Unit 6 – Estimation
Week #9 - Practice Problems
SOLUTIONS

The results of IQ tests are known to be normally distributed. Suppose that in 2007, the distribution of IQ test scores for persons aged 18-35 years has a variance $\sigma^2 = 225$. A random sample of 9 persons take the IQ test. The sample mean score is 115.

1. Calculate the 50%, 75%, 90% and 95% confidence interval estimates of the unknown population mean IQ score.

Answer:

50% CI	(111.6 , 118.4)
75% CI	(109.2, , 120.8)
90% CI	(106.8 , 123.2)
95% CI	(105.2 , 124.8)

Solution:

Let the random variable X = IQ test result assumed normal with:

μ unknown
$\sigma^2 = 225$, known
$\sigma = 15$, known

Confidence interval estimate of the unknown mean is given by:

estimate $\pm$ { critical value} { se of estimate}

where,

estimate = observed sample mean = 115
critical value = $(1 - \alpha/2)100$th percentile Normal(0,1)
se of estimate = standard error of sample mean

$$= \sqrt{225/9}$$
$$= 15/3$$
$$= 5$$

For 50% confidence interval estimate:

$1 - \alpha = (1 - 0.50) = 0.50$

$\alpha/2 = 0.50 / 2 = 0.25$

Therefore want (1 - .25)100th or 75th percentile = 0.6745
The required confidence interval estimate is thus,

estimate $\pm$ { critical value} { se of estimate}
= 115 + { 0.6745 } { 5 }
= (111.6 , 118.4)

For 75% confidence interval estimate:

$1 - \alpha = (1 - 0.25) = 0.75$

$\alpha/2 = 0.25 / 2 = 0.125$

Therefore want (1 - .125)100th or 87.5th percentile = 1.1505
The required confidence interval estimate is thus,

estimate $\pm$ { critical value} { se of estimate}
= 115 $\pm$ { 1.1505 } { 5 }
= (109.2 , 120.8)

For 90% confidence interval estimate:

$1 - \alpha = (1 - 0.10) = 0.90$

$\alpha/2 = 0.10 / 2 = 0.05$

Therefore want (1 - .05)100th or 95th percentile = 1.645
The required confidence interval estimate is thus,

estimate $\pm$ { critical value} { se of estimate}
= 115 $\pm$ { 1.645 } { 5 }
= (106.8 , 123.2)

For 95% confidence interval estimate:

$1 - \alpha = (1 - 0.05) = 0.95$

$\alpha/2 = 0.05 / 2 = 0.025$

Therefore want (1 - .025)100th or 97.5th percentile = 1.96
The required confidence interval estimate is thus,

estimate $\pm$ { critical value} { se of estimate}
= 115 $\pm$ { 1.96 } { 5 }
= (105.2 , 124.8)

2. What trade-offs are involved in reporting one interval estimate over another?

Answer:
For a given probability distribution with a known variance and a fixed sample size,

(i) Increasing the confidence coefficient is at the price of a wider confidence interval.

(ii) Decreasing the width of a confidence interval estimate is at the price of a lower confidence coefficient.

This is apparent in the following summary of the solution to Exercise #1.

Confidence Coefficient	Lower Limit	Upper Limit	Width
.50	111.6	118.4	6.8
.75	109.2	120.8	11.6
.90	106.8	123.2	16.4
.95	105.2	124.8	19.6

3. If it is known that the population mean IQ score is $\mu = 105$, what proportion of samples of size 6 will result in sample mean values in the interval [135,150]?

Answer: << .0001

Solution:
Solve for Prob [$135 < \bar{X}_{n=6} < 150$] using the standardization formula.
Note that $\bar{X}_{n=6}$ is normally distributed with:

$$\mu_{\bar{X}} = 105$$

$$\sigma^2_{\bar{X}} = \frac{\sigma^2}{n} = \frac{225}{6} = 37.5$$

$$SE_{\bar{X}} = \sqrt{\sigma^2_{\bar{X}}} = \sqrt{37.5} = 6.1238$$

Thus,

$$\text{Probability}\,[\,135 < \bar{X}_{n=6} < 150\,] = \text{Probability}\left[\frac{135-105}{6.1238} < \frac{\bar{X}_{n=6} - \mu_{\bar{X}}}{SE_{\bar{X}}} < \frac{150-105}{6.1238}\right]$$

$$= \text{Probability}\,[\,4.89 < \text{Z-score} < 7.34\,] << .0001$$