INT. J. SCI. EDUC., 1999, VOL. 21, NO. 9, 921–944

RESEARCH REPORT

Use of a computer simulation to develop mental simulations for understanding relative motion concepts*

James M. Monaghan, California State University, San Bernardino, USA, and John Clement, University of Massachusetts at Amherst, USA

Think aloud interview protocols from three high school post-physics students who interacted with a relative motion computer simulation presented in a predict–observe–explain format are analysed. Evidence is presented for: qualitative and quantitative difficulties with apparently simple one-dimensional relative motion problems; students' spontaneous visualization of relative motion problems; the visualizations facilitating solution of these problems; and students' memory of the on-line simulation used as a framework for visualization of post-test problems solved off-line. Instances of successful and unsuccessful mapping of remembered simulation features onto target problems are presented. Evidence from hand motions and other indicators suggesting that the subjects were using dynamic imagery in mental simulations during the treatment and post-test is presented. On the basis of these observations, it is hypothesized that for successful students, dissonance between their incorrect predictions and simulations displayed by the computer initiated the construction of new ways of thinking about relative motion, and that the memory of certain simulations acted as an analogue 'framework for visualization' of target problems solved off-line after the intervention. In such cases we find that interaction with a computer simulation on-line can facilitate a student's appropriate mental simulations off-line in related target problems. Implications for design and use of educational computer simulations are discussed.

Introduction

The article reports on case studies from clinical interviews which examined the use of a computer simulation to assist high school physics students in learning relative motion concepts. These case studies are presented to examine not only the measured outcome of a treatment, but learning events and factors which facilitate or hinder learning during the treatment. A central issue is whether simulations can help improve relative motion understanding. Subquestions here include whether students can use, off-line, what they have learned during interaction with a computer simulation on-line, and in particular whether interaction with computer simulations can facilitate mental simulations off-line. Other more general purposes of this study are to study helpful and detrimental modes of thinking that may occur when a computer simulation is used by students, and to identify any hidden pitfalls associated with computer simulation use. Because evidence concerning students' learning processes and reasoning with mental simulations requires a deep level of analysis, this study examines a few students in depth, rather than a more cursory examination of a larger number of students. By this means we hope to identify

International Journal of Science Education ISSN 0950–0693 print/ISSN 1464–5289 online © 1999 Taylor & Francis Ltd
http://www.tandf.co.uk/JNLS/sed.htm
http://www.taylorandfrancis.com/JNLS/sed.htm

factors which affect the efficacy of computer simulation as a pedagogical tool as well as factors salient to learning physics. Once these factors are identified, ways to collect data from larger numbers of subjects may be designed. Following a brief overview of relevant literature is a description of the study, protocols and analysis for three subjects, and a discussion of implications for relative motion understanding and pedagogical uses of computer simulations.

Student difficulties with relative motion problems and imagery

Exploratory research that we conducted in classroom and interview settings indicated that many students display considerable difficulty attaining basic relative motion concepts. Students displayed difficulties visualizing one-dimensional relative motion problems, an inability to solve functionally identical problems presented in different contexts, and difficulty understanding and using relevant technical language. We have noticed that students can have several methods for solving relative motion problems (see also Metz and Hammer 1993), including arithmetic algorithms with triggering rules (e.g. 'if the objects are moving toward each other then you add the speeds') and visualizations via imagery (e.g. 'picturing it in my head I think it will be going faster').

To describe this aspect of our theoretical framework and to say what we mean by 'imagery', we begin with 'the mental invention or recreation of an experience that in at least some respects resembles the experience of actually perceiving an object or an event' (Finke 1989: 2). By 'visualization' we mean imagery of the visual type (as opposed to kinesthetic or aural imagery, e.g.). In some of the following case studies there is evidence for an absence of, or low-confidence use of visualization before interaction with the computer simulations, and the presence of more confidence in visualization of relative motion events following interaction with the computer simulations. This is important, as visualization is a powerful skill used by expert scientists (see Giere 1988, Nersessian 1992, Clement 1994a, b) and is hypothesized by many to play an important role in the transfer of knowledge to new situations.

The above definition of imagery is compatible with a view of knowledge that is active, anticipatory and always partially top-down in its ability to search for and structure perceptions in a noisy and uncertain environment. A knowledge schema that can assimilate perceptual patterns from, e.g. the visual system, can also anticipate certain patterns. For example, knowing what a moving car about to emerge from a tunnel will look like means having a schema that can anticipate such a perception and be ready to assimilate it. In this view, these schemas can generate images, and imagery of a somewhat more complex scene may take place as the coordinated anticipatory activity of knowledge schemas without requiring an infinite regress of 'mind's eyes' to 'view' an internal image. Finke (1990) has also documented the recognition of new wholes from combining two or more images internally and the production of novel images of inventions in the laboratory. Nor do we assume that images must be as vivid or detailed as perceptions to be useful. However, in this paper we will not attempt to mention the issue of the origins or vividness of images. We will investigate whether it is reasonable to link certain indicators from transcripts, e.g. hand motions, to the presence of imagery and mental simulations. This in itself is a difficult but important initial task.

In this paper we are particularly interested in whether students can visualize relative motion relationships because we believe that it may indicate a deep level of conceptual understanding and provide a meaningful basis for choosing arithmetic operations in quantitative problems. Although the existence of mental imagery is increasingly accepted (Kosslyn 1980, Finke 1989), a great challenge is to actually produce evidence for mental imagery from protocols. Unfortunately, mental imagery has a history of being difficult to study because it has relatively few behavioural indicators. One very plausible hypothesis concerning mental simulations is that they depend on dynamic mental imagery. We will take this as part of our theoretical framework here, and we will take the following as evidence for mental simulations: observations that indicate the presence of dynamic imagery near the production of a prediction for an event that the subject has not observed. Thus, a major question addressed in this paper is: 'Is it possible to gather evidence on whether students are able to run imagistic mental simulations on their own after using a computer simulation?'

Clement (1994a, b) proposed several indicators for mental imagery based on observations of expert scientists solving explanation problems. Two of the major indicators were depictive hand motions, where the subject makes movements that indicate shapes, locations or movements of objects in the problem situation; and imagery reports, where subjects use language, e.g. 'imaging' or 'seeing' a scene 'in their head'. These and other indicators will be used here as well.

Previous research on relative motion. Students' relative motion conceptions have been described in several ways in previous research programs. Saltiel and Malgrange (1980: 75) indicated that a preferred frame of reference is implicit in day-to-day life observation of motion, and hypothesized that many people utilize spontaneous ways of reasoning which are inconsistent with the reasoning of a physicist. They posited the existence of a 'natural model' which is in contradiction to a 'kinematic model of the physicist'. Pasne *et al.* (1994) identified the existence of several alternative conceptions in Galilean relativity. Bowden *et al.* (1992) and Walsh *et al.* (1993) presented a hierarchy of conceptual frameworks used by students when approaching relative motion problems. Their analysis of students' interview protocols indicated that most students used a flawed framework which could yield correct answers to some problems, but which lacked the explanatory power of an expert's framework. Aguirre and Erickson (1984) categorized students' incorrect responses and described context-dependent 'inferred rules' applied by students when solving relative motion problems. McCloskey *et al.* (1983) investigated misconceptions concerning the path of a dropped ball. They concluded that perceptual difficulties led to the misconception that a ball dropped by a walking person would fall straight down relative to the ground. Metz and Hammer (1993) categorized a number of problem interpretation and problem solving strategies used by students. In their study, students attempted to solve relative motion puzzles while using a computer 'microworld' (see Papert 1980).

This study focuses on relatively simple problems compared to many of the problems used in the above studies, one-dimensional relative motion problems that should appear near the beginning of a student's study of high school physics. Even though the problems we used were relatively simple, we were still able to document widespread difficulties even after the students had studied a unit on relative motion in their high school physics class. Thus, this study adds to the concern expressed by others about how difficult these concepts are. It also attempts to go beyond a study of difficulties to investigate the results of a teaching intervention.

