THE JOURNAL OF THE LEARNING SCIENCES, 6(1), 61–89
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The Role of Extreme Case Reasoning in Instruction for Conceptual Change

Aletta Zietsman
Department of Physics
University of the Witwatersrand, South Africa

John Clement
Scientific Reasoning Research Institute
University of Massachusetts at Amherst

Although it is common to see extreme case reasoning included in lists of expert heuristics for problem solving, little work has been reported on the role that extreme cases can play in learning that leads to conceptual change. Evidence is presented from video tapes of think-aloud tutoring sessions to document the learning from extreme cases in a unit about levers for seventh graders. The observations support the view that one role of extreme cases is to provide a firm data point or comparison that helps students to establish an ordinal relation between two given variables. Two new additional roles for extreme cases in fostering learning are also identified: (a) their role in *activating an intuition,* often in the form of a *perceptual motor schema,* that is used in *constructing an imageable, intuitively, grounded, explanatory model* as opposed to an empirical rule; and (b) their role in *facilitating the formation of new causal variables.* Pending confirmation of similar effects in other subject areas, these roles are candidates for being included in a set of general learning strategies for science instruction. This illustrates the function that "learning-aloud" studies can play in documenting new types of learning processes and instructional strategies. The study highlights the importance for instructional design of research that uncovers students' existing knowledge structures and natural reasoning processes. The study suggests that explanatory model construction, causal relation construction, and concept formation can result from such instructional designs. The extent to which these three outcomes are evidence for strong conceptual change is also discussed.

Requests for reprints should be sent to John Clement, Scientific Reasoning Research Institute, Hasbrouck Laboratory, University of Massachusetts, Amherst, MA 01003. E-mail: clement@srri.umass.edu

The use of extreme cases as a reasoning strategy by experts has been documented in historical studies (Nersessian, 1992) and expert thinking-aloud studies (Clement, 1989, 1991). The use of extreme case reasoning as an instructional strategy is not as well documented (for an exception, see Zietsman & Hewson's, 1986, description of computer-based instruction that uses extreme cases in dissonance generating situations). This article examines the ways in which extreme case reasoning facilitated learning in a study of students' learning about levers. The purpose of this article is to use qualitative analyses of tutoring transcripts as an empirical base to develop grounded hypotheses for thinking about the roles extreme cases can play in learning and teaching.

The context of this study is a broader program of research and development in which the major goals are to:

1. Identify students' persistent misconceptions, or alternative conceptions as we prefer to call them, that conflict with currently accepted theory.
2. Identify students' positive preconceptions, or anchoring ideas as we call them, that are largely in agreement with the scientist's ideas.
3. Build on the students' positive preconceptions in designing experimental lessons that deal with their alternative conceptions.
4. Use tapes of tutoring trials with the lessons to criticize and improve them.
5. Analyze the processes occurring in successful lessons to develop more general models of learning and teaching principles.

This article is structured as follows. We first review descriptions from prior research of different mechanisms by which extreme cases may contribute to thinking. We next give an overview of the design of the tutoring study and then present protocol evidence to illustrate students' learning. Finally, in our discussions of student learning, we concentrate on the following two issues: (a) asking *whether* the extreme cases in an experimental lesson on levers facilitated learning and (b) constructing hypotheses grounded in case study data that explain *how* the extreme cases facilitated learning.

PREVIOUS RESEARCH ON EXTREME CASE REASONING

Although it is common to mention extreme cases as an expert heuristic in problem solving (Polya, 1954), little work has been reported on the roles that extreme case examples may play in learning. The traditional view of the way in which extreme cases contribute to thinking is as a *check* on a problem solution as follows.

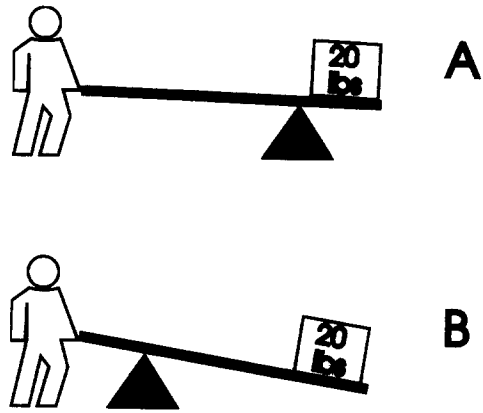


FIGURE 1 Extreme cases providing data points.

A Check on a Problem Solution

When the answer to a problem is expressed as a mathematical function, the correctness of the function can be checked conveniently by plugging in extreme values, such as zero or infinity, for the independent variable (Polya, 1954). Often, one can infer from the physical situation (independently from the mathematics) what the answer should be in this case and see whether the function gives the same prediction.

Generating a "Direction of Change" Functional Relation

A less commonly recognized, but important role in science is to use the extreme case to help generate a less quantitative, direction of change relation or ordinal function of the form "when X increases, Y increases (or decreases)." Such a relation can be inferred from the knowledge of two data points, ($X0$, $Y0$) and ($X1$, $Y1$), and the assumption of a monotonic relation between the variables (that Y always moves in the same direction as X increases). The comparison of the two lever situations in Figure 1 illustrates this point. In general, students know intuitively that it will be very easy to lift the load in Case A and very hard in Case B; hence, one may infer that, as the distance from the load to the fulcrum decreases, the force needed to lift the load would also decrease.

The extreme case can provide one of the "data points" needed to infer the more general direction of change relation. This role should be useful for learning basic ideas in science, in which such relations are ubiquitous. It can be argued that such relations also provide an intuitive underpinning for the understanding of mathematical relations in science. Weld (1990) developed an artificial intelligence (AI)

program that explores some of the issues involved in implementing similar types of extreme case reasoning.

Galileo's (1638/1954) famous extreme case of an object rolling on a plane of zero inclination also provides an intuitive starting point from which to develop and better understand a generalized, mathematical theory of motion. He reasoned that because a ball rolling on a plane that slopes down to the right will accelerate, a ball rolling on a plane that rises to the right will decelerate, and a ball rolling on a plane that is horizontal (parallel to the surface of the Earth) will neither accelerate nor decelerate.

Other Roles of Extreme Cases in Learning

Nersessian (1989, 1992) reviewed some other extreme cases used by Galileo and hypothesized that most thought experiments (of which, according to Nersessian, extreme cases are one type) work via a process of mental simulation. The mental simulation required is linked to perceptual processing and contrasted with propositional processing (Nersessian, 1992). In this article, we provide some support for viewing extreme cases as involving perceptual motor schemas, as opposed to merely invoking verbal rules (e.g., "more *A* means more *B*"). This provides some initial empirical evidence that is very compatible with Nersessian's view.

