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Imagistic Processes in Analogical Reasoning: Conserving Transformations and Dual Simulations

John J. Clement (clement@srri.umass.edu)
Scientific Reasoning Research Institute
College of Natural Sciences and Mathematics
and School of Education
Lederle GRT 434
University of Massachusetts
Amherst, MA 01003 USA

Abstract

The classical theory of analogical reasoning focuses on mappings between discrete symbols as the mechanism of analogy evaluation and transfer. This paper introduces several other analogy evaluation strategies discovered in expert reasoning protocols: bridging analogies, conserving transformations, dual simulations used to detect perceptual-motor similarity, and overlay simulations. These findings provide evidence for the hypothesis that certain analogical reasoning processes can be imagery based.

Earlier work on higher order reasoning has indicated that expert subjects use various methods to generate analogies spontaneously when solving difficult problems (Clement, 1988), and that evaluating the validity of such analogies is essential to using them (Clement, 1989). That is, even if one has generated a confidently understood analogous case, one must evaluate one s confidence in the validity of the analogy relation to have confidence in transferring results to the target. The classical theory of analogical reasoning (Gentner, 1983; Holyoke and Thagard, 1989; Forbus, et al, 1997) focuses on mappings between discrete symbols as the mechanism of analogy evaluation and transfer. This paper examines several other analogy evaluation strategies observed in expert think aloud protocols. The data base for the study comes from professors and advanced graduate students in scientific fields who were asked to think aloud about a variety of problems. This paper focuses on two mathematicians solving physics problems they found difficult. By focusing on problems with which they were unfamiliar (i.e., a problem on the frontier of their own personal knowledge). it is plausible that the thought processes analyzed will share some characteristics with hypothesis formation and model construction processes used on the frontiers of science.

An example of a problem where analogy evaluation is important is the Sisyphus problem in Figure 1A : You are given the task of rolling a heavy wheel up a hill. Does it take more, less, or the same amount of force to roll the wheel when you push at x, rather than at y? Assume that you apply a force parallel to the slope at one of the two points shown, and that there are no problems with positioning or gripping the wheel. Assume that the wheel can be rolled without slipping by pushing it at either point.

One expert subject proposed the analogy that the wheel acts like a heavy lever perpendicular to the slope, with its fulcrum at the point of contact. Intuitively, the lever would be easier to move by pushing at X, suggesting that the same would be true for the wheel. But in the wheel the point of contact is moving, and ordinarily lever fulcrums do not move. In addition some subjects assume that the fulcrum should instead be at the wheel s center. Therefore the evaluation of the validity of the analogy relation (shown as the dotted line between A and B in Figure 1) was in question. This is distinguished from the subject s confidence in his understanding of the analogous case B itself, which was quite high in this case.

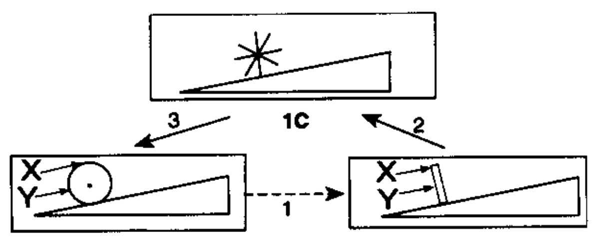

Figure 1: S2 s lever analogy for the wheel

Bridging Analogies

One method for evaluating this analogy used by this subject was the bridging analogy shown in Figure 1C of a spoked wheel without a rim. By breaking the problem of confirming a farther analogy into the problem of confirming two closer analogies. such a bridge can make it easier to develop confidence that the wheel does work like the lever in Figure 1B (a correct analysis). Bridging analogies are defined as occurring when the subject finds or generates an intermediate case which shares features with both the target and source analog. Their value has been documented previously in a number of expert problem contexts and in instructional applications (Clement, 1986). While it can be very helpful to subjects, bridging in itself is an incomplete strategy for analogy evaluation, since each half of the bridge must itself be evaluated. Therefore bridging is most useful in conjunction with other evaluation methods and it adds to, rather than reduces, the number of tasks to be performed.

This raises the problem of why experts bother to consider bridging cases at all, since they seem to create more work.

