Unit 4 – Bernoulli and Binomial Distributions
Week #6 - Practice Problems

SOLUTIONS
Revised (enhanced for q4) 10-29-2008

1. *This exercise gives you practice in calculating "number of ways to choose".*
Suppose my 2008 BE540 class that meets "in class" in Worcester, MA has 10 students.

a. I wish to pair up students to work on homework together. How many pairs of 2 students could I form?

Answer: 45

Solution: 10 choose 2 = $\binom{10}{2} = \frac{10!}{2!\ 8!} = \frac{(10)(9)\cancel{8!}}{(2)(1)\cancel{8!}} = \frac{90}{2} = 45$

b. Next, I wish to form project groups of size 5. How many groups of 5 students could I form?

Answer: 252

Solution: 10 choose 5 = $\binom{10}{5} = \frac{10!}{5!\ 5!} = \frac{(10)(9)(8)(7)(6)\cancel{5!}}{(5)(4)(3)(2)(1)\cancel{5!}} = 252$

2. *This exercise is a straightforward application of a binomial probability calculation.*
A die will be rolled six times. What are the chances that, over all six rolls, the die lands neither ace nor deuce exactly 2 times?

Answer: .08

Solution:

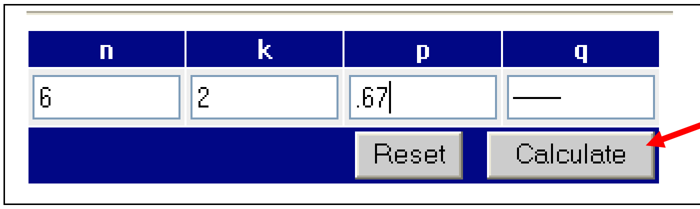
Success on roll of die occurs for event "neither ace nor deuce". This has probability 4/6=.67
Outcome of interest is "exactly 2 successes and 4 failures".
Define random variable X = # successes on six rolls of one die
Binomial number of trials, N = 6
Event probability π=.67
Want Pr [X = 2]

Pr [X=2 | N=6 and π=.67] = $\binom{6}{2}[.67]^2[1-.67]^4 = .07985$

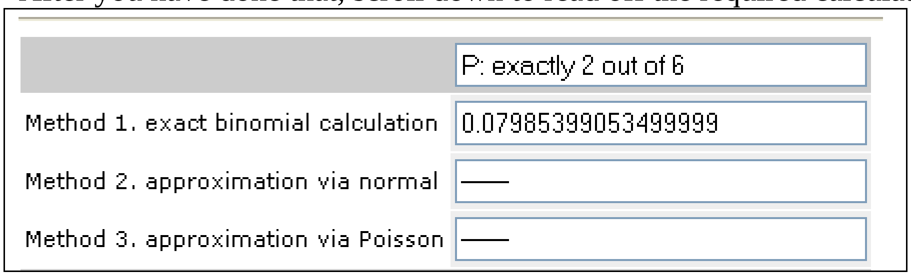
Probability[one scenario of 2 success and 4 failure] = $[4/6]^{2\text{ times}}\,[2/6]^{4\text{ times}} = .0055$

Number of scenarios yielding exactly 2 success and 4 failure] = $\binom{6}{2} = \frac{6!}{2!4!} = \frac{(6)(5)\cancel{4!}}{(2)(1)\cancel{4!}} = 15$

You can also get this using the binomial calculator on line. From the course website, click on the Bernoulli and Binomial unit from the left navigation bar. From there, scroll down and click on Vassar Statistics Binomial Calculator. Scroll down and fill in the following values. Calculate



After you have done that, scroll down to read off the required calculation.

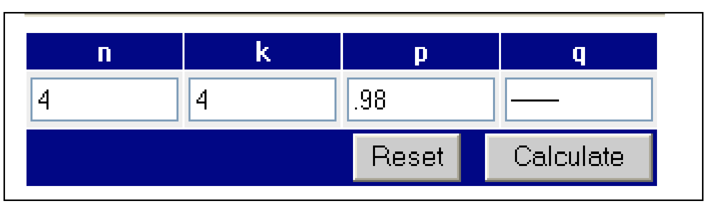


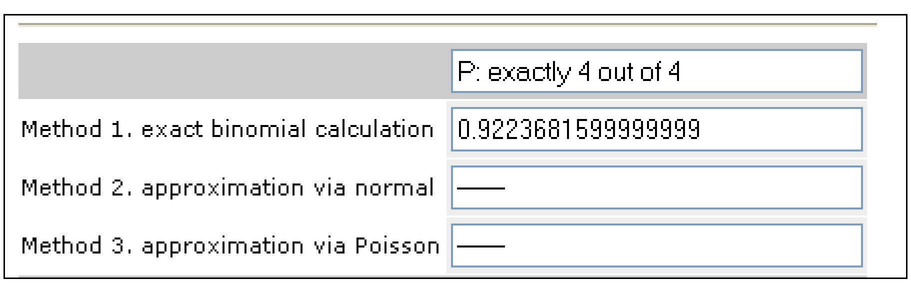
3. *This is also an application of a binomial probability calculation.*
Suppose that, in the general population, there is a 2% chance that a child will be born with a genetic anomaly. What is the probability that no congenital anomaly will be found among four random births?