A number of researchers have explored pedagogical strategies for addressing relative motion alternative conceptions. Camp *et al*. (1994) addressed context sensitivity issues. They indicated that problems which involve motion relative to the ground are easier for most students than problems which involve motion relative to a river. Motion relative to air was considered to be even more difficult than motion relative to water. In their book of lesson guides, relative motion instruction follows a 'bridging analogies' strategy, whereby an attempt is made to start with the easiest cases, and then to show how harder problems are analogous to the easy problems.

Ueno *et al*. (1992: 3) addressed context sensitivity and language issues. They stated that, in ordinary discussion, '"the static ground" as a frame of reference is tacitly considered as natural'. They proposed a 'recontextualization' strategy for teaching relative motion that includes seeing the ground as a moving object within the solar system. Hewson (1984) and Zietsman & Hewson (1986) showed treatment gains in conceptual understanding of a relative motion concept following subjects' use of an extreme case in a computer simulation. Their studies addressed the alternative conception, described by McDermott (1982), that objects which are momentarily next to each other must be travelling at the same speed. Thus, as described by the research programs above, there is a diverse set of difficulties facing those who would learn or those who would teach relative motion concepts.

Pedagogical uses of computer simulations to assist students' learning in science is still a relatively new field. Research by White (1993b) and Lewis *et al*. (1993) proposed guidelines for using computer simulations to assist students' learning that includes the primary goal of developing students' mental models. Gorsky & Finegold (1992) used computer simulations to display the ramifications of students' alternative conceptions – implicitly employing a cognitive conflict technique for approaching students' alternative conceptions. Roth (1995) provided evidence for 11th-grade students developing their ability to use and understand canonical language and conventions following use of Interactive Physics simulation software (see also McDermott 1990, Sachter 1990, BBN Systems and Technologies 1992).

A remaining weakness in the literature is the difficulty of studying the learning process in enough detail to determine where elements of teaching strategies are succeeding or failing to contribute, and to uncover new factors in learning. Also, the social learning processes occurring in large and small group discussions seem integral to classroom teaching in this area, but the video tapes in this mode in our pilot studies did not yield enough information on the learning processes occurring in each student at the grain size addressed in this paper. Therefore, we decided to interview students learning on their own in this particular study in order to increase our ability to track each student's learning process. Such detailed case studies of student learning are rare, but we feel they are needed to make further progress on theories of teaching for conceptual change. Studies of social learning processes in groups are also of obvious importance.

The intervention using a computer simulation discussed in this article shares the intent with Zietsman and Hewson, and Gorsky and Feingold of creating a discrepant event in the hopes of producing cognitive dissonance and curiosity. Unlike the studies above, we will try to exhibit evidence for imagery and mental simulation processes occurring during the intervention as well as after the intervention in the hope that this will provide useful information on the type of learning occurring.

Purpose and general method of investigation

Experimental studies usually focus on comparisons of the outcomes of instruction as opposed to gathering evidence on the understanding or learning process itself. This makes it difficult to know how to improve existing instructional innovations. An important purpose of case study research is to produce initial but viable hypotheses about complex knowing and learning processes. Analyses of 'learning-aloud' protocols make this possible, but it is still a great challenge. We selected some of the best and worst cases of achievement for study in this paper so that we speak to both the potential of such systems and the difficulties that need attention. Thus, we do not claim to examine a random sample, but our purpose here is not to project observation frequencies from a sample to a population. Rather, they are to generate viable hypotheses about learning and understanding processes – to use the case studies to obtain 'existence proofs' for (examples of) certain processes or methodological attainments (Clement, in press). More specifically our major purposes in this article are to use case studies:

- to present some examples of student difficulties with one-dimensional relative motion problems;
- to examine student reactions to depicting a discrepant event via a computer simulation;
- to explore the reasoning processes at work as students apply their new knowledge to target problems off-line;
- to describe changes in students' abilities to visualize relative motion problems after working with a computer simulation.

In addition, an important methodological question is whether it is possible to collect evidence that students are able to run mental simulations of target situations after an intervention. As we are concerned at this stage with formulating new descriptors and hypotheses concerning these topics, we will use descriptive case studies as the appropriate method of study. An overall purpose is to begin to address the question of whether interaction with a computer simulation on-line can facilitate a student's appropriate mental simulations off-line.

Method of study

Subjects

Subjects were enrolled in an introductory, 1-year, algebra-based physics course at a public high school in a Western Massachusetts town. All subjects were interviewed approximately 1 week before their final examination. The relative motion curriculum used by the teacher included class discussion of example problems, homework problems, a laboratory exercise, quiz and an exam. The computer simulation learning experiment that is the subject of this article took place as a supplement several months after completing their unit on relative motion in class.

Pretest/Post-test

Subjects were given identical pre- and post-tests immediately before and after the simulation treatments. Four of the seven test questions used as well as the accompanying questions concerning the student's confidence in his or her answer are listed in the Appendix. Questions had been pilot tested in interviews and had been modified following pilot testing to improve their validity as indicators of relative motion understanding. Although they are apparently very basic, it is important to note that questions such as those used in the pretest are highly problematic for many physics students. Indeed, pretest data from 86 honours physics students, taken from four classes in two different Western Massachusetts high schools and 121 standard physics students from six classes (and three teachers) in the same two schools provides evidence that questions such as those used in the pretest are problematic for many physics students. See table 1[1].

Treatment

Computer simulations were presented to the student as follows. The simulations had been created using RelLab (Relativity Laboratory) software (Horwitz *et al.* 1992). The interviewer controlled each presentation on an Apple Macintosh Powerbook 160 computer. During the course of each demonstration, the interviewer paused the computer simulation and asked the subject to predict what was about to occur. Following the prediction, the student viewed the remainder of the simulation. After this was done, the student stated what he or she had just seen.

We used simulation software in a constrained format to both facilitate learning in a brief intervention and to allow for systematic investigation of students' interactions. (This is a more constrained use of the software than the authors of the software would recommend in classroom use. For another perspective, see de Jong (1991) and Njoo and de Jong (1993) for their recommendations concerning constraining the use of computer simulations by novices.)

The simulation discussed here titled 'cars and plane', displayed the relative motion of three objects: a black car, white car and plane. The default reference frame was the ground. The interviewer ran the simulation once from the ground frame of reference. During this time, the black car moved from the far right of the screen to the left, the white car moved from the far left of the screen to the right, and the airplane appeared from the left of the screen and travelled to the right at a higher speed than the white car. A 'stopwatch' timer on the screen ran concurrently with the animations.

The simulation was then reset and run until the computer stopwatch read approximately 3s. At this point, the interviewer asked the subject to predict the direction and comparative speed of each of the cars when viewed from the airplane frame of reference.

Table 1.

Question	*Honours pretest % correct*	*Standard pretest % correct*
3	22	21
5	52	45
9	63	40

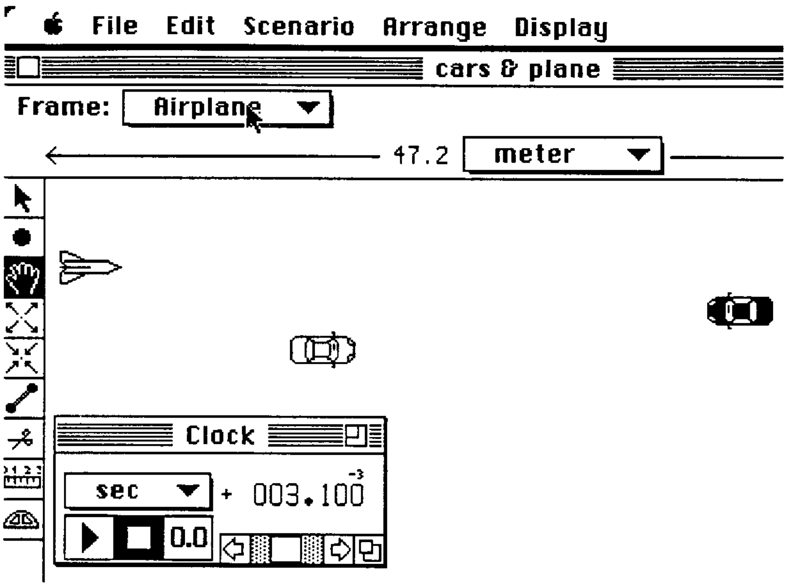


Figure 1.