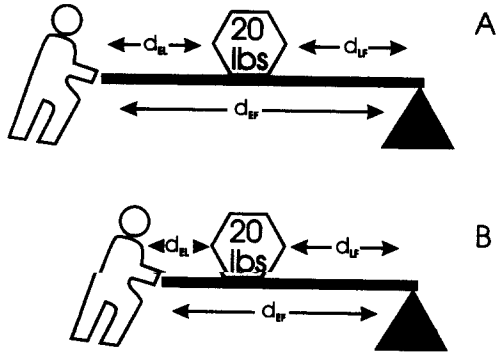
In this article, we also provide evidence for two other roles of extreme cases that appear to be important learning strategies but to our knowledge have not been discussed in the literature. These are the functions of (a) aiding in the construction of a qualitative, explanatory model for a system and (b) fostering the isolation of new causal variables in a system. We expose these roles via excerpts from thinking-aloud tutoring studies with seventh graders learning about the behavior of levers.

CHILDREN LEARNING CONCEPTIONS ABOUT LEVERS

An example of a generic levers question is given in Figure 2. (The labels for variables have been added to the figure and were not seen by the students.)

Students were told that the boards are considered to be light, but strong and inflexible (so as not to affect the outcome) and are hinged to the fulcrums. We refer frequently to the lever arms (dLF and dEF), as well as the load and the effort. The children were asked to refer to the fulcrum as the turning point, a term that was thought to be more descriptive and perhaps more suggestive for them. In pre- and posttest interviews and lesson interviews, we asked questions about such drawings of "generic" levers. In transfer questions in both the lesson and posttest, real levers, such as nail clippers, were also presented for discussion.

FIGURE 2 Example of a generic levers question:
Question: Where would it be easier to keep the board with the 20-lb load on it level?
(a) In case A.
(b) In case B.
(c) The same force would be needed in case A and case B.
Confidence Scale:
(a) Just a blind guess.
(b) Not very confident.
(c) Fairly confident.
(d) I'm sure I'm right.



The data reported in this article comes from a set of studies reported in Zietsman (1991). The set comprised a sequence of lesson design and evaluation activities including diagnostic studies (n = 242), exploratory tutoring (n = 37), and two systematic tutoring studies ($n = 6 \times 2$). Before and after the systematic tutoring sessions, clinical interviews were conducted on pre- and posttests with the all the participants. During the tutoring sessions, students were asked to think aloud as much as possible. Detailed analyses of transcripts from the first systematic tutoring group, focusing on why learning processes did or did not occur, were used to criticize and modify the instruction for the second group of students in tutoring interviews. The focus in this article is on an analysis of protocol data from this last cycle of tutoring interviews (called the experimental group) and specifically from sections dealing with Class II levers (those in which the load is between the fulcrum and the effort). We use this data to formulate grounded hypotheses about whether learning occurred and why.

Exploratory Work: Students' Preconceptions About Levers

Alternative conceptions about levers. Seventh-grade students from three schools in Western Massachusetts were interviewed in the first, more exploratory cycle of the study. The first set of interviews was aimed at identifying the students' preconceptions concerning levers. The term preconception is used to refer to the conceptions students appeared to have just before the study. In the diagnostic question comparing Class II levers shown in Figure 3, the only change in variable, from the expert's point of view, is the effort arm (dEF). One can conclude that, because

$$\text{Effort} \times \text{dEF} = \text{Load} \times \text{dLF},$$

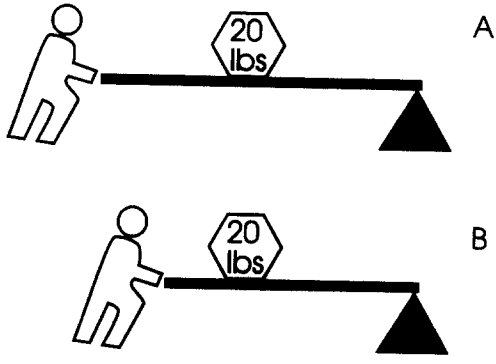


FIGURE 3 Comparing Class II levers: Target question.
Question: Where would it be easier to keep the board with the 20-lb load on it level?
(a) In Case A.
(b) In Case B.
(c) The same force would be needed in Case A and Case B.

the effort required to balance the load in Case B would be greater than that required in Case A.

However, many students exhibited one of two alternative conceptions (misconceptions) in their explanations for this pretest question. The first, held by the majority of students, is illustrated by EE5's explanation that a smaller load-effort (dEL) separation in Situation B resulted in less effort (I = interviewer; gaps in transcript line numbers indicate places where sections of the transcript were not included to save space):

3 EE5: Well, in B I think. [less effort]
5 EE5: its harder ... to control it and stuff [in A]

This "control" idea conflicts with accepted physical theory in this context. Using this "control conception," children appear to think that the closer one's hand is to the load, the easier it is to control—which is true in many everyday situations.

The second alternative conception, observed for the same question, also relates the load-effort separation and the effort (i.e., an increase in dEL) results in a decreased effort:

13 EE2: [It is easier in Case A], because the weight of the 20 lb is more spread out and farther away.
14 I: Farther away?
15 EE2: Farther away from where you would be holding it up and the other 20 lb would be right near you in Case B, and so it'd be harder to hold up.

Students using this conception in Class II lever situations would answer correctly. However, this nongeneralizable or limited conception can only be applied successfully to Class II levers. It gives an incorrect prediction, for example, when applied to the problem in Figure 1. One can therefore not consider this a good

intuitive conception from which to develop a principle of levers that is compatible to the physicist's, who tend never to refer to the distance between the effort and the load at all.

Examples of Positive Preconceptions: Spontaneous, Student-Generated Ideas That We Incorporated Into the Instructional Strategy

A number of students' spontaneous ideas emerged during the exploratory tutoring phase of the research that were eventually used in the final systematic tutoring sequence. We give two examples. The first is the idea that the "fulcrum can help" support the load and comes from Meaghan, when she discussed a problem similar to the problem in Figure 3, except that the triangular fulcrum was replaced by a table, and the boards were of equal length, with the weight closer to the table in B. (M is Meaghan and I is the interviewer.)

1	M:	I think once again it'd be easier in Case B, because
2		when the weight is farther away from the table
3		itself then it puts more stress on the board [pause]
		And [pause]
5		Oh wait. Actually, I think it'd be easier in A,
6		because you're closer to the weight and, like I said
		before with the seesaw, things like that would be
		easier [pause].
9		Uhm ... I'm not sure.
10	I:	What is puzzling you now?
11	M:	It seems that in B it'd be easier because it's
		closer to the table, so the table is holding more
		weight than you are
13		But in A if it would be so, it would change what I
		said for all the other's answers—all the other's.

