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Not all preconceptions are misconceptions: finding 'anchoring conceptions' for grounding instruction on students' intuitions

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This study begins the task of mapping out the domain of valid, potentially helpful beliefs of students and raises the possibility of drawing on these intuitions in teaching conceptual material. Some issues are explored surrounding the identification of such intuitions, referred to as anchoring conceptions or *anchors*. We attempt to: (1) propose some organizing theoretical and observational definitions of the anchor construct; (2) present some initial findings from a diagnostic test designed to uncover anchors for physics instruction; and (3) provoke an initial discussion of the new methodological issues that arise in this domain.

The results of the diagnostic test indicate that a number of group anchors exist. In addition, some unexpected non-anchors were identified. Furthermore, evidence was found indicating that some anchoring examples may be 'brittle', i.e., evidence that the anchor could not be extended analogically to help a student make sense of a target situation.

Finally, it is suggested that further research is needed to construct a theory of anchoring conceptions that would, for example, specify what characteristics would indicate that an anchoring conception can provide the basis for conceptual change via analogical extension.

Introduction

It is now well established that students' preconceptions (ideas held before instruction) often pose strong barriers to understanding in physics (Viennot 1979, Clement 1982, Driver and Erickson 1983, McDermott 1984, Halloun and Hestenes 1985, Brown and Clement 1987 a, b). However, although many preconceptions are detrimental to learning, there may be other preconceptions that are largely in agreement with accepted physical theory. These will be referred to here as 'anchoring conceptions' (or more briefly, as anchors). This study focuses on the possibility of identifying such positive intuitions and explores some of the issues surrounding their potential for use in instruction.

We assume that it is desirable to be able to ground new material in that portion of the student's intuition that is in agreement with accepted theory. When this is possible, it should help students to understand and believe physical principles at a 'makes sense' level instead of only at a more formal level. For example, many students refuse to believe that static objects can exert forces. They refuse to believe the physicist's assertion that a table exerts an upward force on a coffee cup sitting on the table. However, almost all students believe that a spring will exert a constant force on one's hand as one holds it compressed. In teaching that inanimate objects

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can exert forces, this intuition about springs can be built on as an anchor. By working with students to help them see that even 'rigid' objects are springy to some extent, one can anchor the idea of static forces in the student's intuitive conception of springiness.

In this paper we will use the term misconception to refer to students' ideas that are incompatible with currently accepted scientific knowledge. To be sure, misconceptions should be respected as creative constructions of the individual. In some cases misconceptions are also adaptive and successful for dealing with the practical world. They do, however, present significant difficulties in learning a subject like Newtonian mechanics. In these terms, our first hypothesis is that not all preconceptions are misconceptions; rather, some of the students' preconceptions are usable anchoring conceptions.

There are the beginnings of some support for this hypothesis in the literature. Minstrell (1982) described a successful physics lesson that we interpret as being based on an anchoring example. Clement *et al.* (1987) measured significant gains in high school students' understanding of the concept of force using several experimental lessons that were based on anchoring examples such as the hand pushing on the spring. In these lessons other techniques, such as the use of 'bridging analogies' and socratic discussions were used to extend the students' physical intuitions from the anchoring example to the misconceived example by analogy. For example, in the above situation of a coffee cup on a table, an intermediate or bridging case of an object resting on a noticeably flexible board can help students to transfer their intuitive understanding from the anchor of a hand pressing on a spring to the target situation of the coffee cup. Although these lessons also used empirical demonstrations, tutoring studies have shown that significant gains can result from bridging from an anchor without the benefit of empirical demonstrations (Brown 1987).

While considerable research has been conducted on misconceptions in science, very little is known about anchoring conceptions. In order to develop a more systematic approach to promoting conceptual change in certain domains, it could be productive to identify what can be used as anchoring conceptions and what are effective bridging analogies in the domain. Three purposes of this study are to: (1) propose some organizing theoretical and observational definitions of the anchor construct; (2) present some initial findings from a diagnostic test designed to uncover anchors for physics instruction; and (3) provoke an initial discussion of the new methodological issues that arise in this domain.