Conserving Transformations

In this section I present examples of a second evaluation strategy called conserving transformations and argue that it is distinctly different from the commonly cited method of matching discrete features for evaluating an analogical relationship. A paradigmatic case of a conserving transformation (although he did not identify it as such) is Wertheimer's method for determining the area of a parallelogram by cutting one end off and moving it to the other end to form a rectangle. A transformation is an action that changes a system 1 to system 2. If 2 is the same as 1 with respect to a feature or relationship R, then the transformation conserves R. An example of a conserving transformation in the Sisyphus problem occurred when subject S7 changed the problem to an analogous one involving an almost-vertical cliff with gear teeth:

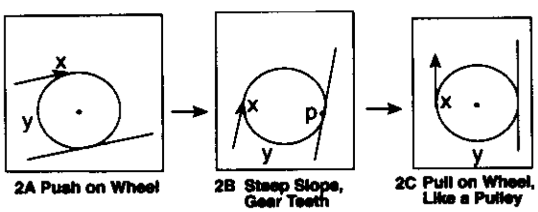

Figure 2: Wheel analogy series of S7

(Brackets in transcripts denote interpretations from viewing tape, while parentheses denote observed actions.)

01 S: Suppose it were tilted steeply and you did that; so steep as to be almost vertical. (Draws Figure 2B). It seems like it [the wheel] would skid out from under you the other way [down along the cliff]. This (moves hands as if turning an object clockwise) would get away from you here [at point p]. Let's assume it's gear toothed [gear teeth on the wheel and the cliff] and that it won't slip.

The change from situation A to B in Figure 2 appears to be a double transformation consisting of: the change of slope, and the addition of gear teeth. One can define the "targeted relationship" as the one for which an explanation or prediction is sought in the target situation (e.g. the relation between the force required and its location on the wheel). In his further work on the problem S7 never questions the validity of these transformations, and assumes that the targeted relationship in the problem situation is not affected by them. One can surmise that this occurs because the gear teeth transformation is a standardized one in physics and both are intuited to be irrelevant to the relationship of interest in the problem, i.e. they are conserving transformations. The origins of this kind of intuition have been studied since Piaget s early conservation experiments but are still poorly understood. (Case 2C will be discussed later.)

The hand motions over the drawing here provide one source of evidence on the use of dynamic imagery. Although the drawing can be an external support for a static visual representation, it does not depict movements, so it is reasonable to hypothesize that the subject is performing a mental imagistic simulation of the wheel slipping down on the cliff. The change in slope simplifies the problem by changing it to one in which forces act mostly along only one dimension.: upward and downward. Since the problem already specified no slipping, the gear teeth do not add new information but may help in imagistically simulating what will happen in the analogous case. Thus they may be an example of what I have called an imagery enhancement strategy (Clement, 1994, 2003).

The transformations appear in this case to be a means of both generating and evaluating the new analogy. Clement (1988) found that of a collection of 31 spontaneous analogies generated by ten experts, a greater number of analogies were generated via such transformations than those generated via an association to another case already in memory. However, the present paper focuses on the possible analogy evaluation function of transformations rather than on their analogy generation function.

Spring problem. A more substantial transformation is illustrated by the passage below from S2's solution to the following spring problem: A weight is hung on a spring. The original spring is replaced with a spring made of the same kind of wire, with the same number of coils, but with coils that are twice as wide in diameter. Will the spring stretch from its natural length more, less, or the same amount under the same weight? (Assume the mass of the spring is negligible.) Why do you think so? Earlier this subject has considered long and short horizontal bending rods made of the same wire as the spring and bent by hanging the same amount of weight on one end as an analogy for the spring problem (Figure 3B). Knowing that the longer rod will bend more suggests to him that the wider spring stretches more. In the following passage he evaluates that analogy by speaking of rolling up the bending rod into a spring (Figure 3C is discussed later):