In line 1, Meaghan gives a response that she marked "I'm sure I'm right" on the confidence scale while she was starting to talk. In line 5, she realizes that this answer is different from her others (five other lever situations, such as see-saws) and this apparently caused dissonance (lines 13–14). In the other answers, she consistently used the "load closer to the effort, easier for the person" misconception (the control conception). In line 11, the idea of "a fulcrum helping," inferred from Meaghan's statement that "it's closer to the table, so the table holds more weight than you are," was recognized for the first time in the study. We in fact eventually built this idea into our instructional sequence. This reinforces our view that hidden natural

TABLE 1
Abbreviations and Coding for Patterns of Speech

Abbreviation	*Patterns of Speech*	*Coding*
c_c	You just have more control over it Its harder to control [weight farther] [Easier closer] because the weight is right there	control conception
share model	[They are] sharing the weight evenly [They hold the same] its spread evenly	share model
f-h model	The thing holds the other 10 lb up This [fulcrum] is there pushing up	fulcrum-helps model
c_{po}	The man has to hold 20 lb, because there is just this [fulcrum] supporting it The other 10 lb is just resting on this [fulcrum]	powerless-object conception
c_l	[Easier] because the weight is ... farther away from where you would be holding it	limited/nongeneralizable conception

knowledge structures can be found and tapped in research driven curriculum development. A summary of conceptions found and typical patterns of speech used to identify them appears in Table 1.

In an even more striking example of a positive naive reasoning process, Brian (also in the exploratory interviews) spontaneously extended the Figure 2a situation into an extreme case. In this excerpt, Brian is thinking about a summary of "what makes it easier to hold the load." (B is Brian and I is the interviewer.)

54 B: Uhm ... what makes it easier? I think it makes it easier if—can I put this on the triangle? [makes drawing]

56 I think its easier when the load is on that end of the board, right near the block because

58 I: OK, let's call that the turning point

59 B: Yeah, near the turning point because then the force of the load, I mean is supported.

61 I: OK so you said one thing: the load is, if the load is near to the turning point—maybe you can write that down, that's one thing.

64 Load near to the turning point. [and writes]

65 I: And do you think, ah—is there anything else that matters?

67 B: Uh how long the board is. Because if you have like, because if you're holding that same board with the force there, and the same force was

right on top of there [indicates load], then
it'd be easier [in the first case]—or you were holding it
71 when the board went all the way to the other
end of the library you'd barely have to hold it
at all.

Brian, as opposed to Meaghan, gave the correct answers throughout his interview but supported by the nongeneralizable conception (i.e., the further the load is from the person, the easier). In the transcript excerpt presented, he differentiates for the first time the load-fulcrum distance as important, reasoning that the load is supported by the turning point (lines 59–60). His imaginary extension of the lever arm to "the other end of the library" (the interview occurred in the library) is a first-class, playful example of an extreme case and is obviously meaningful to him: "You'd barely have to hold it at all." This is one of the early findings that encouraged us to build extreme cases into our instruction. This reinforced our view that hidden natural reasoning processes can be found and tapped in research driven curriculum development.

DESCRIPTION OF LEVERS LESSON

The Class II levers lessons attempted to ground instruction in the students' naive ideas like those previously mentioned that were in agreement with scientific theory. Two teaching techniques employing this grounding principle were used in the lesson: (a) an analogy-based bridging approach (Clement, 1988, 1993) and (b) an extended sequence on reasoning from extreme cases.

Analogical Bridging Sequence

Target question. The diagnostic question in Figure 3 was also used as a target situation in the lesson. The function of the target question was to identify a student's initial and final knowledge state and to give the lesson developers a concrete subgoal in attaining student understanding.

Anchoring example. A much easier case (analogous to the target example) followed the target question and was viewed as an anchoring example. In the anchoring example for the Class II levers lesson, two people are sharing a 20-lb load (see Figure 4). Evidence for the existence of an anchoring example, defined theoretically as an example activating an intuitive knowledge structure in rough

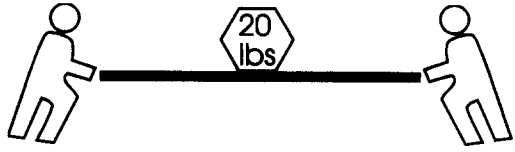
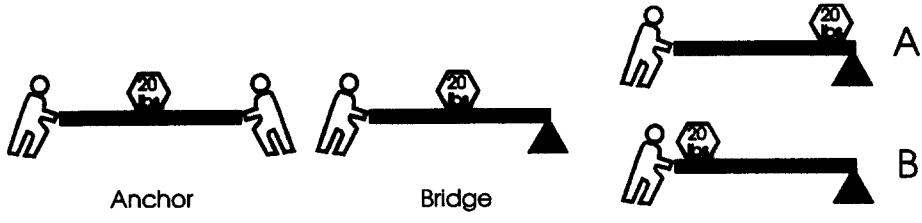


FIGURE 4 Anchoring example. Question: How much does each man have to push up to balance the 20-lb load in the center of the board?



ANALOGICAL BRIDGING EXTREME CASE COMPARISON

FIGURE 5 Initial lesson sequence.

agreement with accepted physical theory (Clement, Brown, & Zietsman, 1989), was provided by the study's diagnostic tests.

Analogical bridging. The bridging situation (illustrated in Figure 5) builds on the students' belief that the two people shared the load in the anchoring example. This sharing idea is extended analogically in the bridging example to a person and a fulcrum sharing the load. (In both the anchor and the bridging examples, the students were asked to estimate "how much of the load each person would hold.")

Initial extreme case. A comparison between two extreme case situations was presented at the end of the sequence to extend the ideas of sharing that may have been triggered by the anchor and bridging situations. Students were asked to judge which was the easier task of holding a 20-lb load, A or B, in Figure 5.

Introduction of terminology and extreme cases revisited. In the earlier exploratory tutoring sessions, the bridging sequence showed promise but was not effective on its own in producing transferable conceptual change in most students. A further problem was the emergence of an inert-objects-do-not-push (powerless-objects) conception that prevented some students from accepting the analogy between the anchoring and bridging cases. This conception is not compatible with a physicist's view—students referred to the fulcrum as having "no power," that it "cannot hold anything"—hence our term the *powerless-objects* conception. It is most likely related to the *normal forces* alternative conceptions discussed in

Minstrell (1982) and Clement (1993). However, all the children responded with a useful physical intuition to the final extreme case comparison (in Figure 6). Hence, extreme cases were used more heavily in the lesson in this study.

The extreme case comparison from this sequence (see Figure 6a) was revisited after a short introduction to different terms, such as *lever arm* and *fulcrum*. A question was added to focus attention on the lever arms as variables ("Does it matter what the distance from the man to the load is?").

The second extreme cases comparison (see Figure 6b) drew attention to the relevant variables in an even more focused way. In this question, the load-effort separation distance (which turned out to be a distracter for most students) is kept constant, thus encouraging the children to reason in terms of the effort and load lever arms.