Anchoring conceptions and anchoring examples

Defining the concept of an anchor involves a number of theoretical and methodological issues. First, how should we define an anchor in terms of internal knowledge structures? In theoretical terms we define an *anchoring conception* as an intuitive knowledge structure that is in rough agreement with accepted physical theory. By intuitive, we mean that it is concrete rather than abstract, and in particular that it is self-evaluated; the strength of the belief is determined by the subjects themselves rather than by appeal to authority. (See Clement (in press) for a discussion of elemental physical intuitions as knowledge structures.)

At the observational level we have found the following definition to be useful: a problem situation is an *anchoring example* for a student if he or she makes a correct response to the problem and indicates substantial confidence in the answer. An observed anchoring example, then, provides one source of evidence for detecting an anchoring conception in the mind of the student, especially when there is reason to believe that the student's answer was not simply memorized by rote.

Results from an anchor diagnostic test

Method

Data collection: The diagnostic test in the appendix was used to search for anchoring examples in the areas of forces from static objects (questions 1 to 6), Newton's third law in dynamic situations (questions 7 to 13), and frictional forces (question 14)*. The test was administered in three Western Massachusetts high schools to students who had not yet taken physics and who were enrolled in chemistry, biology or general science courses. The average age of the three groups of students was as follows: chemistry students—17 years old; biology students—15 years old; and general science students—14 years old.

The test consists of 14 multiple-choice questions (some with multiple parts) designed to identify anchoring examples. For each question students were asked to indicate their confidence on a scale ranging from zero (just a blind guess) to 3 (I'm sure I'm right). The questions included both static and dynamic situations involving forces and their effects. The questions were generated in sessions with researchers and high school physics teachers while attempting to design experimental lessons. (The lessons are discussed in Clement *et al.* 1987.)

Individual anchors: A problem was considered to be an anchoring example for an individual student if he or she gave the correct answer and indicated a confidence level greater than or equal to 2 on the confidence scale shown at the beginning of the appendix. Perhaps such an example should actually be called a 'potential anchor'. As will be shown, not all anchoring examples defined in this way can be used effectively in instruction via transfer. Thus in some contexts it may be useful to split the concept of anchoring example as follows: *potential* anchors are anchoring examples defined as above; *usable* anchors are anchoring examples that can be extended in instructions so that a useful anchoring conception is transferred to other more difficult target situations. It should also be noted that in using a multiple-choice test alone there is no guarantee that students answering correctly have the same anchoring conception in mind as the experimenter. Nevertheless, such a test may be the most cost effective initial means for detecting potential anchors.

Group anchors: Data from the diagnostic test can indicate that a particular example is an anchoring example for a group as well as for a particular student. The percentage of students who answered the problem correctly with a confidence level of 2 or higher is termed the *belief score* for that group. We refer to a problem situation as a *group anchor* for a sample of students if it is an anchoring example for a certain criterion percentage of those students. In using anchors in experimental lessons in introductory physics, our initial impression is that if an example is a confident anchor on a pre-test for about 70% of the students in a sample, most other students will indicate

* The problems in the appendix are grouped by category. Students encountered the problems (as numbered in the appendix) in the following order: 7, 1, 8, 2, 14, 3, 9, 10, 4, 5, 11, 12, 13, 6.

that the idea makes sense to them after a minimal amount of instruction, such as a demonstration. Thus we have considered problems with a belief score of 70% or higher to be group anchors that have excellent potential for use in instruction. This criterion is somewhat arbitrary and was determined by practical considerations in searching for examples that would be useful in instruction.

Results

Anchoring examples: A number of group anchors were discovered as indicated by the results in the appendix, which reports the results for the three student groups combined. For example, 80% of students answered correctly with a confidence level of 2 or higher that a spring pushes up on your hand when you press down on the spring and hold your hand still (question 4). A total of 84% of students answered with high confidence that a rowboat would move to the left when a person stepped out of it to the right (question 7). Further, 74% of students answered with high confidence that a skater pushing another skater to the right would herself move to the left, although not necessarily at the same speed, even though the skaters were of the same weight (question 11). Given that two carts on a smooth floor are pushed apart by a spring not attached to either cart, 83% of the students were confident that the carts would move apart at the same speed (question 9).

Unexpected results: There were also some unexpected results. We were somewhat surprised by the uniformity in results across the three groups of students tested. With only three exceptions, the 15 examples that were group anchors (belief scores of 70% or higher) or close to group anchors (belief scores of 61% to 69%) for one group were also group anchors or close to group anchors for the other two groups. Given this uniformity, we report on the results averaged across the three groups.