Following the student's prediction, the frame of reference was changed to the airplane frame of reference and the remainder of the simulation was presented. (Figure 1 represents a sample 'snapshot' of the computer screen at the moment when the frame of reference had been changed to the airplane frame of reference.)

After the subject viewed the remainder of the demonstration, he or she indicated what had been seen. In some cases, the interviewer asked probing questions to obtain additional information from the subject. These probes included: 'Is that what you had expected to see?', 'Can you explain what you saw?', and 'What did you see?'

Case studies and analyses

In the following, three subjects are examined via case study protocol analysis. These cases provide examples of opportunities and pitfalls in the use of computer simulations.

To frame the analysis of student protocols, we hypothesize that students interact with relative motion scenarios by processing differing combinations of visual and numeric data. Often, students will attend only to numeric data. In many of these cases, algorithms are manipulated in a mechanical fashion. At other times, students will attend to visual information prior to, or along with numeric information. In the problems attempted by students in this study, numeric and directional answers were requested.

Subjects GS8

Pretest. Prior to use of the simulation, this subject displayed considerable difficulty visualizing the physical features of question 9 (truck's speed with respect to the helicopter), as evidenced by the following protocol

Section A

A1 S: OK, number 9, (reads question)... Let's see. (pause)

A2 I: What are you thinking.

A3 S: Um, I'm not quite sure because, I don't know, like when the helicopter gets over the truck, right underneath, the truck is right underneath (holds one hand under the other, both stationary), I'm really not sure what that would cause, like, how it would look.

A4 I: I see.

A5 S: In comparison to the helicopter and the truck.

A6 I: What part is difficult to uh – imagine?

A7 S: Just, them coming in the opposite direction.

A8 I: I see.

A9 S: (pause) Well, I would have to say now the truck, the truck is coming toward the helicopter, it looks as if it is going faster, so I would have to just add them. And get 240 and I'm not very confident on this one.

A10 I: Why not?

A11 S: I'm just not quite sure of uh, just how to do this one. (Laugh)

A12 I: Any uh, like what you said before, that was the part that was difficult, like

A13 S: Yeah, like seeing the truck coming at the helicopter (motions toward himself with right hand), I'm thinking that it would look faster and stuff, so then relative to it is actually going faster, because they are coming at each other (moves hands toward each other).

GS8 gives a correct answer but his confidence is low. There is evidence that suggests that the subject is attempting to visualize the problem yet is having great difficulty doing so. The statement, 'Um, I'm not quite sure because, I don't know, like when the helicopter gets over the truck, right underneath, the truck is right underneath, I'm really not sure what that would cause, like, how it would look' implies that he is trying to visualize the problem. In this section of the interview, he also uses hand gestures that appear to represent vehicles involved in the problem. The statement 'it looks as if it is going faster', in combination with these non-verbal signs, suggests that the student is attempting to visualize the problem.

Treatment. During the treatment, this subject incorrectly predicts the direction of motion of the white car (the car whose direction of motion is different depending on whether the motion is viewed from the ground or the airplane frame of reference). The student accurately predicts which vehicle (the black car or the white car) would appear to be going faster on the computer screen when the simulation is run from the airplane frame of reference.

Section B

B1 S: OK. Hmm, I think that – then the cars will look like – I think, let's see, the black car will still look like it's going faster, because it is moving towards you (moves right hand from right to left).
[correct prediction]

B2 I: OK. What direction will the black car be moving on the screen?

B3 S: It will be moving to the West.

B4 I: And how about the white car?

B5 S: To the East [incorrect prediction]... (points to right)

B6 I: To the right?

B7 S: Yeah.

After viewing the simulation from the airplane frame of reference, the student acknowledges the accuracy of his speed prediction and the inaccuracy of his direction prediction.

C1 I: All right. Here we go. (simulation is run for approximately 3 s) OK. What did you see?
C2 S: OK, I saw the black car moving faster (moves right hand quickly to the left) than the white car and the white car was moving backwards as if the plane (points to computer screen) was going past it. And the black car is moved faster as if the plane was going towards it (moves right hand to the left).
C3 I: Is that what you had expected to see?
C4 S: Yes. I did. I didn't, I didn't expect, uh, the white car (points to computer screen) to be going backwards.
C5 I: OK.
C6 S: But I did expect them to – the black car to be going faster than the white car.

In our ongoing studies, many students make errors similar to GS8 as above. Two plausible explanations for this error are: (i) he believed that direction of travel is invariant across reference frames; and (ii) he never seriously considered the possibility of a direction change.

Post-test. Subsequent to use of the computer simulation, subject GS8 appears to have incorporated a memory of the simulation as a framework for visualization of a target problem performed without the simulation.

D1 S: Hmm, number 9: (reads question) . . . OK, in my mind *I'm seeing* [emphasis added] the truck [from problem 9] as if it were a black car on the screen, on the computer, and again the helicopter [in problem 9] as if it were the plane [in the computer simulation], and uh, I'm just – I noticed that the black car [in the computer simulation] was going very fast towards the plane when it remained stationary, so I'd have to say 240 mph for this, and 'D': I'm sure I'm right [confidence rating].
D2 I: OK.
D3 S: Because of – because of the comparison (repeatedly moves right hand back and forth) – it looked to be going very fast (moves right hand to the left) towards the uh, the plane.
D4 I: OK. You changed your confidence to you're sure you are right.
D5 S: Yeah.
D6 I: Why is that?
D7 S: Because a simulation helped me out in seeing that it was going faster than what it looked to be, going faster than what it actually was, because the helicopter is moving towards it (points pencil).
D8 I: OK. Can you say more about that?
D9 S: No, just that the helicopter was going 200, and the truck was going 40, and, uh, just if you put the helicopter stationary (puts left fist down on the table) like the plane was – the truck will look to be going faster than it actually was.

There is evidence that GS8 uses memory of the simulation as a framework for his visualization of the target problem. Evidence for visualization in the post-test problem here comes from several sources:

- Imagery reports. These are places where the subject makes spontaneous references in the first person to 'seeing' or 'picturing' or 'imaging' a scene, e.g. in line D1, 'in my mind I'm seeing the truck . . .'
- Depictive hand motions. These occur when a hand motion depicts an event in the problem solution, e.g. the subject putting his fist on the table in the last line above.
- References to perceptions. These are similar to imagery reports but not as direct. The subject refers explicitly to the sensation of perception while describing visual or other perceptual aspects of the scene during thinking by using phrases, e.g. 'the truck will look to be going faster' in line D9.

The above indicators give us reason to believe that the subject is using imagery. While none of them are ironclad indicators on their own, together they are easiest to explain by the hypothesis that imagery was involved in the solution, and provide evidence for it. There is also evidence that he used memory of the simulation in an analogue fashion. This indicates that the subject is not simply doing a mapping of correspondences between verbal symbols. Instead, he has an experience that he describes as 'seeing the truck as if it were the plane'. In addition, he now seems to comprehend the idea that when changing to the reference frame of an object O originally described as moving:

(1) the object O no longer moves; and
(2) the velocities of other objects are changed.

As stated in line D9 'just if you put the helicopter stationary (puts left fist down on the table) like the plane was – the truck will look to be going faster than it actually was'.

Note that in the above protocol he is not using terminology that is completely consistent with that of the physicist. Use of words, e.g. 'the truck will look to be going faster than it *actually* [emphasis added] was', suggests that the student still strongly prefers the ground frame of reference. Statements such as the latter one are consistent with findings by Saltiel and Malgrange (1980) and Pasne *et al*. (1994) in which students referred to an object's velocity relative to the ground as its 'proper velocity'. Despite this remaining flaw, we are encouraged that this subject's confidence in his correct solution to this problem has increased markedly.

