Number π by Monte Carlo

I throw points at random.
What is the probability, Perce, that a point finds itself inside the circle?

Introduce small bins:
All bins are identical.

\[ P_{\text{bin}} = \frac{1}{N_{\text{bin}}} \]

Probability for a point to get into a given bin

\[ P_{\text{rel}} = \frac{N_{\text{rel}}}{N_{\text{bin}}} = \frac{A_{\text{rel}}}{A_{\text{total}}} = \frac{A_{\text{rel}}}{1} = \pi / 4 \]

\( N_{\text{rel}} \) is the number of bins covered by the circle.
\( A_{\text{rel}} \) is the area of the circle.
\( A_{\text{total}} = 1 \) is the total area.

\[ P_{\text{rel}} = \pi / 4 \]
I throw a rod of the length \( d \) on a line pad with the interline separation equal to \( d \).

What is the probability that rod crosses a line?

We introduce coordinates:
\( x \in [0, d] \) is the distance from nearest left line of the pad to the nearest end of the rod; \( \theta \in [-\pi/2, \pi/2] \) is the angle between the axis of the rod and a perpendicular-to-line direction.

We thus have a 2D sample space of coordinates \((x, \theta)\), each point of which is equivalent to each other point by symmetry: translational for \( x \) and rotational symmetry for \( \theta \). Hence, we are dealing with the case of classical probability and in complete analogy with the previous example have

\[
\text{P(cross)} = \frac{\text{Across}}{\text{Atotal}}
\]
where Across is the area in the sample space corresponding to rod crossing a line, and A_{total} is the total area of the sample space.

The rod crosses a line when

$$x + d \cdot \cos \theta > d$$

Hence,

$$\text{Across} = d \int_{-\pi/2}^{\pi/2} \cos \theta \cdot \sqrt{\theta} = 2d$$

$$A_{total} = \pi d$$

$$P_{cross} = \frac{2}{\pi}$$