Some examples that we expected to be group anchors were found not to be confident anchors. For example, only 22% of students answered with confidence that a wall exerts a force on your fist when you punch the wall, an example often used in attempts to convince students that static objects such as walls can exert forces (question 3); 59% indicated that the wall does not exert a force on your fist and 36% gave this answer with high confidence.

There were also some cases for which we mistakenly expected certain anchors to be stronger than others. For example, given the situation of a hand pushing down on a spring in question 4, students were asked whether the spring exerts a force on the hand. This was considered to be a good candidate for an anchor, but we had some reservations about how strong an anchoring example it would be. We expected that the upward force would be recognized more intuitively in the case of holding up a 30 pound dictionary on an outstretched hand (question 2). In both cases the subject can imagine feeling the upward force, but the dictionary situation involves a person exerting the force and allows for direct use of kinesthetic intuition. However, the results indicated that the hand-on-spring situation was in fact an anchoring example for more students (belief score of 80%) than the dictionary-on-hand situation (belief score of 65%). One possible reason for the spring being a stronger anchor is that the spring moves up when the hand is removed, whereas this is not so obvious for the hand when the book is removed.

Perhaps the most surprising result from this study was the low belief score for the log exerting a force on Mr T's chest in question 1. We predicted this situation would be a solid anchor for students because of the opportunity to identify with the person in the problem. However, this situation was an anchor for only 53% of the students. A full 30% answered, although some with low confidence, that the log would not exert a force. We are interested in using deeper probes and analysis techniques to determine the origins of these anomalous responses in the future.

Instructional applications and the problem of brittle anchors

Teaching strategies: As mentioned earlier, Clement *et al.* (1987) reported success in using an approach to overcome misconceptions in mechanics that used anchors as a central element in the teaching strategy. The hand on the spring situation, for example, is used as an anchoring example for helping students make sense of the idea that static objects can exert forces. In this approach, the student needs to transfer a central idea from the anchor—the idea of 'the applied force causing deformation causing a reaction force', thereby providing a mechanism for the reaction force.

This causal relationship of applied force causing deformation causing a reaction force is an example of what we call the ***key relationship*** or ***key structure***, the major relationship in the situation that we wish the student to transfer to other situations. As mentioned earlier, this process can be aided by discussions of 'bridging analogies'. Sometimes this can also be accomplished via a ***transformation***: if the student believes that the flexible board pushes up on the book, and sees that the board can be gradually transformed into the more rigid table by making it thicker and thicker, he or she may come to believe that the table pushes up as well.

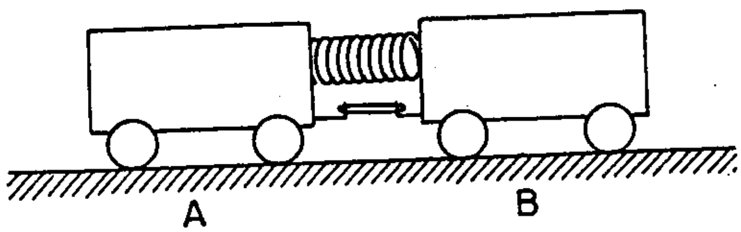
In some cases, however, pilot tutoring has indicated that the strategy of extending anchors via analogies can fail. For some achoring examples, even though students are in complete agreement with the physicist in their predictions about the anchor situation, they refuse to believe that the prediction applies to the target situation. Apparently they cannot transfer the key relationship to the target. In such a case we refer to the anchor as *brittle*.

Brittle anchors: The results of the studies suggest 'brittle' anchors might be especially prevalent for beliefs based on symmetry. As an example of what we mean by 'brittleness', 96% of the students answered correctly that identical carts, pushed apart by a spring suspended between the carts, would move apart at the same speed (belief score = 83%). We had hoped that this would be a useful anchor. However, only 32% said they would move apart at the same speed for the virtually identical, but slightly asymmetrical situation in question 10 in which the spring was attached to one of the carts (belief score = 23%). The asymmetrical problem appeared immediately after the symmetrical one, suggesting that a majority of students saw the minor change (attaching the spring to one cart) as significant. Hence, the symmetrical carts situation may be a *fragile* or brittle anchoring example, in which a small modification changes the students' intuitions about it.