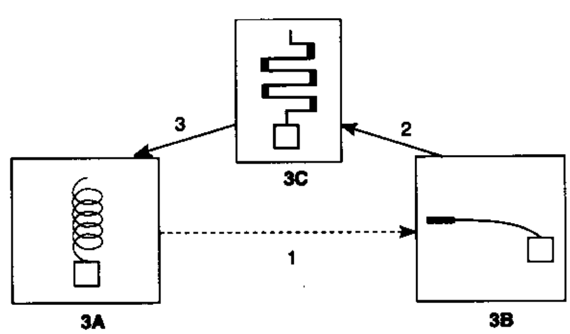

Figure 3: Rod analogy and zigzag bridge of S2

102 S: You can imagine a spring...and you know...there's no difference between the top and the bottom. It's a symmetric situation

105 S: You take your [straight, horizontal] wire, you say 'OK, you think it's the bending that does it. Well, then

let's bend it [by pulling down on one end of the straight wire]. And then let's roll it up [around a vertical axis] to make the spring. And you get a spring which stretches more and more at the bottom. The loops are wider apart!

107 S: Stretch it [a normal spring]: you don't get this increase of the distance between the loops toward the bottom. You just get a uniform stretching. And therefore the stretched spring cannot be understood as a rolled up bent spring.

This argues against bending as the source of stretching. Here the subject describes a very explicit spatial transformation between the spring and the rod. The sequence is: he generates the rod analogy; he simulates bending in the rod; he evaluates the analogy by transforming it back into a spring; there is a conflict with a known property of the spring, and he discounts the rod analogy. This evaluation is extremely valuable in that it gives him information arguing that the conjectured mechanism of bending is invalid. (In fact springs stretch primarily via twisting in the wire, not bending.) Note the imagery report in line 102. These passages suggest the attempt to use a visual transformation to evaluate the validity of a tentative analogy. The evaluation is influential in that it leads to discounting the validity of the analogy. Griffith, Nersessian, and A. Goel (2000) have also designed and investigated a computer program which successfully accounts for a number of features of this protocol and others collected for the spring problem. Transformations played an important role in modifying and improving faulty analogies or models in their program. However, they did not examine the role of conserving transformations as a means for analogy evaluation.

Prior to these sequences the subject had generated not only the analogous rod case, but what appeared to be a complete mapping of symbolic features between the rod case and the spring case. Bending, length, and slope, in the rod were mapped onto stretching, width, and slope in the spring. The relation of <greater length causes greater bending in the rod> had been mapped to the sought-after relation of <greater width causes greater stretch in the spring>. Therefore the transformations above do not appear to be adding any new elements to the mappings. Rather, they seem to be increasing the subject's confidence that he has found an important visual mismatch in the slope feature. They are new ways to *arrive at* the same mappings. That is, the transformations are a *means* to determining a match or a mismatch as the outcome, not just the notation for a mismatch as read off from two different lists. The notion that the transformation should be conserving is quite plausible. If the main mechanism is bending, this "winding up" transformation is locally perpendicular to the bending, therefore it could very well be a conserving transformation. Instead of transferring the result from the base to the target by using an explicit set of correspondences, in the present model this can be simply read off from an image derived from the imagistic results simulated in the base being transformed back to the target. Thus the conserving transformation strategy is a process that can work independently from an explicit feature matching process.

A traditional approach to analogy evaluation focuses on determining that multiple similarities between the base and target are sufficiently important. In contrast, a conserving transformation strategy need only focus on determining that a single transformation from base to target is sufficiently unimportant (irrelevant to the targeted relationship). This may mean that confirmation of an analogy via a conserving transformation can require considerably less work than confirmation via mapping.

Dual Simulation

Case 1. There is evidence in the protocols for a very direct strategy for analogy relation evaluation termed "dual simulation". A brief example that hints at this possibility follows where S2 says:

(Line 23) "Surely you could coil a spring in squares, let's say, and it would behave more or less the same".

There is not very much data in this statement, but it is plausible that the subject created an image of a square spring, simulated the effect of hanging a weight on it, and found this to be similar to the image of hanging a weight on a normal spring. However, the resolution of the perceived similarity appears to be at a low level of detail.

It is doubtful that his conclusion here is from "looking up a fact in memory", because of the novelty of the square case. (Later simulations by S2 with the square coil lead to imagining one side acting like a wrench to twist the next side. This produces an Aha episode with the insight that torsion is a major mechanism of stretching in the spring, and predicts correctly that the wider spring will stretch more, but that is the topic of another study (Clement, 1989)).

