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MISCONCEPTIONS IN GRAPHING

John Clement

Department of Physics & Astronomy
University of Massachusetts
Amherst, MA 01003
and
Technical Education Research Centers
Cambridge, MA, 02138 USA

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Abstract: This paper attempts to organize some recent findings in the literature concerning the errors students make on tasks involving graphs and proposes some initial cognitive explanations for patterns in these errors. Some basic characteristics of a competence model are proposed for knowledge structures used in comprehending and generating graphs. Two types of common misconceptions, treating the graph as a picture and slope-height confusions, are discussed in terms of this model.

Competence Model

The ability to interpret graphs is important for mathematical literacy and for understanding the concepts of function and variable, as well as for developing basic concepts in calculus. I will first propose a partial model of four types of knowledge structures needed to comprehend graphs, shown in Figs. 1a and 1b. This should not be viewed as a Platonic "given", but as a plausible competence model which provides an initial theoretical framework for interpreting students' errors. I will consider some of the conceptions needed to solve the problem of drawing the qualitative shape of the graph of speed vs. time for a bicycle rider coming down from the top of a hill. In particular, I want to incorporate (a) the connection to a real world context and (b) the idea of variation.

Static model. Conception (1) in Fig. 1a is a naive practical representation incorporating everyday knowledge about the problem situation based on one's concrete experience with watching and riding bicycles, including the sensation of speeding up as one rides down a hill. Here I will simply refer to each knowledge structure as a conception, but they can also be thought of as occurring at different levels of representation. Conception (2) represents the idea that at a particular time, the bicycle is at a particular speed. Thus, the subject must have adequately developed concepts for speed and time and must be able to isolate these variables in the problem situation. In conception (3), I show the subject forming a spatial distance metaphor for these variables. The notation here is related to but not equivalent to that of Driver (1973). Time and speed are represented as lengths of line segments. The segments in 3 can then be mapped onto distances in the graph in conception (4). Here the length representing time must be mapped onto the distance of a point from the y axis, and the length representing speed must be mapped onto distance from the x axis.

Thus in this view a graph is essentially a model of a relationship between variables based on a metaphorical representation of variable values as lengths on line segments. The model is analogical in the sense that the relationships between line segments in the model are isomorphic to the relationships between variables in the problem context.

<u>The concept of variation in a dynamic model</u>. One limitation of the model in Fig. 1a is that it only shows correspondences between individual static values of the variables. The notion of variation seems to be missing. What is needed is a representation of correspondences between <u>changes</u> in variables. An attempt to model some aspects of the notion of variation is illustrated in Fig. 1b. Here conception (1) is the same as in Fig. 1a, but in conception (2) we find the variables of change in time and change in speed instead of simply time and speed.