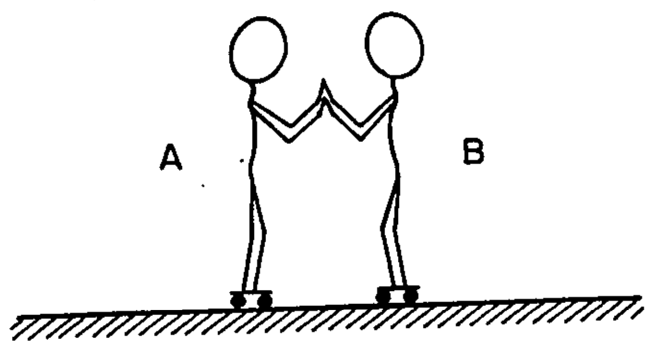
As another example, 97% of the students answered correctly that skaters of equal mass would separate with the same speed if both skaters pushed (question 11). Yet only 41% indicated that they would separate at equal speeds if only one of the skaters pushed on the chest of the other. (The latter problem does not appear in the appendix since it was not intended to be an anchor.) Apparently many students did not think about the same key relationship in the second problem. Thus, we may not be able analogically to extend anchoring examples such as the symmetrical skaters or

Table 1. Symmetrical anchors.

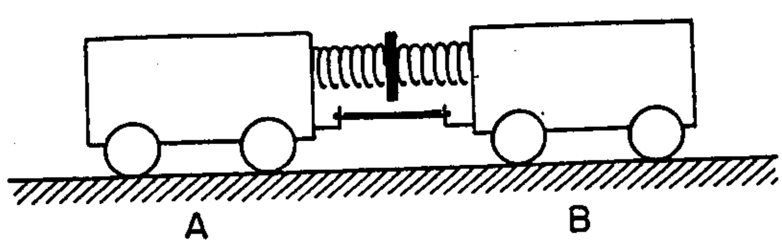
Problem	*Answer*	*Percentage correct (Belief score)**
(9) Carts Problem I	A moves left (part a) B moves right (part b) Same speed (part c)	82 (75) 87 (77) 96 (83)



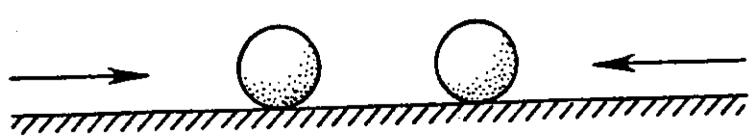
(11) Skaters problem	A moves left (part a) B moves right (part b) Same speed (part c)	86 (77) 84 (75) 97 (83)



(12) Carts problem III	A moves left (part a) B moves right (part b) Same speed (part c)	85 (72) 86 (74) 94 (79)



(13) Billiard ball force problem	Forces are equal	82 (69)



symmetrical carts situations in attempts to help students overcome the misconception represented in the asymmetrical skaters problem.

In effect this means that anchors exist at two levels. At the first level naive students may agree on the correct answer to a particular example. However, this does not guarantee that they have in mind a usable anchoring conception. A second, more solid level is reached only when the anchoring example triggers a conception that is an extendable starting point for building the physicist's conception.

In observational terms we will say that an anchoring example is brittle for a particular student if it cannot be analogically extended to help the student make sense of the target by techniques such as bridging. We interpret this phenomenon as follows. An anchoring example is brittle because:

1. It contains a feature or aspect (such as symmetry) that must be altered in order analogically to extend the anchor to the target.
2. The student considers this aspect to be critical in the following sense: if the situation is changed so that the aspect is altered, the student no longer believes that the key relationship or predicted outcome is valid, even though it is still valid from the physicists' point of view. Even if one attempts gradually to transform the critical feature by small degrees via a bridging strategy, the student resists transferring ideas from the anchoring situation to the target situation.

A requirement for using a bridging approach is that one can always 'split the difference' with a conceptually intermediate situation. Metaphorically this requires a conceptual domain analogous to the real number line, where between any two reals one can always find another real. Thus one would expect to find brittle anchors when the student's conceptual domain is analogous to a discrete number line. In the present examples, it is unfortunate that students consider symmetry to be critical, since between symmetry and lack of symmetry there exists no intermediate state.