Dual simulation depends upon the process of **imagistic simulation** discussed in Clement (2003). That article found evidence for such an internal process from several observation categories for external behavior: **personal action projections** (spontaneously redescribing a system action in terms of a human action, consistent with the use of kinesthetic imagery), **depictive hand motions**, and **imagery reports**. The latter occurs when a subject spontaneously uses terms like "imagining," "picturing," a situation, or "feeling what it's like to manipulate" a situation. In several of the present cases one sees **dynamic imagery reports** (involving movement or forces). None of these observations are infallible indicators on their own, but as multiple instances accumulate, they can be taken as evidence for imagery. Taken together with the subject s new predictions, the observations above can be explained via *imagistic simulations* wherein a somewhat general perceptual motor schema assimilates the image of a particular object and produces expectations about its behavior in a subsequent dynamic image, or simulation.

The process of **dual simulation** can be summarized as follows. Imagistic simulations of the target and the analogous case are each run in as much detail as possible. The dynamic images of the behavior of each system are then compared; and they may be inspected for certain aspects. If their behavior "appears" to be the same, the analogy relation receives some support, depending on the level of certainty in the comparison.

Case 2. More data is present in the following episode of an analogy to a two-dimensional spring made of zig-zagging wire that lies in a single vertical plane, shown in Figure 3C.

23 S2: I wonder if I can make the spring..which is a 2 dimensional spring..but where the action ..isn't at the angles...it's distributed along the length I have a visualization... Here's a .. a bendable bar, and then we have a rigid connector...(draws more bars connected in a zig zag, two dimensional shape). And when we do this what bends...is the bendable bars...and that would behave like a spring. I can imagine that it would.... it would stretch, and you let it go and it bounces up and down. It does all the things.

Here the conjunction of the dynamic imagery reports and the comparison of the two systems gives more support to the hypothesis that a dual simulation is occurring to compare the target and the zig zag cases. The dual simulation appears to establish the analogous case as being relevant and plausibly analogous in that its behavior is similar, at least at a gross level of qualitative behavior, to the target. But this does not tell the subject whether the two systems exhibit the same relationship between width and stretch. Thus in the above cases dual simulation appeared to serve only as a check on the initial plausibility of the analogy.

One then needs to be clear that dual simulation as an analogy evaluation strategy does not necessarily mean confidently simulating the targeted relationship in both base and target. In that case there would be no need for an analogy because the target could have been directly simulated on its own. However, the examples presented indicate that dual simulation can still help one determine whether the target and base are similar with respect to other important behaviors, thereby increasing one's confidence that the analogy is sound (or eliminating the analogy from consideration).

Overlay Simulation

Lever case: There is evidence for the existence of a more precise type of dual simulation that I term "overlay simulation" where the image of one simulation takes place on top of a second image. Although I have separated them in Figure 1 for clarity, S2 actually drew his lever analogy (Figure 1B) directly on top of the wheel (Figure 1A) and compared the movement of the wheel and the lever. This meant that the arrow symbolizing the application of a force by pointing to the top of the wheel was also pointing to the top of the lever. When two separate systems are represented as overlapping in the same external diagram with salient features aligned I term this an *overlay diagram.* This supports the interpretation that internal dynamic images of the two systems and their actions were overlapping in the same way. I call this hypothesized internal aspect here an *overlay simulation* as a special type of dual simulation. Presumably the alignment of key features made it easier for him to compare the expected movements and resistances of the wheel and the lever as he simulated each of them.

Spokes case: Overlay simulation may also be responsible for the power of S2 s "spoked wheel without a rim" bridging analogy shown in Figure 1C. For the spoke that is touching the ground, the spoke can be seen as a lever with its fulcrum at the ground. This means that the entire wheel of spokes can be seen at any one time as equivalent to a single lever, supporting the analogy on the right hand side of the bridge BC in Figure 1. This subject spoke of a tireless, rimless wheel. Again this is shown separately in figure 1C for clarity, but in fact the spokes were inscribed within the rim of a circular wheel in the subject s drawing. So on the other side of the bridge AC, the spokes are envisioned at the same size as the original wheel, and this may make it easy to sense via dual simulation that they behave in the same way as the wheel when a force is applied. In particular, the way the rimless spoked wheel "rocks" on each spoke over a short distance can be seen as similar to the way the original wheel rolls. That is, it appears, especially with many spokes, to have the same kind of motion in a mental simulation and therefore be amenable to the same type of analysis with respect to the causes of motion. Although such arguments must be bolstered mathematically to make them rigorous, as a form of heuristic reasoning, this type of qualitative argument can be quite compelling.