Consistent with the above protocol for post-test question 9, the subject also appears to use memory of the simulation as a framework for visualization while answering post-test question 5. He indicates that he is more confident in his answer to this problem now than he was during the pretest. He says: 'I'm sure that I'm right, because in the simulation I noticed that the white car looked to be going slower when the plane was going by it, so I compared my car to the white car, and the plane to the helicopter, so, that's how I got my answer'. Furthermore, the subject states 'I saw – I saw in my mind, uh, the grey car as if it were the white car, I saw a helicopter as if it were the plane, and, uh, it looked it looked as if, like it was going – the – my car looked as if it was going very slow backwards – or uh, very fast backwards, and making no progress forwards'. Here again we have a clear spontaneous imagery report concerning a moving object. We refer to this as a dynamic imagery report. We take this as evidence that the subject is using dynamic imagery in a mental simulation, as well as from the fact that he is describing vehicle movements as he gives imagery indicators, and displaying some indicators that are inherently dynamic, e.g. hand or pencil motions.

As evidenced by the protocol, following his experience with the computer simulation, the student increases confidence in his answers to relative motion questions, apparently due to an improved ability to visualize this type of problem. Thus it appears that the student is able to map visual features of the computer simulation onto a target problem and run a mental simulation in the absence of the computer. We believe this is an important and desirable outcome that may indicate a significant new type of conceptual understanding for this student.

Subject GS10

Subject GS10 was a high school sophomore who had just completed an introductory, one-year, algebra-based physics class. She was in the honours physics class, but she was rated below average in that class by her teacher. We will examine how GS10 showed progress in correctly choosing an arithmetic operation following use of the computer simulation; it appears that this was due to an improved ability to visualize features of target problems.

Pretest. During the pretest, this student had difficulty determining whether to add or subtract the respective speeds relative to the ground to obtain the speed of a car or truck relative to a helicopter. The student also appears to be having difficulty visualizing aspects of the target problem. Below is protocol which details some of the student's difficulties in solving a relative motion problem:[2]

A1 S: Numbers 5–10 refer to the picture below. In the picture above you are in the grey car. Your speedometer reads 40 mph. What is the car's speed relative to, to a very low flying helicopter going exactly the same direction as your car at a speed relative to the ground on 200 mph. OK.
A2 1: What are you thinking?
A3 S: You want me to say it out loud?
A4 I: Yes, please.
A5 S: All right, well if the speed is relative to the ground 200 mph and um, speedometer reads forty miles per hour so that's relative to the ground also. Um, I'm trying to think, I can't remember, how you would figure this out, um, What is your car's speed relative to a low flying helicopter going exactly the same direction as your car at a speed relative to the ground at 200 miles per hour so, if this is the grey car and it's going this direction (draws vector), and it's going forty miles per hour and this is the helicopter going the same direction going 200 mph (draws vector). Then I think the velocity equals just 200 um, yeah, relative speed equals the first take away the second I think. Which equals, um, um, What is the car's speed relative the, oh, it's relative to the ground, so are they asking like what you see, or what you would see if you were on the ground?
A6 I: What's the question say?
A7 S: Because it says the speed relative to the ground of 200 mph, oh OK, what's your car's speed relative to a very, so year so it's 60 miles per hour. Because the um, helicopter is going 200 mph but you're also going forty, so, oh I'm not sure if it's subtracted or added.
A8 I: What are you thinking?
A9 S: Well I'm thinking that um, we learned this a while ago and um, well if the helicopter is already going 200 and then the car is also going forty then it would be 200 plus forty. Because you add, since they are both relative to the ground, and if you were looking at the car it would be going faster, so it's 240 mph. All right, yeah, I think that's what it is. I don't know.
A10 I: OK, what, what are you concerned about?
A11 S: Um, I'm just concerned about like my process of figuring that out.
A12 I: OK, what part of it?
A13 S: Um, well, I known like I've learned like the relative speed in class but, *I'm not sure like if you're supposed to um subtract or add*, [clear reference to previous instruction, emphasis added] because I know it's um one of the speeds is A and the other one is B but I'm not sure if you're supposed to subtract or add when they're going the same direction, That's the only thing I'm concerned about.
A14 I: I see.
A15 S: so I just decided that that's plus because I know that they are going the same direction and they wouldn't, it would be subtracted if they were going in opposite directions. So.

A16 I: I see. How's your confidence?
A17 S: Um, Uh, Somewhere between not very confident and fairly confident.
A18 I: OK.

There is evidence that the student is having difficulty remembering how to apply a learned algorithm. Although she attempts to use an external visual representation (arrows or vectors) at one point, her focus seems to be on 'whether to add or subtract' and there is relatively little other evidence for attempts to visualize the events of the problem. One interpretation of her solution is that she is devoting most of her effort to attempting (unsuccessfully) to recall a verbal rule for 'when to add and when to subtract' in such problems.

Treatment. During the treatment, like GS8, GS10 incorrectly predicts the direction of travel of one of the objects on the screen (the white car) when viewed from an unfamiliar (airplane) frame of reference.

B1 S: Oh, OK! So, uh, so its like assuming I was in the airplane looking down?
B2 I: Uh, you could, uh –.
B3 S: Can I assume it that way too?
B4 I: Uh, well, we're in the airplane frame of reference.
B5 S: OK! So, I just – well. I think this one [white car] is going faster anyway, so relative to that [airplane] it still is going faster in this in – in the same direction as the airplane. And this one [black car], uh, looks like its going slower than this one [white car] and also slower than it would if we're looking on the ground because it is going the opposite direction of the airplane.
B6 I: OK! OK. So, uh, what direction will the white car be going on the screen?
B7 S: Uh, that way (subject points to the right)
B8 I: To the right?
B9 S: Yeah!
B10 I: And the black car?
B11 S: To the left

In addition to predicting the wrong direction of travel for the white car above, she also incorrectly predicted the comparative speeds of the black car and the white car, relative to the airplane. However, her reference to being 'in the airplane looking down', and her hand motion near the end of the episode suggests that she is in fact attempting to image at least part of the situation.

Following interaction with the simulation below, GS10 expresses surprise since she had predicted a different outcome, and this provides evidence that she is having some cognitive dissonance. The impact of this anomalous data may be to motivate and constrain her construction of more correct view in lines C8–C12.

C1 I: OK. OK, well lets try that out. [simulation is continued from the airplane frame of reference]
C2 S: Oh, that's right! (surprised tone)
C3 I: OK –
C4 S: (laugh) That's neat!
C5 I: What did you see?
C6 S: (laugh) Uh, the – OK, since, uh, – the other – the airpl – the car going in the same direction as airplane [the white car], uh, it – the airplane went by quicker. It looked like the airplane was going by quicker and it looks like it [white car] was going, actually, backwards –
C7 I: I see.
C8 S: – the in – and see it's [white car] still going in to the right but from the airplane perspective it's going backwards, because uh, the airplanes' velocity is so

much higher, and uh, it – it's like coming upon that car right slower than... [it's coming upon] the one [black car] coming towards it [airplane]! –
C9 I: I see.
C10 S: And uh, the airplane – it looks like the uh, the other car [black car] that is... was actually going to the left was going faster in they're both going the same way to the left.
C11 I: OK.
C12 S: But the black car was going much faster since it was coming towards the airplane at the same time as airplane is going – you know what I mean?
...
C13 I: Uh, OK. OK, and what did you – what actually did you see on the screen?
C14 S: Well, I saw them both going on to the left, but the black car was going much faster, uh, then the white car, the clear car.
C15 I: OK.
C16 S: Uh. But I know from watching the other thing the car – the white car was actually going to the right. But –
C17 I: OK.
C18 S: – since it was going in the same direction as the airplane [relative to the ground] it looked like it was going the other direction.
C19 I: I see.
C20 S: And since the black car was going to the left as airplane was going to the right, then its velocity looked faster.
...
C21 I: OK. Uh, did you expect what happened?
C22 S: Uh, no (laugh) From what I told you before I said that they'll go the opposite directions, but –
C23 I: OK.
C24 S: It was wrong –

Evidenced by her expression of both surprise and pleasure, (S: (laugh) *That's neat*! [emphasis added]) she appears to readily agree with the output and apparently views it with great interest. It is certainly plausible that if the student originally considered direction of travel to be invariant across reference frames, she now would challenge that belief. In line C8 she is able to give an explanation for the anomaly that she observed, and this is one indication that she has constructed a new understanding of the change in reference frame.