Participants and Procedure

Grade 7 students from one school participated in the systematic tutoring. All completed the diagnostics tests, and from the test results a list of students holding alternative conceptions in conflict with accepted physical theory were identified. From the list, a pool of students were selected by their teacher to represent three levels of ability with half boys and half girls. One student from each of these six subpools was assigned randomly to a tutoring group and to a control group. This means that a stratified sample of six students was interviewed in each of the two conditions discussed in this article.

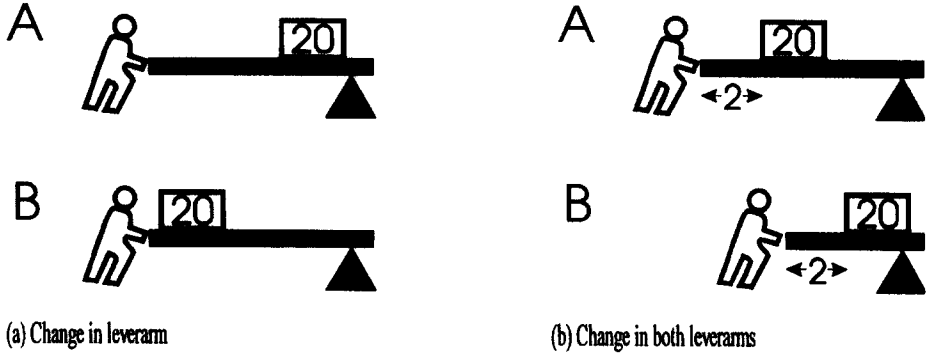


FIGURE 6 Extreme cases revisited sequence.
Questions for each comparison:

1. Where would it be easier to keep the board with the 20-lb load on it level?
   (a) In Case A.
   (b) In Case B.
   (c) The same force would be needed to keep each board level.
2. Does it matter what the distance from the man to the load is?

Pre- and posttests were given before and after instruction for the experimental group. The same pre- and posttests were given on separate days without intervening instruction to a control group.

QUANTITATIVE RESULTS

Although the primary results discussed in this article are qualitative, a brief examination of prepost gains provides evidence for the overall effectiveness of the lessons. Scores were assigned to the students' answers and their confidence in the answers as follows: (a) positive and negative values to correct and wrong answers, respectively; (b) a number (1–4) to the confidence level (rated from a "guess" to "sure I'm right" on a 4-point scale); and (c) multiplying the confidence level number with the appropriate symbol to indicate a correct (or not) answer. Thus, a student who guessed a wrong answer would score –1 on a question, whereas a student who was sure that he or she was right about a wrong answer, would score –4. Gain scores as the difference between post- and prescores were calculated for each student and summed for each group. Using this method, the total score for the experimental group was 77 and the control group's was 5.

Although the numbers of students were small, the Mann–Whitney test was applied to the hypothesis that the control and experimental group were identical with respect to their gains on the pre- and posttests. Using a criterion ($p < .01$), the difference between groups was significant ($U = 4, p = .007$). This gives us an initial reason to believe that learning and conceptual change occurred during the treatment. (The Appendix gives the individual scores for the experimental and control group students.)

However, in this article we focus on an qualitative examination of transcripts for evidence of key learning episodes. Through such case study analyses, we may be able to make grounded hypotheses about the efficacy of particular methods and why they were effective. We restrict our attention to the first section of the lesson, that dealing with Class II levers. We begin these analyses in the next section.

TRANSCRIPT ANALYSES: EVIDENCE THAT LEARNING OCCURRED IN THE LEVERS LESSON

A Case Study Illustrating One Student's Learning Path

As an illustration of one student's reasoning, we present a protocol in Figure 7 providing evidence that learning occurred in the Class II levers lesson. (The preconception we have described as control conception is abbreviated "cc" in Figure 7.)

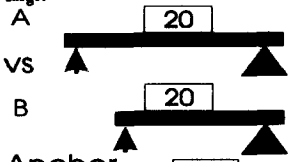
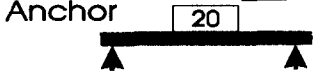
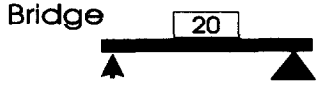
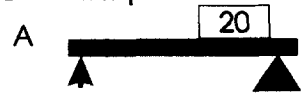
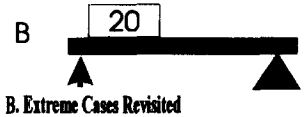
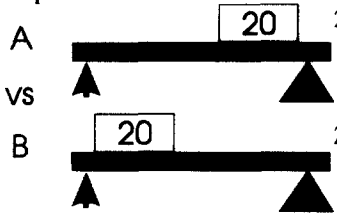
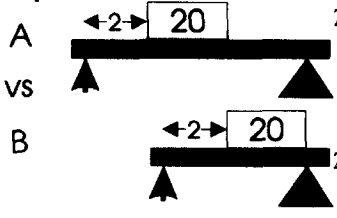
Lesson Element		Protocol Excerpt	Conception (inferred)
A. Analogical Bridging			
Target A VS B		001 ... B [easier] ... when 003 you're near the weight	C_c.
Anchor		017 10 lbs ... if they're each pushing 10 lbs. then together they push 20 lbs	share
Bridge		027 I guess it's 10 lbs again? 029 ... the thing [fulcrum] hold the other ... 10 lbs up.	fulcrum helps model
Extreme Case Comparison			
A VS		043 ... A, the weight is on this thing [fulcrum] resting on it? 044 But, here [B] he is holding the weight ... I think person B though ... [it is]	fulcrum helps model C_c.
B		048 easier for him to keep the weight from moving the board.	C_c.
B. Extreme Cases Revisited			
Comparison 1 A VS B		200 Ohhh. Well can I change my mind ... if I answered this [B] can I answer A? 204 ... the more the weight is on the turning point, and he's [a] holding less of it.	Change fulcrum-helps model
Comparison 2 A VS B		244 [For B] the weight is almost on the turning point so [he] would not have to hold very much of it ... 246 ... because the turning point is holding it up.	fulcrum-helps model

FIGURE 7 Protocol excerpts illustrating learning path for Student EE5.

Student EE5 used the control conception in her explanation for the preinstruction target question: She believed that a shorter Class II lever would make the task easier, because one would have more control over the load (line 005, Figure 7).

Initially, the analogical bridging sequence proceeded more or less as we intended: She was able to extend the anchoring example to the bridging situation and had apparently constructed the sharing model of the fulcrum intended (lines 017–029, Figure 7). However, her reasoning about the extreme case comparison situations in the bridging sequence regressed back to the control conception. This indicates that this new sharing conception is fragile and only applied in cases close to the anchor at first—it is in need of range extension and consolidation. She then used the control conception consistently until the second extreme case comparisons where she changed her mind (line 204, Figure 7). Her change of mind appeared to be abrupt, and it is apparent from the protocol that she is fully aware of this change.