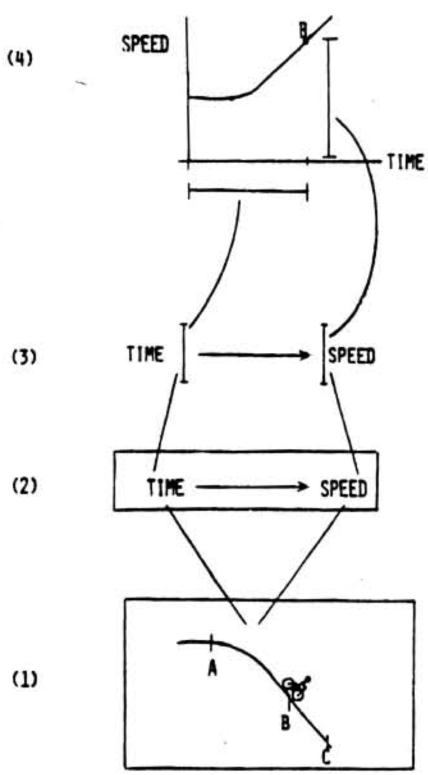

FIG 1A

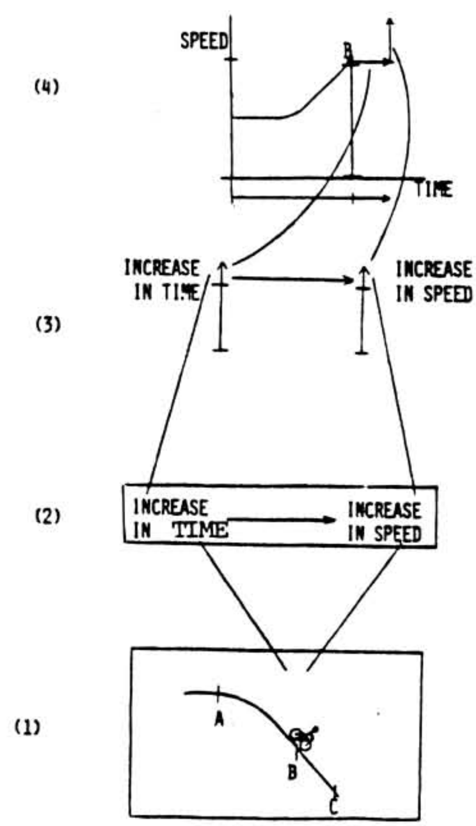

FIG 1B

Conception (2) in Fig. 1b embodies the idea that as the time elapsed increases the speed of the bicycle increases. It is this idea that would seem to carry the basic notion that "the speed is increasing with time" in which the variables are truly thought of as varying. In conception (3) the correspondence of direction of change in time with direction of change in speed is represented in terms of the spatial metaphor of line segments. An increasing value of speed is represented by an increasing length of line segment. Thus whereas the metaphor for the value of a variable was length, the metaphor for an increase in value is an increase in length, and the metaphor for continuous increase in value is a continuously growing line segment. In conception (4) the increases in the two variables are mapped onto changes in the x and y coordinates of the graph as the graph point moves. Thus motion of a point on the graph in two dimensions is used to represent changes in two variables at once. These more dynamic concepts in Fig. 1b are thought of as existing in conjunction with the more static concepts in Fig. 1a.

In this case we are still dealing at a very basic conceptual level, since we are only representing the qualitative concept of the direction of change in two variables. In summary, this competence model proposes some of the basic knowledge structures that are needed to comprehend qualitative graphs, and indicates that even at this low level, the knowledge required can be somewhat complex.

Graphing Misconceptions

Slope-height confusion. In a study of science-oriented college students who anticipated difficulty in taking science courses, McDermott, et. al. (1983) asked the following question referring to the graph in Fig. 2: "At the instant t = 2 seconds, is the speed of object A greater than, less than, or equal to the speed of object B?" As many as half the students answered incorrectly. One interpretation of this error is that students are mistakenly using the graphical feature of height instead of slope to represent speed. The model proposed here for the cognitive source of this error is a misplaced link between a successfully isolated variable and an incorrect feature of the graph, as illustrated in Fig. 2, solution 1. However, as McDermott points out, it is difficult to assign a single cause to this error since students have been observed to confound the physical concepts of speed and relative position at a conceptual level in other tasks which do not involve graphs. This second interpretation is illustrated in Fig. 2, solution 2.

The height-for-slope error has also been reported by Janvier (1978). In one of the problems in his extensive thesis on graphing, he asked students to draw graphs of height vs. time for the water level in different jars being filled with water. In one task he showed them the graph (A) in Fig. 3 for a wide jar and asked them to draw the graph for a narrower jar being filled from the same water source. Some of the students drew the parallel line of dots (B) in Fig. 3 instead of a line (C) with an increased slope. Other errors not discussed here, such as height-for-difference, slope-for-height, and slope-for-curvature substitutions also fall into this general category.

Treating the Graph as a Picture.

In another type of error the student appears to treat the graph as a literal picture of the problem situation. This error can occur, for example, when students are asked to draw a graph of speed vs. time for a bicycle traveling over a hill. In classroom observations of a college science course we noted that many students would simply draw a picture of a hill. This can happen even when the student first demonstrates the ability to describe the changes in speed verbally.

This type of error has been discussed by Monk, (1975), Kerslake (1977), Janvier (1978), and McDermott, et.al. (1983). For example, Janvier interviewed students solving a problem about a graph of speed vs. distance travelled for a race car going around a track with a number of curves in it. Many students erred when asked if they could tell how many curves were in the race track by looking at the oscillating graph. They tended to confuse this with the number of curves in the graph itself.