(Like GS8, GS10's statement that the car 'was *actually* going to the left' [emphasis added] is inconsistent with a physicist's terminology. This quote suggests that the student views motion relative to the ground as *actual* motion; presumably motion relative to another reference frame is not conceived to be actual motion (see Saltiel & Malgrange 1980, Ueno *et al.* 1992, Ueno 1993, Pasne *et al.* 1994). Or, possibly, the student's vocabulary has not yet caught up to her conception of relative motion. The conflict between colloquial speech and technical terms may provide consternation for students (see Ueno *et al.* 1992)).

Post-test. Following use of the computer simulation, there is evidence that the subject performs an appropriate mapping of simulation features to a target case.

D1 S: OK. Uh, number 5. (Reads question)... Um, OK, now the helicopter is going this way (labels diagram with vertical vector and 200;) at 200mph, and you also going this way at 40mph (puts 40mph on the diagram), – uh (looks at static graphic on computer screen), lets see. From the simulation it showed that like (points right hand holding pencil laterally to the right) when the plane was going the same direction as the car that was going the same direction, that looked like the car that was going the same direction was going opposite direction (moves right hand holding pencil laterally to the right) [clear memory of simulation]. So, if this car – the car

(draws vertical vector on diagram) probably *look like it's going that way* [emphasis added] at – uh, – so you probably have to subtract, so 200 minus 40 equals 160mph (writes 200 – 40 = 160mph and circles answer)
D2 I: What are you thinking?
D3 S: I'm thinking that since this is going the opposite direction than... then uh,... *it* [*the car*] *slows down* [from the helicopter frame of reference] instead of speeds up, *the mph that it looks like from the helicopter* [emphasis added], because *it is going the opposite direction, that looks like it is from the helicopter*, because you're going faster than it, so I subtracted 200 which is the mph of the helicopter from the 40mph of the car to give me 160mph that it looks like it is going [in] the opposite direction.
D4 I: How is your confidence?
D5 S: Uh, fairly confident –
D6 I: OK. *Why are you more confident?* [emphasis added]
D7 S: Because from watching the screen *I'd seen like a simulation and an actual, like, picture of it, instead of like something like on the paper*. [emphasis added]

Clearly, subject GS10 made quantitative gains in answering the problem. Not only did the student obtain the correct answer, but she also indicated that she was more confident in her correct post-test answer than she was in her incorrect pretest answer. The subject's statement that the computer simulation provided an 'actual... picture of it' suggests that she found the computer intervention to be a useful tool to introduce a representation of the problem scenario. It is interesting that apparently through use of a qualitative simulation, the student made gains in her ability to solve a quantitative problem.

There are several indications in this segment providing evidence that the subject was using imagery. These include the following.

(1) Depictive hand or pencil motions that are not simply pointing to a word or picture, but that appear to indicate movements of objects. They may also appear near oral references to those movements and appear over a drawing of the problem. Although the drawing provides an external visual representation, it cannot provide a dynamic representation of motion. We interpret these motions as expressions of and evidence for internal motion imagery.
(2) Reference to perceptions. The subject refers explicitly to the sensation of perception while describing visual or other perceptual aspects of the scene during thinking by using phrases such as 'the car probably looks like it is going that way' in line D1.
(3) Self-projection. These occur in phrases like that in line D3 above: 'because *you're* going faster than it'. (emphasis added) where the subject uses a personal pronoun indicating that he or she is imaging projecting themselves into a particular object in the problem. (Although problem 5 suggests thinking about 'your car's speed', at this moment the language indicates that the student has apparently projected herself into the helicopter.) Whereas abstract ideas may be 'view independent', visual imagery usually occurs from a particular point of view (Kosslyn 1980). For example, it is extremely difficult to imagine all sides of a house at once. We can see this type of language as originating in her imagining what it is like from the point of view of the helicopter. In fact, the wording of the problem talks about 'your car', and indeed her language early in the solution in line D1 suggests that she accepts the car point of view: 'Ok, now the helicopter is going this way at 200mph, and you're also

going this way at 40 mph'. But later her point of view appears to shift to the helicopter in line D3.

This contrasts with her language from the pretest on the same problem – when she talks about the use of numerical algorithms, these contain no such personal pronoun indicators of projected points of view.

> GS10: Pretest A5: 'I think the velocity equal just 200 um, yeah, relative speed equals the first take away the second I think . . .
> A13: because I know it's um one of the speeds is A and the other one is B but I'm not sure if you're supposed to subtract or add when they're going the same direction . . . so I just decided that that's plus because I know that they are going the same direction and they wouldn't, it would be subtracted if they were going in opposite directions'.

Again, while none of these indicators is an ironclad indicator on its own, they give us reason to believe that the subject is using imagery. And again, there are accompanying indicators that the imagery is dynamic when subjects are describing vehicle movements as they display imagery indicators, and displaying some indicators that are inherently dynamic, e.g. hand or pencil motions.

The three imagery indicators listed above are also present in the transcript of this subject's predictions during the treatment. One might argue that she simply drew an analogy between the relative motion case presented during the treatment and the target problem in a verbal manner that does not require visualization. But the way in which GS10 refers back to the computer simulation while making horizontal pencil motions, and then solves the above problem while making vertical pencil motions, suggests that imagery was involved in both sections, and that she was able to transfer visual ideas from her memory of the computer simulation to the post-test problem. This suggests that the solution was reached not just by means of a verbal analogy; based on the above observations, we hypothesize that it involves imagery, and the use of dynamic mental models. Thus, there is evidence that she was better able to visualize the problem scenario because of the treatment.

Subject GS11

Pretest and treatment. Subject GS11 displayed correct reasoning in answering pretest questions 1 and 3. During both answers, he indicated that the snowball would be thrown 'about the same', when the chair lift was moving relative to the ground as when it was stationary relative to the ground, because in each case the snowball travelled the 'same distance' [relative to the chair lift].

During the treatment, the subject incorrectly predicted the direction of movement of the white car when viewed from the airplane frame of reference. He initially expressed concern upon seeing the simulation but, after a brief pause, stated 'Oh, yeah, I get it now' and explained that the movement of the plane effected the change of speed of the cars when viewed from the plane frame of reference.

Post-test. Following use of the simulation, the subject inaccurately changes his correct pretest answers for questions 1 and 3. Below, he describes how he reached his answers to post-test problems 1 and 3.

A1 S: (Reads question 1) Uh, uh, I'm trying to think what was on the computer, uh, uh, say, uh, it's harder, 'B' [correct answer is 'A', 'the same']. (pause) Confidence, my – I think I'm fairly confident. I thought that because, I'm trying to think of – when the plane was not moving and this car was, it [the black car] went like a lot faster. I don't know maybe, think maybe, you need to throw harder.
A2 S: (Reads question 3)... you could throw it softer [correct answer is 'the same']. I'm starting to see this now, I think. And, uh, – I'll say 'D' – I'm sure, I'm right [confidence rating]. Uh –, I think this because – I'm in one spot throwing it, right. (moves right hand forward) And uh, and uh I'm moving from that spot, and the chair behind me is going to be – is going towards me, (moves right hand left to right) and it is going to be at that spot pretty soon, so I wouldn't, like, I can just like lob it and all of a sudden the chair will run in to it (moves both hands left to right) because it is going into the snowball, so that's why I thought that. And also I see from (points to computer) watching the computer how you can get that – just you gotta think it out...
A3 I: How so? What are you thinking about?
A4 S: Uh, well, I saw, I see in the – I don't know, I just – the computer kind of influenced me, by watching it's changing what I think about this kind of stuff. As when you asked me what I said what I think would happen, nothing ever happened. And, uh, I don't just know. I just, now that I think about it, I just, you know, kind of logical that you don't need to throw it the same as if it was turned off, I think.
A5 I: OK. Uh, and which, which, uh, simulation or both are you thinking about?
A6 S: Um, I'm thinking of the – the plane.
A7 I: OK.
A8 S: That because uh – I don't know, it's just that, when that car (places left hand horizontally above desk) looked like it was going reverse,
A9 I: OK.
A10 S: it was (moves left hand to the left) going slower. That's how I think I got that.

