Starting from the differential form
\[ d\Omega = -s dT - p dV - N dM, \]
we can produce \( 2^3 = 8 \) different forms (including the original one). Each of the forms leads to \( C_3 = 8 \) Maxwell-type relations. Below we consider each form with its 3 relations.

1) \[ d\Omega = -s dT - p dV - N dM \]

\[
\left( \frac{\partial S}{\partial V} \right)_{T,M} = \left( \frac{\partial p}{\partial T} \right)_{V,M} \tag{1.1}
\]

\[
\left( \frac{\partial S}{\partial M} \right)_{T,V} = \left( \frac{\partial N}{\partial T} \right)_{M,V} \tag{1.2}
\]

\[
\left( \frac{\partial p}{\partial M} \right)_{T,V} = \left( \frac{\partial N}{\partial V} \right)_{M,T} \tag{1.3}
\]

With the intensive-extensive analysis Eq. (1.3) becomes:

\[
\left( \frac{\partial p}{\partial M} \right)_{T} = \frac{N}{V} \equiv n, \tag{1.3'}
\]

because \( P(M,T,V) \equiv P(M,T) \), \( N(V,M,T) \equiv V \cdot n(M,T) \).
Eq. (1.1) can be written as
\[
\left( \frac{\partial P}{\partial T} \right)_m = \frac{S}{V} = S \tag{1.1}
\]
where \( S \) is the entropy density.

Indeed, \( S'(V,T,m) = V \cdot S(T,m) \) is the only way to write \( S'(V,T,m) \) as an extensive quantity.

Eq. (1.2) can be written as
\[
\left( \frac{\partial S}{\partial M} \right)_t = \left( \frac{\partial Y}{\partial T} \right)_m \tag{1.2'}
\]

3.) \[ d(U+TS) = TdS - pdV - N d\mu \]

\[
\left( \frac{\partial T}{\partial V} \right)_{s,m} = - \left( \frac{\partial P}{\partial s} \right)_{V,m} \tag{2.1}
\]

\[
\left( \frac{\partial T}{\partial M} \right)_{s,V} = - \left( \frac{\partial N}{\partial S} \right)_{M,V} \tag{2.2}
\]

\[
\left( \frac{\partial P}{\partial M} \right)_{V,s} = \left( \frac{\partial N}{\partial V} \right)_{M,S} \tag{2.3}
\]
3) \[ d(u_2 + p_N) = -s dT - p dV + \mu dN \]

\[ \left( \frac{\partial S}{\partial V} \right)_{T,N} = \left( \frac{\partial P}{\partial T} \right)_{V,N} \quad (3.1) \]

\[ \left( \frac{\partial S}{\partial N} \right)_{T,V} = -\left( \frac{\partial M}{\partial T} \right)_{N,V} \quad (3.2) \]

\[ \left( \frac{\partial P}{\partial N} \right)_{V,T} = -\left( \frac{\partial N}{\partial S} \right)_{N,T} \quad (3.3) \]

4) \[ d(\frac{u_2 + TS + p_N}{E}) = T dS - p dV + \mu dN \]

\[ \left( \frac{\partial T}{\partial V} \right)_{S,N} = -\left( \frac{\partial P}{\partial S} \right)_{V,N} \quad (4.1) \]

\[ \left( \frac{\partial T}{\partial N} \right)_{S,V} = +\left( \frac{\partial M}{\partial S} \right)_{N,V} \quad (4.2) \]

\[ \left( \frac{\partial P}{\partial N} \right)_{V,S} = -\left( \frac{\partial N}{\partial V} \right)_{N,S} \quad (4.3) \]
5) \[ \frac{d}{dt} \left( \frac{N}{P} + PV + \mu N \right) = -sP + VdP + \mu dN \]
\[ \mu N = c \]

\[ \left( \frac{\partial S}{\partial P} \right)_{T,N} = - \left( \frac{\partial V}{\partial T} \right)_{P,N} \quad (5.1) \]

\[ \left( \frac{\partial S}{\partial N} \right)_{T,P} = - \left( \frac{\partial M}{\partial T} \right)_{N,P} \quad (5.2) \]

\[ \left( \frac{\partial V}{\partial N} \right)_{P,T} = \left( \frac{\partial M}{\partial P} \right)_{N,T} \quad (5.3) \]

On the basis of extensive-intensive analysis:

\[ V(N, P, T) = \frac{N}{W(P, T)} \quad , \quad m(N, P, T) = m(P, T) \]

\[ S(N, T, P) = \dot{S}(T, P) \cdot N \], where \( \dot{S} \) is the entropy per particle.