Pulley case: As a third example the case of the pulley analogy in Figure 2C was also used by S7 in the Sisyphus problem. He believes that perhaps the push needed at X on the wheel is smaller than at Y, similar to a pulley where the force applied to the end of the rope need only be half of the weight of the wheel. As part of an attempt to evaluate that analogy, S7 speaks and gestures as if alternating between seeing the same drawing (Figure 2C) as a wheel and a pulley, referring to it differently as one or the other in alternate fashion. Continuing from segment 01 above:

05 S7: What it feels like is the weight of it [wheel in Figure 2B]-; is pretty close to parallel with what you ve got if you go roll it with a complete vertical. It now begins to feel like a pulley...(Draws Figure 2C) What the vertical is over here no longer matters perhaps but we'll say it's er, gear toothed again.

06 S: ...And <u>you're over here pulling</u> like this [at x]. That <u>feels like you're on the outside of a pulley pulling up</u>

07 S: And since you say it doesn't slip, then this thing over here (points to line in upper right of Figure 2C and adds upward pointing arrowhead to it) must be providing the other half of it, something it feels, in which case it's a classic pulley; no, it can't be classic pulley. But it's, like a classic pulley in which now you only need half of the force. If the weight of the thing is 10 lbs. here, it feels like 5 would work here (writes 5 on upper left of C) and 5

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over here (writes 5 on upper right) as though it were a pulley So let's imagine it is a pulley.

08 S: [In] this new point of view, it feels like working at X [on the edge of the wheel] is better [than at the center].

The personal action projections and alternating references to both the wheel and the pulley systems while staring at the same diagram 2C provide initial evidence for an overlay simulation here that compares the system of rolling the wheel straight up a vertical cliff to the pulley system. Presumably it is easy in an overlay simulation to switch rapidly between simulations of the two cases, (which happens at least five times above). Again, although the imagery is probably assisted in this case by the drawings, the drawing cannot be providing perceptions of forces or motions involved, and so I hypothesize that these are imagined via imagistic simulations. Some evidence for kinesthetic imagery is indicated by personal action projection phrases like feels like you re on the outside of a pulley pulling up and you re over here pulling in the transcript, and such imagery is clearly not already enacted in the static drawings.

Later he expresses some reservations about the pulley analogy however: This rope wrapping around here..doesn t feel to me necessarily like...pushing (moves hand . to r.) on the outside of a wheel. But in the passage below he appears to reevaluate the analogy positively by (1) generating a bridging analogy; and (2) using overlay simulations by simulating different systems in alternating fashion using the original wheel drawing. Therefore this final example is more complicated because it combines these two strategies.

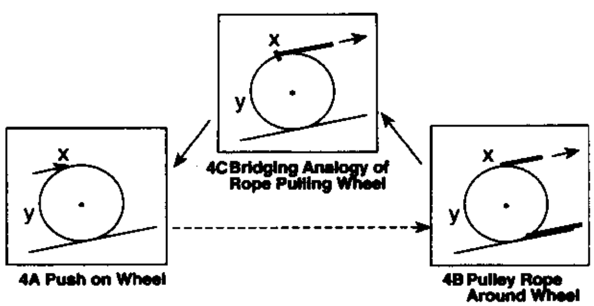

Figure 4: Second analogy series of S7

162 S7: (Looking at Figure 4A) I ve got my full (holds both hands out as if pulling a rope and shakes them slightly) power available- and where would I apply that? My instinct tells me [it is easier to apply force at] X again but that er, but again it's in terms of a pull and not a push. I'd have to get a grip.(closes eyes) Assuming that s not a problem, then pulling should be the same as pushing.. Seems clear that- (silently holds both hands out as if pulling a rope for 4 sec.)...So we attach a rope to one of the teeth [as in 4C but staring at the same Figure 4A], now it becomes more like the pulley problem (holds r. hand out as if pulling a rope for 3 sec) the teeth at the bottom are playing the role of-; the pulley doesn't look so bad after all. And you hang on for all you re worth up there, to keep it from rolling; to keep it balanced.