This example provides evidence for learning that occurs during the time a participant is thinking about an extreme case. In line 204 (Figure 7), she responded with the correct answer and appeared to think in terms of a model of some of the weight being supported by the fulcrum, thereby reducing the effort. This is a very different conception from the control conception: There is evidence that she had considered her old way of thinking and a new way of thinking and that she preferred the new way, and that the new conception is applied later to other questions and used the following day on the posttest. The extreme case in line 200 appears to be a critical point in this protocol after which the student is able to overcome the control conception.

Other Patterns of Learning

The other students in the experimental group showed patterns of learning that are somewhat similar to that of Student EE5. Table 2 summarizes both the correctness of their answers (indicated by plus or minus signs) and the conceptions used (as inferred from the students' explanations) across the lesson:

c_c	control conception
share	idea that the load is shared in the symmetrical anchoring and bridging examples
f-h	(fulcrum-helps model) explanatory model of the fulcrum's action in lever situations
c_{po}	powerless-objects conception
c_l	a conception with limited use—an explanation for one lever class only

TABLE 2
Maps of Students' Learning: Class II Levers Lesson

	Levers Lesson: Class II Levers							*Posttest*
Student	*Target*	*Anchor*	*Bridge*	*Extreme Cases*	*Target*	*Extreme Cases*		*Target*
E1	–	+	+	+	–	–	–	–
	c_c	share model	share model	f-h model	c_c	c_c	c_c	c_c
E2	+	+	–	+	+	+	+	+
	c_l	share model	c_{po}	f-h model	f-h model	f-h model	f-h model	f-h model
E3	–	+	–	+	–	+	+	+
	c_c	share model	c_{po}	f-h model	c_c	f-h model	f-h model	f-h model
E4	–	+	+	–	–	+	+	+
	c_c	share model	f-h model	c_c	c_c	f-h model	f-h model	f-h model
E5	–	+	+	–	–	+	+	+
	c_c	share model	f-h model	c_c	c_c	f-h model	f-h model	f-h model
E6	+	–	–	+	+	+	+	+
	c_l	c_{po}	c_{po}	share model	c_l	f-h model	f-h model	f-h model

Note. See Table 1 for explanation of abbreviations.

There is evidence that all but one of the children had constructed more expert-like knowledge of levers by the end of the Class II levers lesson. The summary shows that most students started using more expert-like knowledge in an apparently consistent fashion from the first extreme cases comparison or the extreme cases revisited sequence onward. Although the anchor plus bridge appeared to facilitate temporary conceptual change in three students (EE1, EE4, and EE5), all reverted back to their preconceptions (or the powerless-objects conception) in the first extreme case comparison or in the postinstruction target question.

It appears therefore, from these maps of the children's progress during the lesson, that the extreme case situations were effective instructional components of the Class 2 levers lesson. The posttest target question was asked at the end of the 2-day-long intervention, and one can therefore not attribute the learning suggested in the posttest explanations to the extreme case components only. However, inspection of maps of students' explanations across the complete intervention shows that the first significant breakthrough for most students apparently occurred while considering the extreme case situations just described (Zietsman, 1991).

EVIDENCE ON HOW THE EXTREME CASES FOSTERED LEARNING: STUDENTS' CONSTRUCTION OF EXPLANATORY MODELS

In this section, we propose some hypotheses for why learning occurred as a result of the extreme case reasoning. For example, we present evidence that the extreme situations fostered the construction of an explanatory model that the fulcrum helps or supports some of the load in the lever situations. As an introduction to this discussion, we clarify our use of the term *explanatory model.*

Terms for Referring to Explanatory Models

The general term *mental model* is often used to denote a connected system of conceptions that produce declarative expectations about a certain target domain. We use the more specific term explanatory model as a particular type of mental model defined (with respect to a set of target situations to be explained) as

> a conception of (schema for) a structure hidden in (not directly observable in) the target situation. It is intended to explain (and often predict) behavior patterns observable in the target. For example, the physicist can explain the law of levers by appealing to the model of forces and torques in a force diagram and torque diagram.

A number of scholars have argued that such explanatory models in science are distinguished from empirical laws that merely describe or summarize patterns of observations (Campbell, 1920; Harre, 1961; Hesse, 1966). Campbell's often cited example is that merely being able to make predictions from the empirical gas law, stating that PV is proportional to T in terms of macroscopic, measurable quantities, is not equivalent to understanding the theoretical explanation for gas behavior in terms of an imageable model of billiard-ball-like molecules in motion. Unlike the empirical law, the explanatory model provides a description of a hidden process that explains how the gas works and answers "why" questions about where observable changes in temperature and pressure come from. This qualitative model can also be extended and used as the foundation for a separate quantitative model that gives more detailed explanations and predictions.

In the view of Hesse (1966) and Harre (1961), scientific explanatory models are usually based on an analogy to a conception of a familiar situation. In our study, we hypothesize that these conceptions are physical intuitions based on perceptual motor schemas. A perceptual motor schema is a nonverbal interpretation and control structure that is active over a period of time and that assimilates, generates expectations about, and sometimes manipulates objects. Schmidt (1982) described such schemas (or "motor programs") as general in the sense that a single program can produce a large variety of responses depending on the values of certain input parameters. The perspective that a motor schema can have generality through a pattern of actions and expectations over time with parameters adjusted to a particular situation in a process of tuning has precedents in Piaget (1955), Neisser (1976), and Schmidt (1982). (By perceptual motor, we mean perceptual or motor or both. We use Schmidt's description then as one example of how such a schema might be general.) More details on this topic are given in Clement (1994a, 1994b).

We suggest that some explanatory models can be developed as students think about particular key instructional examples that activate certain useful physical intuitions of this kind. We define an intuitively anchored explanatory model as an explanatory model that is based on or constructed from a familiar schema. This can be considered to be a recursive definition. Perceptual motor schemas are assumed to be familiar in a primitive sense and can therefore serve as the anchor for a hierarchy of models. We suspect that intuitive anchoring is a desirable feature of explanatory models, which, among other things, makes them more memorable and helps give the student a feeling of conviction about them. For example, in the case of the anchor for levers, the intuition about two persons (at each end of the board) sharing the load in the center of the board is used to develop the model of the fulcrum supporting part of the load in a real lever. This is an explanatory model (albeit a very primitive one) because the students must project a hidden relation of force or support into the fulcrum. It is an intuitively anchored model because its origin is tied to a familiar schema—in this case a perceptual motor schema.

