}] \to y{=}x\}$

71. $\forall x\{Px \to \exists y\exists z\{y{\neq}z \,\&\, \forall w\{Fwx \leftrightarrow [w{=}y \vee w{=}z]\}\}\}$

72. $\sim\exists x\{Hx \,\&\, \exists y\forall z\{[Rz \,\&\, Lzx] \leftrightarrow z{=}y\}\}$
$L[\alpha,\beta]$: α *lives in* β

73. $\exists x\forall y\{[Py \,\&\, \forall z\{Cz \to K[y,p(z)]\}] \leftrightarrow y{=}x\}$

74. $\exists x\exists y\{x{\neq}y \,\&\, [Sx \,\&\, \forall z\{Pz \to Rzx\}] \,\&\, [Sy \,\&\, \forall z\{Pz \to Rzy\}]\}$
$R[\alpha,\beta]$: α respects β [Always write transitive verbs in active voice.]

75. $\exists x\forall y\{\{Py \,\&\, \forall z\{Sz \to Rzy\}\} \to y{=}x\}$

76. $\forall x\{Sx \to \exists y\exists z\{y{\neq}z \,\&\, [Py \,\&\, Rxy] \,\&\, [Pz \,\&\, Rxz]\}\}$

77. $\exists x\forall y\{\{Sy \,\&\, \forall z\{Pz \to Ryz\}\} \leftrightarrow y{=}x\}$

78. $\forall x\{Sx \to \exists y\forall z\{[Pz \,\&\, Rxz] \leftrightarrow z{=}y\}\}$

79. $\exists x\exists y\{x{\neq}y \,\&\, [Sx \,\&\, \forall z\{Pz \to Rxz\}] \,\&\, [Sy \,\&\, \forall z\{Pz \to Ryz\}]\}$

80. $\forall x\{[Px \,\&\, \forall yRyx] \leftrightarrow x{=}l\}$

81. $\forall x\{[Px \,\&\, \forall y\{Sy \to Ryx\}] \leftrightarrow [x{=}a \vee x{=}b]\}$

82. $\forall x\{[Px \,\&\, \forall y\{Dyx \to Sxy\}] \leftrightarrow [x{=}a \vee x{=}h]\}$
D[α,β] : α is a disciple of β

83. $\forall x\{[Fx \,\&\, \forall y\{[Cy \,\&\, Exy] \to S[x,p(y)]\}] \leftrightarrow x{=}a\}$
E[α,β] : α is enrolled in β

84. $\forall x\{[Ax \,\&\, \forall y\{Cy \to K[x,p(y)]\}] \leftrightarrow x{=}p(u)\}$

85. $\forall x\{\forall y\{Rxy \leftrightarrow y{=}m(x)\} \to \sim Hx\}$

86. $\sim\exists x\{\forall y\{\sim Rxy \leftrightarrow y{=}m(x)\} \,\&\, Hx\}$

87. $\forall x\{\forall y\{Rxy \leftrightarrow y{=}x\} \to \sim\exists yRyx\}$

88. $\forall x\{\forall y\{\sim Rxy \leftrightarrow y{=}x\} \to \sim\exists yRyx\}$

89. $\forall x\{Mx \to \forall y\{[Wy \,\&\, \forall z\{\sim Ryz \leftrightarrow z{=}x\}] \to \sim Rxy\}\}$

90. $\forall x\{[Hx \,\&\, \forall z\{Pz \to Uxz\}] \leftrightarrow [x{=}j \vee x{=}k]\}$

91. $\forall x\{[Px \,\&\, x{=}s(x)] \leftrightarrow x{=}1\}$

92. $\forall x\{x{=}s(x) \leftrightarrow [x{=}0 \vee x{=}1]\}$
U = numbers

93. $\forall x\{[Px \,\&\, \forall y\{E[y,c(x)] \to Ryx\}] \leftrightarrow x{=}h(c)\}$
$E[\alpha,\beta]$: α is an employee of β
$c(\alpha)$: the company of α

94. $\forall x\{[Fx \,\&\, \exists y\exists z\{y{\neq}z \,\&\, Py \,\&\, Kxy \,\&\, Pz \,\&\, Kxz\}] \leftrightarrow x{=}j\}$

95. $\forall x\{[Sx \,\&\, \exists y\exists z\{y{\neq}z \,\&\, Py \,\&\, Ryx \,\&\, Pz \,\&\, Rzx\}] \leftrightarrow x{=}j\}$