The relation between \( S \) and \( \dot{S} \) is as follows:

\[ (\dot{S} \cdot N \equiv \dot{S} \equiv \dot{S} \cdot V) \implies \left[ \dot{S} = \frac{\dot{S}}{N} \right] \]

Hence,

\[ \left( \frac{\partial M}{\partial T} \right)_P = \dot{S} \quad (5.2') \]

\[ \left( \frac{\partial M}{\partial P} \right)_T = \frac{1}{m} \quad (5.3') \]

This is basically the same as (1.3').
6.) 
\[ d \left( \frac{U_2 + TS + PV + \mu N}{T S + P N} \right) = T d S + V d P + M d N \]

\[ \left( \frac{\partial T}{\partial N} \right)_{s, p} = \left( \frac{\partial M}{\partial P} \right)_{N, P} \]  
\[ = (6.1) \]

\[ \left( \frac{\partial T}{\partial P} \right)_{s, N} = \left( \frac{\partial V}{\partial S} \right)_{P, N} \]  
\[ = (6.2) \]

\[ \left( \frac{\partial V}{\partial N} \right)_{P, S} = \left( \frac{\partial M}{\partial P} \right)_{N, S} \]  
\[ = (6.3) \]

7.) 
\[ d \left( \frac{U_2 + TS + PV}{T S} \right) = T d S + V d P = N d M \]

\[ \left( \frac{\partial T}{\partial P} \right)_{M, S} = \left( \frac{\partial V}{\partial S} \right)_{P, M} \]  
\[ = (7.1) \]

\[ \left( \frac{\partial T}{\partial M} \right)_{s, P} = -\left( \frac{\partial V}{\partial S} \right)_{M, P} \]  
\[ = (7.2) \]

\[ \left( \frac{\partial V}{\partial M} \right)_{P, S} = -\left( \frac{\partial M}{\partial P} \right)_{M, S} \]  
\[ = (7.3) \]
By extensive-intensive analysis:

\[ T(s, p, m) = T(p, m), \quad N(s, p, m) = \frac{s}{S(p, m)} \]

Hence,

\[ (\frac{\partial T}{\partial m})_p = -\frac{1}{S} \quad (7.2') \]

And this is basically (5.21)

Similarly,

\[ (\frac{\partial T}{\partial p})_m = \frac{1}{S} \quad (7.1') \]

And this is basically (1.11)

8.) Finally,

\[ d(u + pv) = -sdT + vdp - ndm \quad (\star) \]

So, the left-hand side is not a function and we cannot use (\star) to produce new Maxwell-type relations. Eq. (\star) can be written as (divide by \( V \))

\[ -sdT + dp - ndm = 0 \quad (\star \star) \]

Eq. (\star \star) establish the relations between the derivatives of intensive variables. From this equation we immediately get (by fixing one of the three variables):

\[ (\frac{\partial P}{\partial m})_T = n, \quad (\frac{\partial P}{\partial T})_m = 1, \quad (\frac{\partial m}{\partial T})_p = -\frac{s}{n} = -\frac{S}{n} \]
What happens if we formally use Eq. (1) to derive Maxwell relations? We will get the relations in terms of derivatives of the form:

\[
\left( \frac{\partial \text{intensive}_1}{\partial \text{intensive}_2} \right)_{\text{intensive}_3}
\]

For example,

\[
\left( \frac{\partial S}{\partial P} \right)_{\mu, T}
\]

All these derivatives are ill-defined, because fixing two intensive variables we inevitably fix the third one. The inverse counterparts of these derivative are identically equal to zero. For example,

\[
\left( \frac{\partial P}{\partial S} \right)_{\mu, T} = 0,
\]

because \( P(S, \mu, T) = P(\mu, T) \) — independent of \( S \).