This student, as displayed above, has regressed in his ability to answer questions 1 and 3. It is worth noting that he was sure that he was right when he correctly answered these problems during the pretest. Following the simulation, his self-report indicates that he is similarly confident in his post-test answers. However, his answers do not agree with scientifically accepted ones. The student indicates that use of the 'cars and plane' simulation affected his answer. Based on protocol for questions 1 and 3, it appears that the student is analogically mapping the black car and the plane of the simulation onto the snowball and the chair lift of the target problem, respectively – a mapping that is inappropriate from the teacher's perspective.

It is very possible that the student mapped features of the simulation onto the chair lift problem in the following rule-like manner: a person on the chair lift will throw the ball either slower or faster depending on the motion of the chair lift just as objects went slower or faster when viewed from a non-ground (airplane) frame of reference.

Additionally, subject GS11 appears to have lost confidence in his pretest ability to solve the problems because 'nothing ever happened' (see line A4 above) in the way that he predicted when the simulations were conducted. He stated while answering post-test problem 3 that he was 'beginning to see it now'. 'You just gotta think it out'. It appears, therefore, that the student had become dissatisfied with his pretest ability solve relative motion problems. This dissatisfaction may partially explain his desire to change the pretest answers.

Like subjects GS8 and GS10, GS11's protocol displays evidence for visualization during the post-test. Indicators include depictive hand motions, references to perceptions, self-projection and imagery. Clearly, during the post-test, GS11 is able to recollect features of the 'cars and plane' simulation. He also appears able to visualize the simulation and is able to visualize during his solution to the post-test problem; however, the scenario that he visualizes is scientifically inaccurate. GS11 appears to have attended to features of the simulation (the relative speeds and directions of objects before and after the change of reference frame) and generated a faulty rule for solving the problems.

Discussion

Our major purposes in this article were to use case studies: to present some examples of students difficulties with one-dimensional relative motion problems; to examine student reactions to a computer simulation in this area containing a discrepant event; to describe changes in student abilities to visualize relative motion problems after working with simulation; and to investigate the reasoning processes at work as students apply their new knowledge to target problems off-line. We were also concerned with whether evidence that students are running mental simulations could be collected in order to address the question of whether interaction with a computer simulation on-line can facilitate mental simulations off-line.

Student difficulties

As displayed by GS8 and GS10's pretest protocol and by GS11's post-test protocol, apparently simple one-dimensional relative motion problems are difficult for students, even after physics instruction. Recall that the pretest these students did so poorly on before our supplemental intervention was actually given after they had studied a unit on relative motion in their physics class. Consistent with the findings of other authors (see Hammer 1994 for a review), there is a tendency for many physics students to rely on a poorly understood algorithm to solve problems. This almost 'blind' application of an algorithm is particularly evident in the pretest protocol given by subject GS10. Another problem identified was very low confidence and/or competence in one's visualization skills in relative motion. This difficulty was exemplified by subject GS8's pretest protocol.

Effect of intervention

Concerning the effect of the computer simulation, as evidenced by the reported increase in confidence by GS8, and the correct change of answer by GS10, it appears that interaction with the simulation in a predict–observe–explain format had a positive effect on their ability to solve target problems. In the case of GS10 there was evidence for dissonance caused by the simulation after an inaccurate prediction. Such dissonance episodes may have both motivated and constrained the students' subsequent learning in a positive way (see Dykstra *et al.* 1992 for a similar result). Both students were then able to apply their knowledge to quantitative as well as qualitative aspects of post-test problems.

Visualization

We have identified several indicators providing evidence that two of the subjects, GS8 and GS10, were using imagery during the intervention and more importantly during the post-test. These include the following.

(1) Depictive hand or pencil motions.
(2) References to perceptions – using phrases, e.g. 'the car probably looks like it is going that way'.
(3) Self-projection: occurring in phrases, e.g. 'because I would be going faster than it.'
(4) Imagery reports where the subject reports imagining or 'seeing' a situation.

Together these are easiest to explain by the hypothesis that imagery was involved in their thinking about the post-test problems. Furthermore, we argued that the subjects used dynamic imagery in mental simulations during the post-test from the fact that they described vehicle movements as they exhibited imagery indicators, gave dynamic imagery reports of 'seeing' moving objects, and displayed other imagery indicators that are inherently dynamic, e.g. most hand or pencil motions occurring over drawings.

Indeed, both students indicated that visualization of post-test problems was facilitated by memory of the computer simulation. It appears that in both cases, memory of the simulation functioned as a framework for visualization, in which ideas from the recollection of the computer simulation were transferred to the target problem, yielding a way that the students could mentally simulate the problem. If this view is correct, then we can say that the two subjects solved post-test problems off-line via dynamic mental simulations, and were not dependent on the computer to run the simulation. This seems crucial if students are to take deep understandings away with them after using a computer simulation. Thus, there is evidence that appropriate mental simulations of target problems can be fostered by interventions consisting of using a computer simulation in a predict–observe–explain activity.

Our hypothesis then for why the intervention was successful for these two subjects is as follows.

(1) They were initially stimulated to develop a new understanding by the recognition that their prediction was false after viewing the simulation.
(2) They then constructed some new understandings by explaining the simulated event, especially for the case of the car whose apparent direction of travel reverses.
(3) In the post-test or target problem, there is evidence that they were able to use their memory of the simulation in an analogue fashion to generate correct imagery in the target situation.
(4) There is also some evidence that this imagery was dynamic and allowed them to run a qualitative mental simulation of the target situation.
(5) In quantitative problems, the subjects used qualitative comparisons of relative speeds or directions of travel to determine appropriate additions or subtractions to perform at a quantitative level.

Instructional failures

However, protocol from GS11 indicates that our constrained manner of using this computer simulation for purposes of this study is not a panacea for alleviating difficulties that students may have in approaching relative motion problems. Indeed, GS11 illustrates the fact that students may actually regress following the use of a computer simulation. Examination of this case leads to several hypotheses concerning the difficulties that may be encountered by students who use recollection of a simulation to guide their solution of target problems. First, the student may not understand the base, i.e. the computer simulation. This may be due to a lack of conceptual understanding of concepts and/or terms central to the computer simulation; without such understanding the student may not be able to accurately process the feedback provided by the simulation. Second, the student may inaccurately transfer knowledge gained during interaction with the simulation to an 'analogous' case which is not truly analogous. Subject GS11 appeared to display this latter type of difficulty on post-test problems 1 and 3. GS11 also may have regressed due to insecurity concerning his ability to solve relative motion problems following feedback on inaccurate predictions during the treatment (see Stavy 1991 for discussion of a related result). Additionally, the failure of this student to accurately solve post-test problems that he had accurately solved during the pretest, despite indicators for post-test visualization, indicates that students may visualize scenarios that are scientifically inaccurate. This reveals that students may generate inaccurate visualizations even when they are willing to visualize a problem, and they can do so with high confidence. These difficulties indicate areas where further work is needed in our particular intervention, and may serve as useful caution signs for other simulation projects.