The potential brittleness of symmetrical anchors becomes important in attempts to develop a more principled way of generating anchoring examples. As shown in table 1, every one of the four symmetric situations in the diagnostic test was an anchoring example, with the exception of the colliding billiard balls, which came within one percentage point (belief score = 69%). Thus, although one could reliably predict that most students will answer correctly for symmetrical forces in symmetrical situations, some of these examples may be of little use in a teaching via analogy approach since, for many students, the key elements of the situation will no longer be present once the symmetry is broken. (See Brown and Clement (in press) for an analysis of a protocol involving a brittle anchor.) Although the test has given us a warning signal here, teaching experiments in which the examples are discussed are needed to determine whether a particular anchoring example is brittle.

Conclusions

Critique

It is likely that scores from an anchor diagnostic test given at the beginning of a physics course may in some cases be misleading later in the year. We have the impression from classroom observations that group anchor scores may rise during the year even when students have not had direct practice on the test questions.

Experiences in the laboratory and related topics in the course may produce this effect. Thus, the most appropriate time to test for anchors may be just before beginning a unit in which an anchor is needed.

Also, it should be noted that even when only 30% of the students believe an anchor with high confidence, it is still an anchor for those students, and therefore may be useful in instruction for them. This suggests that the use of multiple anchors in the classroom or individualized anchors in computer courseware may be able to reach different students in different ways. (See Schultz *et al.* 1987 for a description of a computerized teaching program that uses different anchors and bridges for different students.)

In addition, very short instructional interventions may in some cases raise belief scores significantly. If this is true, such examples that were not group anchors on a diagnostic test could still be used effectively in instruction. These are interesting issues for future research.

Implications

In conclusion, we have described some initial attempts to systematically map out the domain of positive, potentially helpful preconceptions. These results, in combination with numerous studies conducted on students' alternative conceptions, strongly indicate that physics students cannot be considered 'blank slates'. Fortunately, some of the students' prior knowledge can be helpful to learning if anchoring conceptions can be tapped and used appropriately (Brown 1987, Clement *et al.* 1987).

The situations in this study that were predicted to be anchors, but which turned out not to be, indicate that examples which teachers and curriculum developers take for granted as 'obvious' and helpful may be seen in a very different way by students. Research is required to determine whether the base-level examples used in textbooks and lessons make sense to students. Teachers can participate in this research task by collecting data on diagnostic questions in the classroom, or more informally, by having students vote in class on whether examples make sense to them. With this kind of feedback, teachers as well as researchers should be able to find more effective anchoring examples.

The use of anchors is an example of a more general educational strategy of starting from what the student already knows. As such, we suspect that anchoring examples may be important in teaching other subject areas besides physics. Although we have proposed some initial organizing theoretical and observational definitions, further research is needed to specify a theory of anchoring conceptions. Such a theory would attempt to answer at least two questions: (1) what are the characteristics of a strong anchoring conception (perhaps related to the origins of physical intuition), and (2) what characteristics indicate that an anchoring conception can provide the basis for conceptual change via analogical extension?

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References

BROWN, D. 1987, Using analogies and examples to help students overcome misconceptions in physics: A comparison of two teaching strategies. *Dissertation Abstracts International*, Vol. 49, p. 473A (University Microfilms No. 8805897).

BROWN, D. and CLEMENT, J. 1987 a, Misconceptions concerning Newton's law of action and reaction: The underestimated importance of the third law. In J. Novak (ed.), *Proceedings of the Second International Seminar: Misconceptions and Educational Strategies in Science and Mathematics*, Vol. III (Cornell University Press, Ithaca, New York), pp. 39–53.

BROWN, D. and CLEMENT, J. 1987 b, *Overcoming misconceptions in mechanics: A comparison of two example-based teaching strategies*. A paper presented at the annual meeting of the American Educational Research Association, Washington, DC.

BROWN, D. 1987, Using analogies and examples to help students overcome misconceptions in physics: A comparison of two teaching strategies. *Dissertation Abstracts International*, Vol. 49, p. 473A (University Microfilms No. 8805897).