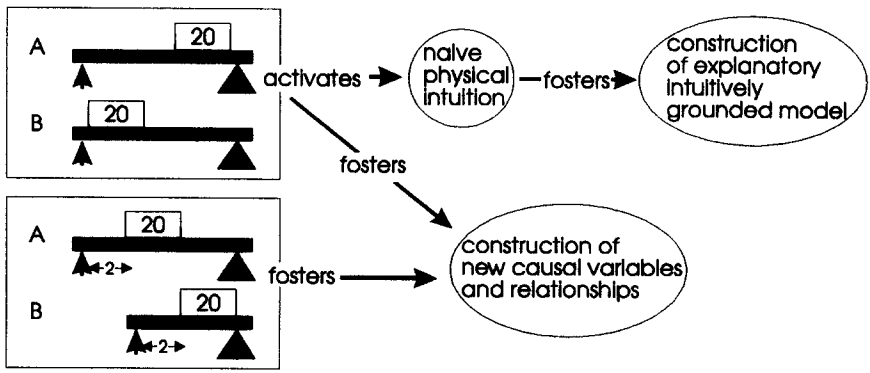


FIGURE 8 Learning facilitated by reasoning from extreme cases.

HOW LEARNING OCCURRED: REASONING FROM EXTREME CASES

In Figure 8, we outline hypotheses about ways in which the extreme case situations facilitated the construction of students' models (and thus new conceptions about the target situation).

The two major functions—the construction of an explanatory, intuitively anchored model and the construction of new causal variables—are now discussed.

Extreme Cases Fostered Model Construction

The intuition that was mentioned first by most students (when comparing the situations in Figure 8), is that Person B must "hold the whole 20 lb," an intuition probably informed by the children's prior experiences. Protocol examples of this kind of thinking are:

82 EE2: This man is gonna have to do 20 lb in Case B. Cause it's [the load] right on him.

98 EE3: I think here [B] the person would have to push all 20 lb.
100 Because it's [load] so close to him, that's where he has to hold it.

206 EE5: Yeah, [A] has to push up less.
208 But this guy [B] is holding the whole 20 lb.

A second or "fulcrum-holds-more" intuition could be seen as related to the first intuition. The following protocol excerpts also illustrate the fulcrum-holding idea:

83	EE2:	And this man [A] will probably have to do 5 to 1 because it [the load] is so far away from him and the block is holding up most of the weight
111	EE3:	I think that this man [A] will [find it easier]. The weight is closer to the turning point and that might help keep it up a little.
76	EE4:	Person A [will find it easier], because the weight is closer to the triangle on this one.
77	I:	And what difference does that make?
78	EE4:	the triangle has more pressure on it.
202	EE5:	Well, I think it'll be easier for him [A]
204		The more the weight is on the turning point, and he is holding less of it.

It is surprising that this intuition was triggered by merely considering the extreme case examples, particularly because half of the students had declared a few minutes earlier that the fulcrum could not "hold any of the weight" in the bridging example. Why are they suddenly able to attribute such an action to the same object? We suggest that the extreme case activates a dormant support or sharing conception previously built up from experience.

We propose that all the students mentioned previously have constructed an explanatory, intuitively anchored model of the *fulcrum helping:* explanatory in the sense that the reduced effort is explained in terms of what is to the child a hidden function of the fulcrum helping (previously inert and not exerting a force) and anchored in the naive intuitions described earlier.

In contrast to explanatory models, an empirical rule predicts the behavior of a target by specifying a relation between observable features of the target. Siegler (1978) described children's ideas about a closely related device, the balance beam, in terms of empirical rules. For example, one simple but not always correct rule would be observed when children are considering only the weights on either side of the fulcrum. If the weights are the same, the system will balance—if not, the side with the greater weight will go down. Such a rule does not represent an explanatory model because it refers only to an association between two observable features.

Figure 9 illustrates a schematic way to think about the use of empirical rules and explanatory models in the levers context. Figure 9a shows a representation for an

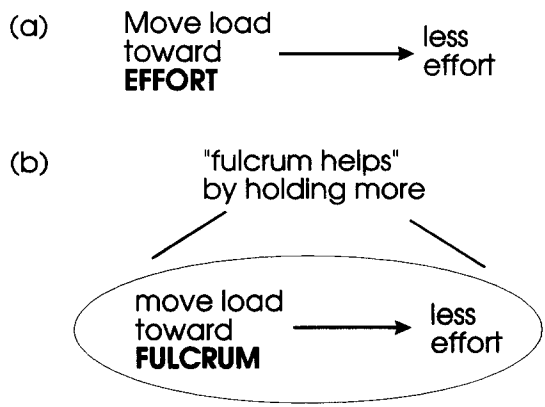


FIGURE 9 Rules and models for levers.

empirical rule that refers only to a correlation between observable features. The "fulcrum helps by holding more" conception in Figure 9b can be thought of as an explanatory model because it refers to a nonobserved factor that provides more detail in the causal chain in the form of an intermediate, mediating, hidden mechanism. Although it is a rather primitive one, this is a model that the students can project into their view of the system. As such, it gives an explanation for why moving the load toward the fulcrum leads to a smaller effort on the part of the person.

In coming to view an idea this simple as an explanatory model we are stretching the use of the term explanatory model to include more primitive cases than in our previous usage (Brown & Clement, 1989). This perceptual motor schema is the simplest, most elementary example of an explanatory model that we have studied, and as such it is an attempt on our part to connect two ideas in philosophy of science and cognitive studies of children.

Using Extreme Cases to Construct New Causal Variables and Relations

Using a new variable. In the pretest interviews, students focused strongly on a variable that experts ignore—the distance between the effort and the load. Students related the effort to the effort-load separation distance in two ways: (a) an increase in dEL results in a decreased effort and (b) a decrease in dEL results in a decreased effort.

We inferred from the protocols that the students were aware of, at most, the following variables before instruction: (a) the force exerted by the person, (b) the load, and (c) the effort-load separation distance. They never referred to the variable dLF (load-fulcrum distance) that is used by experts. Examples follow (the inferred variable is given at the end):

16	EE1:	because then the weight's closer to you.	load
		So, its probably easier to lift if	force
		the weight is closer to you.	dEL
14	EE2:	[B] ... because the weight of the 20 lb is	load
15		farther away from where you'd be holding it up	force
27	EE3:	[B] ... the weight is I guess	load
		closer to you	dEL
		maybe it's easier to hold	force
6	EE4:	[B] ... because you're closer to this [load]	load
			dEL
21	EE5:	[B] ... when you're closer to being underneath it	dEL
23		I think its easier to hold it	force
			load
5	EE6:	The man in A does not have to lift directly	force
		the 20 lb load	load
		[he] is lifting farther away from the 20 lb	dEL

The second comparison of lever situations involving extreme cases (Figure 8) was designed to initiate dissonance between the students' preinstruction connection of the effort-load separation distance and the magnitude of the effort required and the obviously crucial load arm differences in Cases A and B.