Implications

Simulations such as the one used in this study appear to be promising as instructional tools, allowing construction and manipulation of images from numerous viewpoints. Students GS8 and GS10 exhibited evidence for dynamic imagery use during the intervention simulations and also during post-test problems, on which they improved. This suggests that the students were able to visually understand the examples in the computer simulation and draw appropriate analogies to new situations to make inferences. Gick and Holyoak (1983) view the establishment of such sets of analogous exemplars as important steps toward forming a general schema or model, and we believe that it is possible that the students discussed here were beginning to form a general model for relative motion ideas. Although we do not have direct evidence for it in the present data, we suspect that these two subjects were beginning to understand at least two general and central features of reference frames: that when changing to the reference frame of an object O originally described as moving:

(1) the object O no longer moves; and
(2) the relative velocities of all other objects are changed.

Forming a visualizable model with such general properties would comprise what White (1993a) refers to as an 'intermediate abstraction' – i.e. a representation that is more general than single examples, but which is not as abstract as mathematical formalisms. Such visualizable models are considered by many to be central in the thinking of practicing scientists; there is growing evidence that they may also be central for understanding in science students.

We believe that this provides an interesting way to view the role of computer simulations in instruction that is somewhat different from the way usually described, as follows. A simulation can:

(1) provide an experience that produces dissonance;
(2) provide an initial key exemplar of the behaviour of a system that stimulates students to construct an explanation for the behaviour;
(3) provide a 'framework for visualization' as a key exemplar that can allow the student to transfer ideas to similar but different problems;
(4) eventually lead, as experience with different examples is gained, to the formation of a general, schematic model that relies on mental simulation rather than computer simulation.

Critique and call for further research

If this last view has merit, it leads to questions like: What makes a computer simulation memorable enough but basic enough that it can act as a 'key exemplar'? We need to explore this idea more carefully with reference, e.g. to Minstrell's (1996) idea of 'benchmark' examples in physics teaching, and perhaps to Kuhn's (1977) idea of paradigmatic exemplars in science.

At the methodological level, imagery has always been extremely difficult to study because it has so few behavioural indicators, none of which are very direct. From these case studies we cannot predict the frequency with which these will be exhibited. However, the cases do serve to provide 'existence exemplars' establishing the possibility of such indicators, which was our purpose here, rather than the purpose of providing estimated average frequencies of such indicators over a particular population. The development of methodologies for recognizing imagery indicators would give us a powerful tool for detecting the form of student's knowledge structures and aid in discriminating between rote algorithms and conceptual understanding.

Appropriate application of experience gained through interaction with computer simulations is a complex skill. Its full development would undoubtedly require more support than was offered in our interventions. Further research should be conducted to identify and describe additional factors that affect students' ability to solve and visualize problems following computer simulations in order to develop improved pedagogical strategies for using them.

An implication of these results for curriculum development is that formative evaluations that include data on the conditions under which students visualize target problems following use of a simulation can and should be an early component of development projects. Such data should guide and improve both the developer's design of educational computer simulations and the teacher's design of learning activities which employ them.

References

Aguirre, J. and Erickson, G. (1984) Students' conceptions about the vector characteristics of three physics concepts. *Journal of Research in Science Teaching*, 21, 439–457.

BBN Systems and Technologies (1992) Reconceptualization and the role of technology in teaching and learning relativity. Proposal number P93-LABS-C-042, submitted to Education and Human Resources Directorate, National Science Foundation.

Bowden, J., Dall'Alba, G., Laurillard, D., Martin, E., Marton, F., Masters, G., Ramsden, P., Stephanou, A. and Walsh, E. (1992) Phenomenographic studies of understanding in physics: Displacement, velocity and frames of reference. *American Journal of Physics*, 60, 262–268.

Camp, C., Clement, J., Brown, D., Gonzalez, K., Kudukey, J., Minstrell, J., Schultz, K., Steinberg, M., Veneman, V. and Zietsman, A. (1994) *Preconceptions in Mechanics: Lessons Dealing with Student's Conceptual Difficulties* (Dubuque: Kendall-Hunt).

Clement, J. (1994a) Imagistic simulation and physical intuition in expert problem solving. In *The Sixteenth Annual Meeting of the Cognitive Science Society* (Hillsdale, NJ: Lawrence Erlbaum).

Clement, J. (1994b) Use of physical intuition and imagistic simulation in expert problem solving. In D. Tirosh (ed.), *Implicit and Explicit Knowledge* (Hillsdale, NJ: Ablex), 204–244.

Clement, J. (in press) Analysis of clinical interviews: Foundations and model viability. In A. E. Kelly and R. Lesh (eds), *Research Methods in Mathematics and Science Education* (Englewood Cliffs, NJ: Erlbaum).

Dykstra, D. I., Boyle, C. F. and Monarch, I. A. (1992) Studying conceptual change in physics. *Science Education*, 76, 615–652.

Finke, R. A. (1989) *Principles of Mental Imagery* (Cambridge, MA: MIT Press).

Finke, R. (1990) *Creative Imagery: Discoveries and Inventions in Visualizations* (Hillsdale, NJ: Lawrence Erlbaum).

Gick, M. L. and Holyoak, K. J. (1983) Schema induction and analogical transfer. *Cognitive Psychology*, 15, 1–38.

Giere, R. (1988) *Explaining Science* (Chicago: Chicago U. Press).

Gorsky, P. and Finegold, M. (1992) Using computer simulations to restructure students' conceptions of force. *Journal of Computers in Mathematics and Science Teaching*, 11, 163–178.

Hammer, D. (1994) Epistemological beliefs in introductory physics. *Cognition and Instruction*, 12, 151–183.

Hewson, P. W. (1984) Microcomputers, conceptual change and the design of science instruction: examples from kinematics and dynamics. *South African Journal of Science*, 80, 15–20.

Horwitz, P., Feurzeig, W., Shetline, K., Barowy, W. and Taylor, E. F. (1992) *RelLab* [Computer program]. (Cambridge, MA: Bolt Beranek and Newman, BBN).

de Jong, T. (1991) Learning and instruction with computer simulations. *Education and Computing*, 6, 217–229.

Kosslyn, S. (1980) *Image and Mind* (Cambridge, MA Harvard University Press).

Kuhn, S. T. (1977) Second thoughts on paradigms. In S. T. Kuhn (ed.), *The Essential Tension. Selected Studies in Scientific Tradition and Change* (Chicago: The University of Chicago Press).

Lewis, E., Stern, J. and Linn, M. (1993) The effect of computer simulations on introductory thermodynamics understanding. *Educational Technology*, Jan., 45–58.

McCloskey, M., Washburn, A. and Felch, L. (1983) Intuitive physics: The straight-down belief and it's origin. *Journal of Experimental Psychology: Learning, Memory and Cognition*, 9, 636–649.

McDermott, L. C. (1990) Research and computer-based instruction: opportunity for interaction. *American Journal of Physics*, 58, 452–462.

McDermott, L. C. (1982) Problems in understanding physics (kinematics) among beginning college students – with implications for high school courses. In M. B. Rowe (ed.), *Education in the 80's: Science* (Washington, DC: National Education Association), 106–128.

Metz, K. and Hammer, D. (1993) Learning physics in a computer microworld: In what sense a world? *Interactive Learning Environments*, 3, 1–15.

Minstrell, J. (1996) A classroom environment for learning: Guiding students' reconstruction of understanding and reasoning. In L. Schauble and R. Glaser (eds), *Innovations in Learning: New Environments for Education* (Mahwah, NJ: Erlbaum Associates).

Nersessian, N. (1992) How do scientists think? In R. Giere (ed.), *Cognitive Models of Science* (Minneapolis: U. of Minnesota Press), 3–44.

Njoo, M. and de Jong, T. (1993) Exploratory learning with a computer simulation for control theory: learning processes and instructional support. *Journal of Research in Science Teaching*, 30, 821–844.

Papert, S. (1980) *Mindstorms: Children, Computers, and Powerful Ideas* (New York: Basic Books).

Pasne, S., Ramadas, J. and Kumar, A. (1994) Alternative conceptions in Galilean relativity: frames of reference. *International Journal of Science Education*, 16, 63–82.