CLEMENT, J. 1982, Students' preconceptions in introductory mechanics. *American Journal of Physics*, Vol. 50, pp. 66–71.

CLEMENT, J. with the assistance of BROWN, D., CAMP, C., KUDUKEY, J., MINSTRELL, J., PALMER, D., SCHULTZ, K., SHIMABUKURO, J., STEINBERG, M., and VENEMAN, V. 1987, Overcoming students' misconceptions in physics: The role of anchoring intuitions and analogical validity. In J. Novak (ed.), *Proceedings of the Second International Seminar: Misconceptions and Educational Strategies in Science and Mathematics*, Vol. III (Cornell University Press, Ithaca, New York), pp. 84–97.

CLEMENT, J. Use of physical intuition in expert problem solving. In D. Tirosch and S. Strauss (eds.), *Implicit and Explicit Knowledge, an Educational Approach* (Ablex Publishers, Norwood, NJ) (in press).

DRIVER, R. and ERICKSON, G. 1983, Theories-in-action: Some theoretical and empirical issues on the study of students' conceptual frameworks in science. *Studies in Science Education*, Vol. 10, pp. 37–60.

HALLOUN, I. and HESTENES, D. 1985, The initial knowledge state of college physics students. *American Journal of Physics*, Vol. 53, pp. 1043–1055.

MCDERMOTT, L. 1984, Research on conceptual understanding in mechanics. *Physics Today*, Vol. 37, pp. 24–32.

MINSTRELL, J. 1982, Explaining the 'at rest' condition of an object. *The Physics Teacher*, Vol. 20, pp. 10–14.

SCHULTZ, K., MURRAY, T., CLEMENT, J. and BROWN, D. 1987, Overcoming misconceptions with a computer-based tutor. In J. Novak (ed.), *Proceedings of the Second International Seminar: Misconceptions and Educational Strategies in Science and Mathematics*, Vol. III (Cornell University Press, Ithaca, New York), pp. 434–447.

VIENNOT, L. 1979, Spontaneous reasoning in elementary dynamics. *European Journal of Science Education*, Vol. 1, pp. 205–221.

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Appendix: Anchor diagnostic and results.

For each question students were asked to mark their confidence on the scale below:

0	1	2	3
Just a blind guess	Not very confident	Fairly confident	I'm sure I'm right

Each choice is marked with two numbers as follows: Percentage of correct answers (percentage of correct answers with high confidence = belief score). The data were collected from a total of 137 students ($n = 137$). * indicates a correct answer.

Forces from static objects

1. *Log problem*: Mr T rides on the front of a runaway boxcar which then runs into a *stationary* car carrying a single large log. Mr T's chest meets the log head on, starting the log car in motion. During the collision the log:

*70 (53) Exerts a force on Mr T's chest.
30 (17) Exerts no force on Mr T's chest.

2. *Dictionary problem*: When you hold a very heavy 30 pound dictionary perfectly still in your hand, gravity exerts a downward force of 30 pounds on the dictionary. When holding it perfectly still, your hand:

*80 (65) Pushes up on the dictionary.
20 (14) Does not push up on the dictionary.

3. *Fist problem*: You hit a brick wall as hard as you can with your fist. When your fist hits the wall:

*40 (22) The wall exerts a force on your fist.
59 (36) The wall does *not* exert a force on your fist. The wall is just in the way.

4. *Hand on spring problem*: You push down on a bed spring with your hand. After you push the spring down 4 in, you hold the spring down, keeping your hand still. When holding your hand still against the pushed down spring, does the spring push back up on your hand?

*93 (80) Yes
7 (4) No

5. *Roller skate wall problem*: You are on roller skates and stand facing a wall. Your face is very close to the wall, and the tips of your skates are pointed forward. You then quickly extend your arms, pushing as hard as you can on the wall. When you push on the wall you:

*75 (64) Move to the left and roll for a ways
21 (14) Move to the left for a very short distance
2 (1) Stay where you are

6. *Tennis ball problem*: A tennis ball hits a brick wall and bounces off. When the ball hits the wall:

*39 (27) The wall exerts a force on the ball causing it to change direction.
58 (36) The wall does not exert a force on the ball. The wall is just in the way.