96. $\forall x\{[Sx \,\&\, \forall y\{[Py \,\&\, Rxy] \leftrightarrow y{=}j\}] \leftrightarrow x{=}k\}$

97. $\forall x\{Px \to \exists y\exists z\{y{\neq}z \,\&\, [Vy \,\&\, Ryx] \,\&\, [Vz \,\&\, Rzx]\}\}$

98. $\forall x\{Px \to \exists y\{Vy \,\&\, \forall z\{[Pz \,\&\, Ryz] \leftrightarrow z{=}x\}\}\}$

99. $\forall x\{Px \to \exists y\{Vy \,\&\, \forall z\{[Pz \,\&\, {\sim}Ryz] \leftrightarrow z{=}x\}\}\}$

100. $\forall x\{\exists y\forall z\{Rxz \leftrightarrow z{=}y\} \to \exists yRyx\}$

101. $\forall x\{\forall y\{Rxy \leftrightarrow y{=}x\} \to \exists y\forall z\{Rzx \to z{=}y\}\}$

102. $\forall x\{\exists y\forall z\{Rxz \to z{=}y\} \to {\sim}\exists yRyx\}$

103. $\exists x\forall y\{[Fy \,\&\, \forall z\{Pz \to Kyz\}] \leftrightarrow y{=}x\}$

104. $\exists x\{\forall y\{Fy \leftrightarrow y{=}x\} \,\&\, \forall y\{Py \to Kxy\}\}$

105. $\exists x\forall y\{[Sy \,\&\, \exists u\forall v\{[Pv \,\&\, Kyv] \leftrightarrow v{=}u\}] \leftrightarrow y{=}x\}$

106. $\exists x\{\forall y\{Sy \leftrightarrow y{=}x\} \,\&\, \exists y\forall z\{[Pz \,\&\, Kxz] \leftrightarrow z{=}y\}\}$

107. $\mathrm{Cj} \,\&\, \forall \mathrm{x}\{[\mathrm{Cx} \,\&\, \mathrm{x}{\neq}\mathrm{j}] \to \mathrm{Tjx}\}$
$T[\alpha,\beta]$: α is taller than β

This sentence has the form "Jones = ...". However, the 'is' in this sentence cannot be symbolized by '=' in simple identity logic, because the second term contains a sentence; this requires description logic, which allows singular-terms that contain sentences; the form in description logic is:

$j =$ the unique *x* such that: $Cx \,\&\, \forall y\{[Cy \,\&\, y{\neq}j] \to Tjy\}$

108. $Rj \,\&\, \forall x\{[Rx \,\&\, x{\neq}j] \to Fjx\}$

109. $\forall x\{[Ax \,\&\, Vx \,\&\, [c(x){=}s(100)]] \to \exists y\{Pyx \,\&\, Fy\}\}$
$s(\alpha)$: sentence number α ; the αth sentence

100. $\forall x\{Cx \to \{[v(a(x)){=}t \,\&\, v(c(x)){=}f] \to v(x){=}f\}\}$
t : the truth value T f : the truth value F $v(\alpha)$: the truth value of α

111. $\forall x\{Cx \to \{v(x){=}t \leftrightarrow \forall y\{Cyx \to v(y){=}t\}\}\}$
$C[\alpha,\beta]$: α is a conjunct of β

112. $\forall x\{Dx \to \{v(x){=}f \leftrightarrow [v(d_1(x)){=}f \,\&\, v(d_2(x)){=}f]\}\}$
$d_1(\alpha)$: the first disjunct of α
$d_2(\alpha)$: the second disjunct of α
[Here is a case where subscripted expressions are actually useful; recall official rules of formation.]

113. $\forall x\{[Fx \,\&\, T[f(x)]] \to \exists y\{Fy \,\&\, x{=}n(y)\}\}$
$f(\alpha)$: the first symbol of α
$n(\alpha)$: the negation of α
$T[\alpha]$: α is a tilde [α is an instance of the tilde symbol]

114. $\forall x\exists y[x{=}y]$

115. $\forall x\{\forall y[x{=}y] \to \forall x\forall y[x{=}y]\}$

116. $\forall x[x{=}x]$

117. $\forall x\forall y\{[x{=}y] \to [y{=}x]\}$

118. $\forall x\forall y\forall z\{[x{=}z \,\&\, y{=}z] \to x{=}y\}$

119. ${\sim}\exists x\forall y[y{=}x] \to {\sim}\exists x\forall y[x{=}y]$