In addition, we had hoped that the fulcrum-helps model would be reinforced again. The last question was difficult for the children, because both lever arms are changed. The expectation was that they would rely on the fulcrum-helps model to explain Case B in particular—and this did happen. The students also focused on variables never mentioned before and constructed new relations between variables.

For example, Student EE3's reasoning about the preinstruction target question can be contrasted with her explanation for the extreme case comparisons (summarized in Figure 10).

The first extreme case comparison (lines 098–106, Figure 10) appears to lead her away from using a distracting variable (dEL). The result appears to be the formation of a new causal relation (i.e., the effort decreased; lines 102–104, Figure 10) because the load-fulcrum distance (dLF) decreased, as well as the application of a new variable, namely the load-fulcrum variable (dLF). This is the first time she has used this variable in a problem solution.

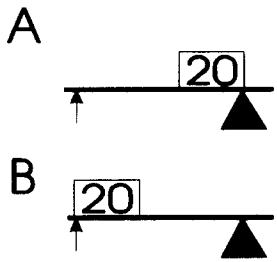
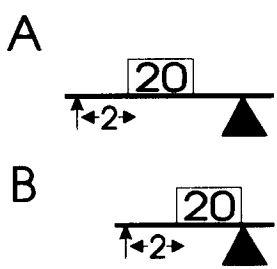
Lesson Element		Protocol Excerpt	Variables and Relationships
Extreme Case Comparison 1			
	098E$_3$:	I think here the person [B] would have to push 20 lbs	E >>
	100	...because it's [load] so close to him.	d_{EL} <<
	102	and here [A]... the person would have to push less.	E <<
	104	it's [load] closer to the think	
	105 I:	Closer to the?	
	106E$_3$:	Closer to the turning point	d_{LF} <<
Extreme Case Comparison 2			
	111E$_3$:	[Easier for B]... because the weight is closer to the turning point and therefore that might keep it up a little.	E << d_{LF} << thus f-h
		Whereas this person [A], the dis-tance here is so far away	d_{LF} >>
	112 I:	Distance from the load to the turning point you are pointing to?	
	113E$_3$:		
	114 I:	Right, from the, ah, yeah.	
	115E$_3$:	And what difference does that make? Well, I guess, ah, if it's (load) closer to the turning point then that might help keep it up.	d_{LF} << thus: f-h

FIGURE 10 Constructing new variables, constructing new relations.

In the second extreme case comparison (lines 111–115, Figure 10), the extreme case appears to not only draw out an intuition about the causal relation by providing a means for comparison but also the comparison encourages a focusing on the new (dLF) variable. In this example, the extreme case is used in combination with a second strategy of controlling for the distracting variable dEL. However, we saw that the new variable (dLF) actually was first applied in Comparison 1 for this student, in which no such control was used.

Five of the six students referred to the load-fulcrum distance (dLF) for the first time during an extreme case example in the instruction and four of these were in the second extreme case comparison only. It appears most likely that the role of the extreme case is to draw out the physical intuition that having the load close to the fulcrum shifts some of the job of support to the fulcrum. This interpretation is supported by the fact that all five students used the fulcrum-helps model. We

conclude from such cases that it is possible for an extreme case to be an aid in the construction of a new variable.

Articulating a new relation. In lines 111 and 115 (Figure 10), we see expressions close to a statement of a functional relation, such as: The closer the weight is to the fulcrum, the more it is supported by the fulcrum; for example, "because the weight is closer to the turning point and therefore that might keep it up a little" (line 111). These appear to emerge from the consideration of an extreme case.

In summary, we appear to have evidence for all three cognitive benefits of using an extreme case: the formation of new models, new variables, and new functional relations.

DISCUSSION

What Processes Underlie Learning From Extreme Cases?

In this section, we offer a theoretical interpretation for how extreme cases are fostering learning that is capable of explaining the case study data. The first possibility—the formally stated process given in the second section of this article on Extreme Case Reasoning—was: Such a relation can be inferred from the knowledge of two data points ($X0$, $Y0$) and ($X1$, $Y1$), and the assumption of a monotonic relation between the variables. However, the Grade 7 participants know nothing of monotonic functions, and therefore are almost certainly not deducing conclusions in this rigorous fashion.

A somewhat less formal view of extreme case reasoning is as follows: Participants possess information in long-term memory that allows them to look up an episodic memory of a an extreme case as a prior event in their experience (data point). They can then compare this memory to that of an ordinary case with respect to the two (presumably causally related) variables X and Y. Monotonicity takes the form of a natural, implicit assumption that simply amounts to trying the simplest relation first. In the case of levers, these causally related variables may be distance of the load from the fulcrum and force of effort required. The comparison leads to an ordinal relation of the form: An increase in A leads to an increase in B.

Schema activation leading to an ordinal function. However, we suspect that our students' conclusions have a more meaningful and direct semantic basis than the look up of memorized data points. We hypothesize that the drawing of the weight at the extreme, as near to the fulcrum as possible, activates a support schema, helping schema, or both as applied to the fulcrum. Such a schema allows a primitive form of mental simulation to occur in which the participant can imagine having to exert less force on the other end of the board. The activation of a general schema means that this can in effect take place as a thought experiment, without the

participant needing to have a particular episodic memory of the situation shown in the drawing. It may be that a monotonic function is present implicitly in the students' perceptual motor schema that generates the new simulation of the situation. That is, the rough, qualitative, functional relation is embodied in the schema itself and is an emergent feature of its activity. (A simpler example is that an estimator for "how hard to lift" is built into schema for picking up objects: The larger the object, the larger the force that is needed; Mounoud & Bower, 1974.) Once the extreme case triggers the helping or supporting schema, that provides a direct way of simulating the situation that is capable of providing implicit information that can be described explicitly by the statement that the closer the load is to the turning point, the more it will help to support it.

Formation of a new variable. The activation of this schema makes the fulcrum the new reference point for describing the position of the load, allowing the closeness to the fulcrum to become a variable. This then allows the construction of a new relation in terms of this variable. The following are the subprocesses we see as consistent with the transcript of E3 in Figure 10 just discussed.

1. Activating one or more perceptual motor schemas for helping or supporting, as applied to the fulcrum supporting part of the block. In our terms, this provides a primitive new explanatory model of a hidden influence in the situation.
2. The student uses the schema to simulate comparisons of values of effort for the extreme case in comparison with another case.
3. The schema also encourages the student to focus on a new variable.
4. The student is then capable of expressing a new ordinal relation in terms of the new variable.