Figure 4C shows how a rope attached to the edge of the wheel at X can be seen as an intermediate bridging case between the original problem and the pulley case in 4B. Although I have drawn three cases in Figure 4 for clarity, in fact S7 used only Figure 4A while talking about the three cases: the pushed wheel, the pulley, and the rope attached to the tooth at X on the wheel. One can hypothesize that the internal overlay simulations create a context whereby the alignment between trajectories and forces in imagistic simulations of different cases, as well as the evaluation of the validity of the analogies between the cases, can be more easily made.

163 S7: Seems a lot easier than getting down here behind it [at "Y" in Figure 4A] and pushing. Why? because of that coupling pulley effect. It seems like it would be a lot easier to hold it here [at "X"] for a few minutes (Holds hands in pulling position) than it would be to get behind it yeah, my confidence here is much higher now, that it's right [easier to push at X] And so the pull--it just felt right with the pulley feeling. Now pushing (lays extended finger on paper pointing up slope at X in Figure 4A and moves it toward X) uh,.. it s got to be the same problem

178 I: Do you have a sense of where your increased confidence is coming from?

179 S: It s the pulley analogy starting to feel right.

The subject s thinking here appears to determine whether the forces on the edge of the wheel and on the rope from the pulley feel the same as he performs an imagistic simulation of each case. The bridging case in Figure 4C of a rope tied to the wheel at point X appears to serve the purpose of setting up two pairs of cases (base:bridge B:C and bridge:target C:A) that are closer to each other than AB. In other words the bridging case creates two analogy pairs that are more perceptually similar. This may be an important advantage if the evaluation of each pair is being done via a dual simulation of the cases. This provides one answer to the earlier question of why bridging can be useful to a subject even though it seems to add more work in creating additional analogy relations.

In fact the underlined references to feeling forces, personal action projections, and hand motions in the above passages provide evidence for the involvement of kinesthetic imagery and for dual imagistic simulations when he is comparing two cases. One can hypothesize that Figure 4A is acting as an overlay diagram for an overlay simulation. The use of an overlay diagram and references that the wheel problem solution felt right with the pulley feeling. supports the hypothesis that dual simulations are being used to evaluate these analogies. Thus this last example illustrates the combined use of overlay simulation (as a special kind of dual simulation) and bridging as analogy evaluation strategies.

Conclusion

In summary, rather than a single process for mapping elements in a discrete symbolic representation, a number of additional processes for evaluating an analogy relation have been identified, namely: bridging analogies, conserving transformations, dual simulations to detect dynamic similarity, and overlay simulations. Roughly, conserving transformations work by allowing the subject to detect the causal, perceptual motor irrelevance to a targeted relationship, of making a transformation on a case. Dual simulations work by allowing the subject to detect a causal, perceptual motor similarity between base and target. Overlay simulations are a special type of dual simulation in which the image of one case is overlaid and aligned on top of the other case to make comparisons more precise. An intermediate bridging case is a higher order strategy that can facilitate making one of the above processes easier to perform. The relationship of these strategies to discrete feature mappings is still unclear, but when subjects can articulate such mappings, that may add another important kind of precision to the process of analogy evaluation.

Implications. These findings add to previous evidence (Casakin and Goldschmidt 1999; Clement 1994, 2003: Craig, Nersessian and Catrambone, 2002; Croft and Thagard, 2002; Trickett and Trafton, 2002) for formulating the general hypothesis that many analogical reasoning processes can be imagery based. Also, the wheel problem transcript provides evidence that imagery and runnability are transferred from base to target. Clement (2003) extended this theme by examining evidence for the transfer of imagery and runnability from source analogues to explanatory models and hypothesized that this may be an important source of model flexibility, providing an argument for the importance of such processes. The importance of bridging analogies as an instructional technique has been documented previously (Clement, 1989), and the same may very well be true for conserving transformations (Wertheimer, 1959), and overlay simulations/animations. Thus much work remains to be done in this area.

Acknowledgments

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