Roth, W. M. (1995) Affordances of computers in teacher–student interactions: The case of interactive physics.™ *Journal of Research in Science Teaching*, 32, 329–347.

Sachter, J. E. (1990) Explorations into the spatial cognition of children using 3-D computer graphics. In I. Harel (ed.), *Constructionist Learning: A 5th Anniversary Collection of Papers Reflecting Research Reports, Projects in Progress, and Essays by the Epistemology & Learning Group* (Cambridge, MA: MIT Media Laboratory), 217–247.

Saltiel, E. and Malgrange, J. L. (1980) 'Spontaneous' ways of reasoning in elementary kinematics. *European Journal of Physics*, 1, 73–80.

Stavy, R. (1991) Using analogy to overcome misconceptions about conservation of matter. *Journal of Research in Science Teaching*, 28, 305–313.

Ueno, N. (1993) Reconsidering p-prims theory from the viewpoint of situated cognition. *Cognition and Instruction*, 10, 239–248.

Ueno, N., Arimoto, N. and Yoshioka, A. (1992) *Learning physics by expanding the meta-context of phenomenon*. Paper presented at the annual meeting of the American Educational Research Association, San Francisco, CA.

Walsh, E., Dall'Alba, G., Bowden, J., Martin, E., Marton, F., Masters, G., Ramsden, P. and Stephanou, A. (1993) Physics students' understanding of relative speed: a phenomenographic study. *Journal of Research in Science Teaching*, 30, 1133–1148.

White, B. Y. (1993a) Intermediate causal models: a missing link for successful science education. In R. Glaver (ed.), *Advances in Instrucational Psychology* (Hillsdale, NJ; Lawrence Erlbaum Associates), Vol. 4, 177–252.

White, B. Y. (1993b) ThinkerTools: Causal models, conceptual change, and science education. *Cognition and Instruction*, 10, 1–100.

Zietsman, A. I. and Hewson, P. W. (1986) Effect of instruction using microcomputer simulations and conceptual change strategies on science learning. *Journal of Research in Science Teaching*, 23, 27–39.

Notes

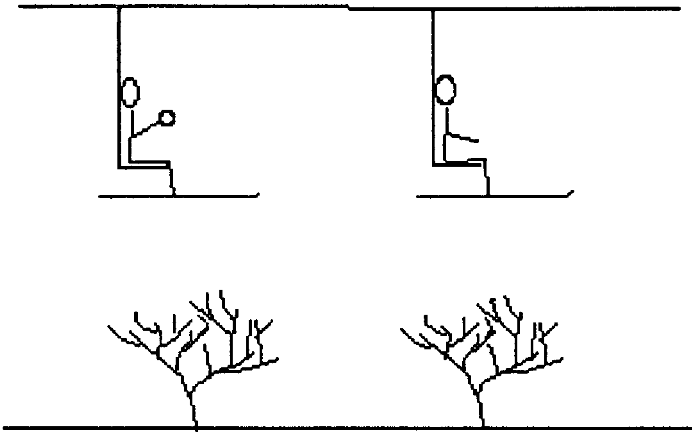
* An earlier form of this paper was presented at the Third International Seminar on Misconceptions and Educational Strategies in Science and Mathematics, Cornell University, Ithaca, New York, August 1993. This research was supported by National Science Foundation grant MDR 9150002. Any opinions expressed are those of the authors and not necessarily of the National Science Foundation.

1. The classroom students had the advantage of selecting a multiple-choice answer for questions 5 and 9. Also, choice (d) of question 3 was not available for the classroom students.
2. Please see the Appendix for a listing of this relative motion problem.
3. Several subjects did not readily understand the wording of diagnostic questions which asked for velocity 'relative to' another object. Many students seemed to prefer that a reference frame be termed a 'point of view' or a 'perspective'.
4. In the classroom diagnostic test, this question read: 'What is your car's speed relative to a very low flying helicopter? Relative to the ground, the helicopter is going exactly the same direction as your car, at a speed of 200mph'.
5. The word 'you' was replaced with 'your position' in the classroom diagnostic test.

Appendix

Pretest/post-test questions

Number 1 features the ski-lift shown below:



Assume that if the ski-lift were turned off, you could throw a snowball and hit the person in the chair ahead of you. Also assume that the ski-lift is over a flat part of the mountain.

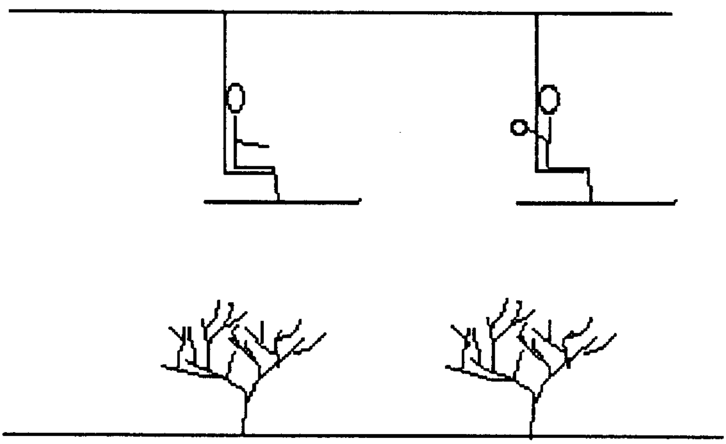
(1) How hard would you have to throw the snowball from one chair on a ski-lift to another chair ahead of you on the ski-lift when the ski-lift is in operation?

(a) About the same as if the ski-lift were turned off.
(b) Harder than if the ski-lift were turned off.
(c) Softer than if the ski-lift were turned off.
(d) It's impossible to hit the chair that is ahead of you when the ski-lift is in operation.

(2) What is you confidence in your answer?

(a) Just a blind guess.
(b) Not very confident.
(c) Fairly confident.
(d) I'm sure I'm right.

Number 3 features the ski-lift shown below:



Assume that if the ski-lift were turned off, you could throw a snowball and hit the person in the chair behind you. Also assume that the ski-lift is over a flat part of the mountain.

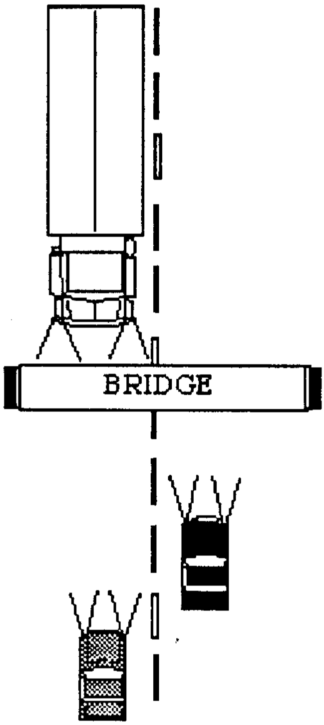
(3) How hard would you have to throw the snowball from one chair on a ski-lift to another chair behind you on ski-lift when the ski-lift is in operation?

(a) About the same as if the ski-lift were turned off.
(b) Harder than if the ski-lift were turned off.
(c) Softer than if the ski-lift were turned off.
(d) It's impossible to hit the chair that is behind you when the ski-lift is in operation.

(4) What is your confidence in your answer?

(a) Just a blind guess.
(b) Not very confident.
(c) Fairly confident.
(d) I'm sure I'm right.

Numbers 5–10 refer to the picture below:[3]



In the picture above, you are in the grey car. Your speedometer reads 40mph.

(5) What is your car's speed relative to a very low flying helicopter going exactly the same direction as your car, at a speed relative to the gound of 200mph?[4]

(6) What is your confidence in your answer?

(a) Just a blind guess.
(b) Not very confident.
(c) Fairly confident.
(d) I'm sure I'm right.

(9) The white truck is travelling toward you.[5] If the truck's speedometer reads 40mph, what is the truck's speed relative to the helicopter?

(10) What is your confidence in your answer?
(a) Just a blind guess.
(b) Not very confident.
(c) Fairly confident.
(d) I'm sure I'm right.