Topic 5 – The Normal Distribution
Week #8 - Practice Problems
SOLUTIONS

1. Suppose the distribution of GRE scores satisfies the assumptions of normality with a mean score of μ=600 and a standard deviation of σ=80.

   a. What is the probability of a score less than 450 or greater than 750? **Answer**: **.0608**
   Solution: Probability { score < 450 OR score > 750 }

$$\begin{aligned} &= \text{pr}[X < 450] + \text{pr}[X > 750] \\ &= \text{pr}\left[Z < \left(\frac{450\text{-}600}{80}\right)\right] + \text{pr}\left[Z > \left(\frac{750\text{-}600}{80}\right)\right] \\ &= \text{pr}[Z < -1.875] + \text{pr}[Z > +1.875] \\ &= 2\text{pr}[Z > +1.875] \\ &= 2(.0304) \\ &= .0608 \end{aligned}$$

   b. What proportion of students have scores between 450 and 750? **Answer**: **.9392**
   Solution: Proportion of students with scores between 450 and 750

$$\begin{aligned} &= \text{pr}[450 < X < 750] \\ &=1 - \text{pr}[x < 450 \text{ or } X > 750] \\ &= 1 - .0608 \\ &= .9392 \end{aligned}$$

---

   c. What score is equal to the 95th percentile? **Answer**: **731.2**
   Solution: For Z~Normal(0,1)

$$\text{pr}\left[Z_{.95} < 1.645\right] = .95$$

From $Z = \dfrac{X - \mu}{\sigma}$ substitute

$$1.645 = \frac{X_{.95} - 600}{80}$$

Thus, $X_{.95} = \sigma Z_{.95} + \mu$

$= (80)[1.645] + 600$

$= 731.6$

---

2. The Chapin Social Insight Test evaluates how accurately the subject appraises other people. In the reference population used to develop the test, scores is normally distributed with mean μ=25 and standard deviation σ=5. The range of possible scores is 0 to 41.

a. What proportion of the population has scores below 20 on the Chapin test? **Answer**: **.1587**

Solution: $\text{pr}(X < 20) = \text{pr}\left[Z < \frac{20 - 25}{5} \right] = \text{pr}[Z < -1] = .1587$

b. What proportion has scores below 10? **Answer**: **.0014**

Solution: $\text{pr}(X < 10) = \text{pr}[Z < \frac{10 - 25}{5}] = \text{pr}[Z < -3] = .0014$

c. How high a score must you have in order to be in the top quarter of the population in social insight? **Answer**: **28.35**

Solution: $pr(Z > 0.6745) = .25$

Thus, $X = \sigma Z + \mu$

$= (5)[0.6745] + 25$

$= 28.3725$

---

3. A normal distribution has mean μ=100 and standard deviation σ=15 (for example, IQ). Give limits, symmetric about the mean, within which 95% of the population would lie:

Solution: First obtain an interval for Z~Normal(0,1)

$$\text{pr}\ [-1.96 < Z < +1.96] = .95$$

Next, recall that the standard error, SE, of $\overline{X}$, is related to σ via $se[\overline{X}] = \sigma / \sqrt{n}$

And the mean of $\overline{X}$ is $E[\overline{X}] = \mu$

Thus, the standardization formula can be manipulated to yield a formula for $\overline{X}$ in terms of Z.

From $Z = \dfrac{\bar{X} - E[\bar{X}]}{SE[\bar{X}]}$, solve for $\bar{X}$.

$$\bar{X} = \{se(\bar{X})\}Z + \mu$$
$$= \left(\sigma / \sqrt{n}\right)Z + \mu$$

Now we can make a little table

	Mean	SE	Lower limit	Upper limit
Z	0	1	-1.96	+1.96
X	100	15	$(15)(-1.96)+100=70.6$	$(15)(+1.96)+100=129.4$
$\bar{X}_{n=4}$	100	15/2	$(15/2)(-1.96)+100=85.3$	$(15/2)(+1.96)+100=114.7$
$\bar{X}_{n=16}$	100	15/4	$(15/4)(-1.96)+100=92.65$	$(15/4)(+1.96)+100=107.35$
$\bar{X}_{n=100}$	100	15/10	$(15/10)(-1.96)+100=97.06$	$(15/10)(+1.96)+100=102.94$

a. Individual observations. **Answer**: **70.6, 129.4**

b. Means of 4 observations. **Answer**: **85.3, 114.7**

c. Means of 16 observations. **Answer**: **92.65, 107.35**

d. Means of 100 observations. **Answer**: **97.06, 102.94**

e. Write down an expression for the width of the limits symmetric about the mean, within which 95% of the population of means of samples of size n would lie.

Solution:

Width of limits symmetric about the mean is therefore

= (Upper endpoint) - (Lower endpoint)

$$= \left[\frac{\sigma}{\sqrt{n}}(Z)+\mu\right] - \left[\frac{\sigma}{\sqrt{n}}(-Z)+\mu\right]$$

$$= \frac{2\sigma Z}{\sqrt{n}}$$

Thus, the width gets smaller as the sample size n gets larger. On to confidence intervals in unit 6!