In making the bicycle problem error of drawing the shape of the hill, the student appears to be making a figurative correspondence between the shape of the graph and some visual characteristics of the problem scene, as shown in Fig. 4. This simplistic process contrasts to the relatively complicated process in the competence model shown in Fig. 1. Notice that in Fig. 1 the student must differentiate between and coordinate at least two separate images: (a) the problem

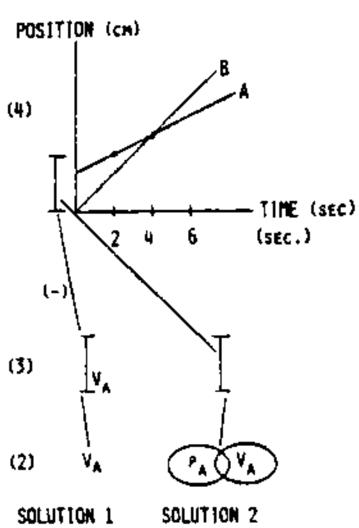

FIG. 2

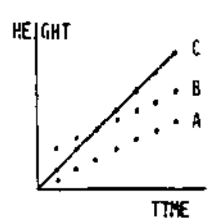

FIG 3

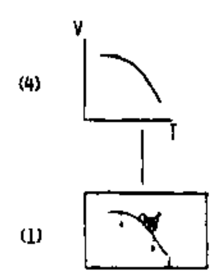

FIG 4

situation and (b) the graph. Students making the error apparently have difficulty in maintaining this differentiation.

<u>Two types of graph-as-picture errors</u>. In classroom observations with college remedial mathematics students, we found that many students generating a graph such as the one shown in Fig. 5 would say incorrectly that the two cars represented in the graph were passing each other at the point where the graphs cross. Here they seemed not to be drawing a whole picture, but to be mapping a local visual feature of the problem scene (same location) onto a similar feature of the graph. However this response could alternatively be interpreted as confounding the physical concepts of same speed and same location, as in the problem studied by McDermott et. al. mentioned earlier.

As part of an effort to develop problems elucidating specific misconceptions more clearly, David Brown of our group and I developed the following Intersection problem referring to the graph in Fig. 6.:

> Two cars (A & B) are driving toward an intersection. One car is coming from the north on Main Street, the other car is coming from the east on Green Street. Below is a graph showing each car's distance from the intersection versus time. At what time or times are the cars at the same location?

This problem does not involve speed as a variable and so avoids the alternative interpretation referred to above. When we gave it to 16 students studying remedial level mathematics in college, 13 said that 2 and 6 were times when the cars would be in the same location. Here the students seem to be focussing on the local feature of the graphs being in the same location and treating this as representing the cars being in the same location. These point by point visual correspondences can occur even if the student is not treating the graph as a complete picture of the paths the cars are driving on.

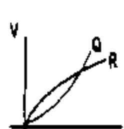

FIG. 5

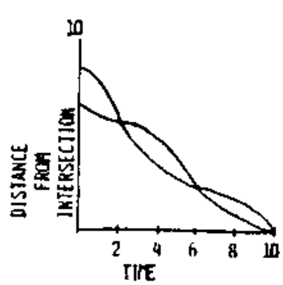

FIG. 6

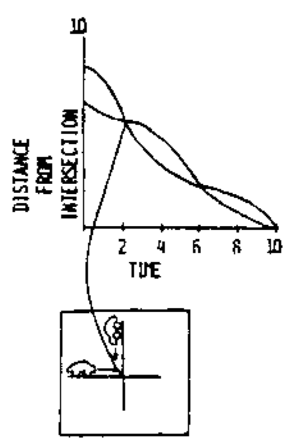

FIG 7

We can now distinguish between two types of graph-as-picture errors. We refer to the bicycle error as a global correspondence error. Here the shape of an entire problem scene is matched to the shape of the entire graph in a global manner, as shown in Fig. 4. This contrasts with the intersection problem error, which we call a feature correspondence error. Here a specific visual feature of the problem scene (path intersection) is matched to a specific feature of the graph, as shown in Fig. 7.

Conclusion

Our partial taxonomy of misconceptions in graphing now looks like:

1) Link to Incorrect Graph Feature
   a) Height for Slope   b) Slope for Height
   c) Height for Difference   d) Slope for Curvature

2) Graph as Picture
   a) Global Correspondence   b) Feature Correspondence

More generally, we can refer to all of the above as non-standard symbolization strategies. Non-standard symbolization strategies also occur in the area of algebra word problems (Clement, 1982). In addition students can confound physical context variables, which can occur in problems with or without graphs.

Space has allowed a discussion of only two of the many observable types of graphing errors. A competence model portraying both static and dynamic aspects of graphing concepts has allowed us to describe different possible sources for the errors. This model is still limited in its present form however, and its extension, refinement, and empirical validation are important tasks in our current research program.

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