This set of subprocesses also appears to be consistent with EE5's transcript in Figure 7. It provides an initial account for how the three identified benefits of extreme case reasoning are realized together in these transcripts. In this view, we have emphasized the idea of an *explanatory model*—the activation of a schema that interprets situations and projects imagined elements into it—rather than the idea of an algorithm for inferring a relation from two pairs of data points. The hypothesized process is *more complex than a simple look-up algorithm* in the following ways:

1. One of the variables has not previously been defined or even attended to by the participant, and this variable may need to be isolated explicitly and defined. (One way this could happen is if information in an analog representation is converted to linguistic information.)
2. This is consistent with the view that a perceptual motor schema may be the source of the physical intuition used in these extreme cases. Such schemas can

embody implicit functions to vary expectations and actions in accord with different situations (Schmidt, 1982).

3. The idea of the fulcrum helping is the participant's semantic interpretation of it—a meaning that he or she projects into the situation—as opposed to just an algorithm for calculation or the memory of an event.

4. The participants do not mention values of force or data points for the extreme cases. This supports the view that their comprehension of the ordinal relation between distance from the fulcrum and weight supported by the fulcrum is very direct, rather than an inference from two known data points.

5. The entire inference in this view is dependent on the perceptual motor schema. Such schemas may be quite different from a list of facts (see Clement, 1994a, 1994b; Neisser, 1976; Schmidt, 1982).

Extreme cases as schema activators. The following question remains: Why was this schema not activated by the nonextreme cases? The answer to this question may be related to the small domain size of naive conceptions noted by diSessa (1985). The range of situations that activate this and many more of the naive student's conceptions is either displaced from or simply much narrower than that of the expert scientist. Our task then becomes helping students to extend the activation range of the positive intuitions they have (in agreement with the physicist) from intuitive extreme case or anchoring situations to other more difficult situations. This is a major strategy taken in these lessons.[1]

CONCLUSIONS

Roles for Extreme Cases

The purpose of this article was to use qualitative analyses of tutoring transcripts as an empirical base to develop models for thinking about the roles and mechanisms that can make extreme cases a powerful learning tool. We began with the question of how extreme cases may be used for more than a check in simple problem solutions by playing a role in students' learning. We have confirmed that one role of extreme cases is to provide a firm comparison that helps one to establish a causal, direction-of-change relation (ordinal function) between two given variables. We have also identified two new roles for extreme cases in fostering learning, namely:

[1]In other cases, the schema may already be activated by the original problem. The extreme case may then serve as a situation that provides a clear contrast for a mental simulation using the schema—one that draws out implicit knowledge in a clear and unambiguous way with high confidence. The central view of extreme cases working by drawing implicit knowledge out of an existing schema remains the same.

(a) their role in activating a perceptual motor schema that is used to construct an explanatory model and (b) their role in contributing to the formation of new causal variables. In the case studies, these processes appear to be prerequisites for the desired role of forming and articulating a new ordinal relation. Pending confirmation of similar effects in another subject area, these roles are candidates for being included in a set of general learning strategies for science instruction.

These two additional mechanisms for learning via extreme cases add support to the following hypothesis. The conceptual change facilitated by this instructional sequence did not simply consists of the formulation or confirmation of a statement of the form: A causes an increase in B. Rather, it includes the construction of a new way of seeing lever situations in the sense that the students (a) are sensitive to new concepts or features of levers that they did not see earlier and (b) are able to imagine a hidden visualizable explanatory mechanism and project it into lever situations. This knowledge restructuring resembles the strong conceptual change requirements of Carey (1986) and Vosniadou and Brewer (1987): (a) the construction of new core conceptions, (b) establishment of new relations between conceptions, and (c) explanation of a different domain of phenomenon. We have provided some evidence for this, although it is debatable whether the students' introduction of the fulcrum as a significant, new element of the levers can be construed as evidence of the third requirement.

How Extreme Cases Lead to New Knowledge

We have hypothesized, on the basis of transcript data, that many extreme cases work by making it possible to tap implicit knowledge in perceptual motor schemas as the source of physical intuition involved. This view contrasts with views of extreme case knowledge as episodic memories and with more formal views of algorithms for making inferences from data points. In this article, we have provided some empirical support for the view of the role of extreme cases as activating flexible perceptual motor schemas, as a process prior to the formation of verbal rules (e.g., more A means more B). This view is very compatible with Nersessian's (1992) speculation that extreme cases in the history of science involve mental simulations in a thought experiment. It helps explain how students appeared to make breakthroughs from thinking about one or two examples rather than having to look for a pattern of observable events in many examples.

Implications for Teaching

The use of extreme cases and explanatory models of hidden mechanisms have been much more powerful teaching strategies than we had expected for this younger age group. We were surprised by these results because these strategies often appeared

to be more powerful than the analogical reasoning techniques we were counting on originally. Surprises like these testify to the power of descriptive, learning-aloud studies to criticize and improve teaching strategies.

Implied recommendations for curriculum development and teaching from this study are the following:

1. Draw out and listen to students' naive explanations to identify their preconceptions that conflict with the scientist's conceptions.
2. Use extreme cases as one way to tap positive preconceptions in students that agree with the scientist's view.
3. Investigate (in our case, in a research project) whether extreme cases have been chosen, so that they actually do activate schemas in students that allow them to make accurate predictions. A strong finding from our study is that one cannot assume that whatever makes sense to the teacher or researcher will also make sense to students.
4. Investigate whether students are seeing or attending to the same variables that the teacher or researcher are in the proposed situations.
5. Temporary, intuitively worded labels for variables, such as *load arm* and *turning point,* may help this process.
6. Design some extreme cases to create dissonance with alternative conceptions.
7. Allow time for and support the process of converting case comparisons into general statements of qualitative functions of the form: Increasing A leads to an increase in B.

Further Research Needed

There are a number of questions we have not been able to address with this data, such as: Now that we have some models for thinking about the roles and mechanisms that make extreme cases important in learning, can we design an experiment showing that when important extreme case examples are replaced by ordinary examples, less learning occurs? How do experts and students pick good extreme cases? Because many possible extreme cases could be generated, how do experts, teachers, and students pick the best one for a given circumstance? Sometimes an extreme case moves the example too far by moving it into a regime that behaves very differently from the intended problem. How do people evaluate whether the inferences they make from extreme cases are valid? These are among the many interesting questions for future work in this area.

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APPENDIX
Experimental and Control Groups: Pre- and Postgain Scores

Experimental Group Gain		*Control Group Gain*	
Student	*Summed Score*	*Student*	*Summed Score*
EE1	+12	C1	+ 4
EE2	+21	C2	0
EE3	+16	C3	0
EE4	+ 1	C4	+14
EE5	+18	C5	− 6
EE6	+ 9	C6	− 7
Group total	+77	Group total	+ 5

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