Newton's third law in dynamic situations

7. *Rowboat problem*: Two small boats float freely on a perfectly calm pond. They are three feet apart. When Suzie jumps from boat 1 into boat 2, *boat 1* will

7 (6) Move to the right
*91 (84) Move to the left
2 (1) Remain stationary

8. *Log problem II*: An insane criminal has captured Mr T and gives him a choice—he can be on the moving boxcar in drawing 1, or he can be on the stationary boxcar in drawing 2. In words, the two situations are:

1. The boxcar (moving at 20 mph) hits the stationary log car, starting the log car in motion.

In both situations, Mr T's chest meets the log head on. *Both cars are free to roll*, and *both weigh one ton*.

Mr T's chest would:

8 (4) Feel more force in situation 1.
38 (22) Feel more force in situation 2.
*53 (43) Feel the same force in both situations.

9. *Carts problem I*: Two identical carts resting on a smooth level floor are tied together by a rope. Between the two carts there is a compressed spring. The spring is not attached to either cart. The rope is cut and the spring stretches to its normal length and falls to the ground. When the rope is cut in the middle:

1. 8 (3) A moves to the right (→)
*82 (75) A moves to the left (←)
8 (4) A remains stationary
2. 87 (77) B moves to the right (→)
6 (0) B moves to the left (←)
6 (4) B remains stationary
3. 0 (0) A moves faster
2 (1) B moves faster
*96 (83) Both move at the same speed

10. *Carts problem II*: Two identical carts resting on a smooth level floor are tied together by a rope. Cart A has a spring *attached* to it, which presses up against cart B as shown below. The spring is *not* attached to cart B. The rope is cut and the spring stretches to its normal length. (Note: Because the attached spring adds a little weight to cart A, a small extra weight is added to cart B to make their weights equal again.) When the rope is cut in the middle:

1. 5 (3) A moves to the right (→)
*70 (55) A moves to the left (←)
23 (13) A remains stationary
2. *86 (69) B moves to the right(→)
5 (1) B moves to the left (←)
7 (4) B remains stationary
3. 17 (11) A moves faster
50 (29) B moves faster
*32 (23) Both move at the same speed

11. *Skaters problem*: Two roller skaters of *equal weight* and *equal strength* are facing each other *standing still*. The floor is very smooth and both skaters can roll easily. Both roller skaters hold their skates straight, so *both are free to roll* forward or backward. Please answer the following three questions for the case when *both push with the same effort*.

1. 3 (1) A moves to the right (→)
*86 (77) A moves to the left (←)
9 (5) A remains stationary
2. *84 (75) B moves to the right (→)
6 (4) B moves to the left (←)
8 (6) B remains stationary
3. 0 (0) A moves faster
0 (0) B moves faster
*97 (83) Both move at the same speed

12. *Carts problem III*: Two identical carts resting on a smooth level floor are tied together by a rope. Each cart has an identical spring *attached to it*. As shown below these springs press up against a board that is between the carts. The rope is cut and the springs stretch to their normal lengths, allowing the board to fall to the ground. When the rope is cut in the middle:

2. *86 (74) B moves to the right (→)
   4 (2) B moves to the left (←)
   6 (2) B remains stationary

3. 1 (0) A moves faster
   1 (1) B moves faster
   *94 (79) Both move at the same speed

13. *Billiard ball force problem*: Two *billiard balls* that *weigh the same* move toward each other with equal speeds and collide head on. Which of the following is true at the moment they collide?

*82 (69) Each ball exerts a force on the other, and the two forces are equal in size.
14 (11) The two balls exert forces on each other, but the forces are not necessarily equal in size.
1 (1) Neither ball exerts a force on the other.

Frictional forces

14. *Hairbrush problem*: You have two identical hairbrushes shown in drawing 1 below. You clamp one down tightly on a table, and pull the other one across it so that the bristles mesh. The bristles bump and bend each other as shown in the magnified drawing 2 below. You pull the top brush to the right. Does the *upper* brush exert a force on the *lower* brush?

3 (1) No.
*85 (50) Yes, it exerts some force to the right on the lower brush.
12 (8) Yes, it exerts some force to the left on the lower brush.

Does the *lower* brush exert a force on the *upper* brush?

42 (23) No.
13 (4) Yes, it exerts some force to the right on the upper brush.
*44 (21) Yes, it exerts some force to the left on the upper brush.