Answer: .92

Solution:

Success occurs for event "no congenital anomaly". This has probability =.98
Outcome of interest is "exactly 4 successes and 0 failures".
Define random variable X = # successes among four random births
Binomial number of trials, N = 4
Event probability π=.98
Want Pr [X = 4]





4. *This is a slightly harder application of a binomial probability calculation.* Suppose it is known that, for a given couple, there is a 25% chance that a child of theirs will have a particular recessive disease. If they have three children, what are the chances that at least one of them will be affected?

Answer: .58

Solution:

The event being investigated is that of a "particular recessive disease"

Event probability π=.25

The number of trials considered is N=3

There is more than one way to reason out the solution.

One approach:

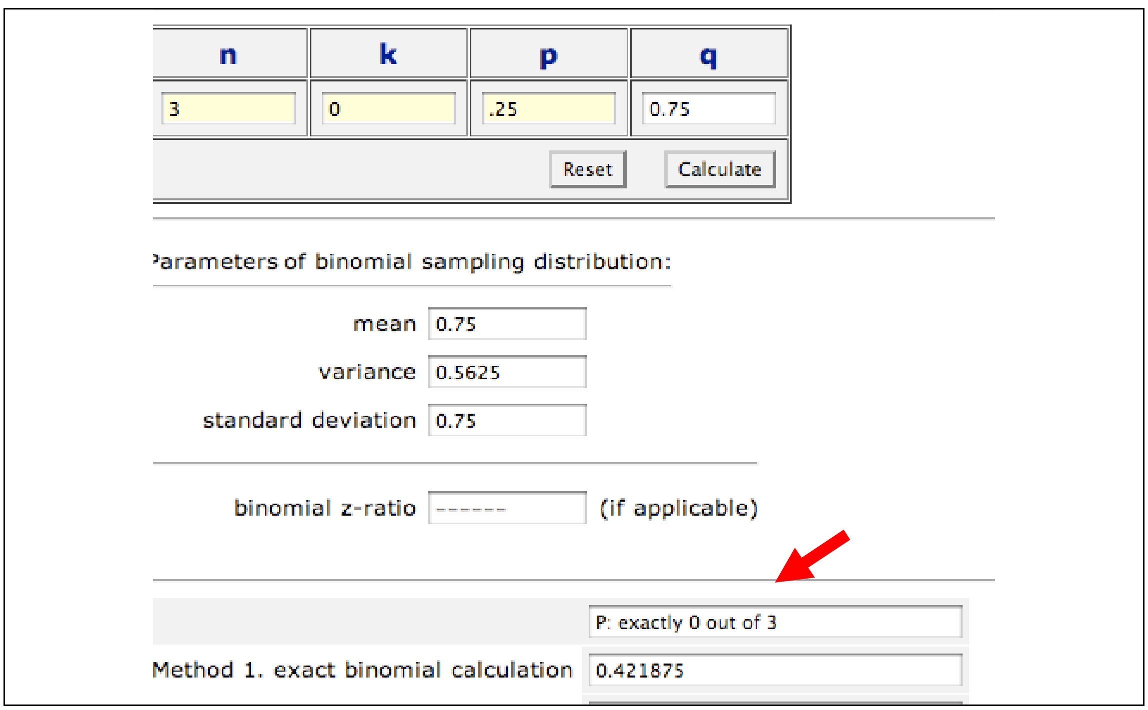
"chances that at least one will be affected"

= 1 – "chances that zero will be affected"

= 1 – Probability [X=0] for X distributed Binomial (N=3, π=.25)

= 1 - .421775

= .58



Another approach:

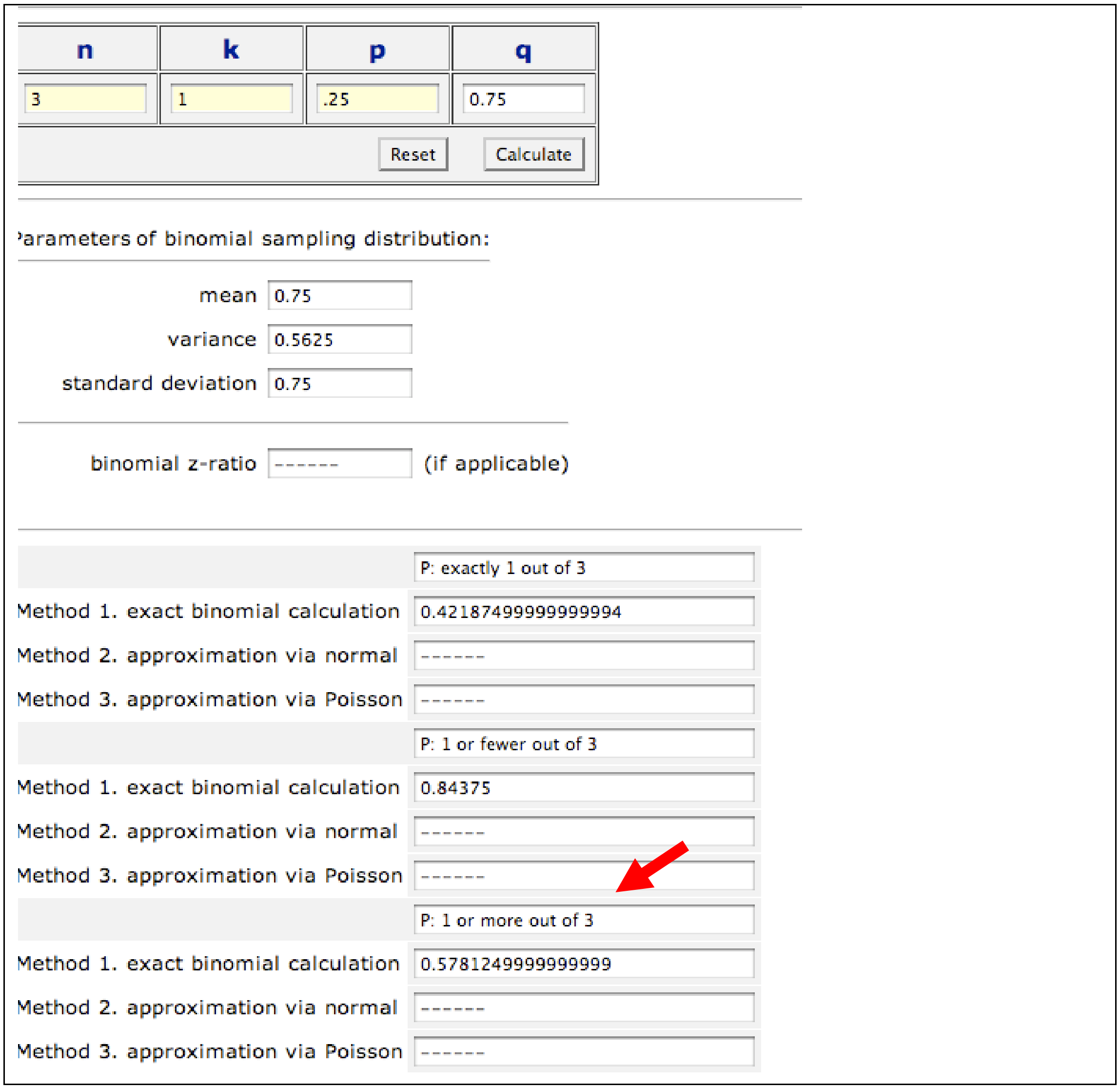
"chances that at least one will be affected"

= "chances that one or two or three will be affected"

= Probability [X=1] + Probability [X=2] + Probability [X=3]

for X distributed Binomial (N=3, π=.25)

= .58



n	k	p	q
3	1	.25	0.75

Reset Calculate

Parameters of binomial sampling distribution:

mean	0.75
variance	0.5625
standard deviation	0.75
binomial z-ratio	------ (if applicable)

	P: exactly 1 out of 3
Method 1. exact binomial calculation	0.42187499999999994
Method 2. approximation via normal	------
Method 3. approximation via Poisson	------
	P: 1 or fewer out of 3
Method 1. exact binomial calculation	0.84375
Method 2. approximation via normal	------
Method 3. approximation via Poisson	------
	P: 1 or more out of 3
Method 1. exact binomial calculation	0.5781249999999999
Method 2. approximation via normal	------
Method 3. approximation via Poisson	------

Yet another approach:

"chances that at least one will be affected"

= 1 - "chances that zero are affected"

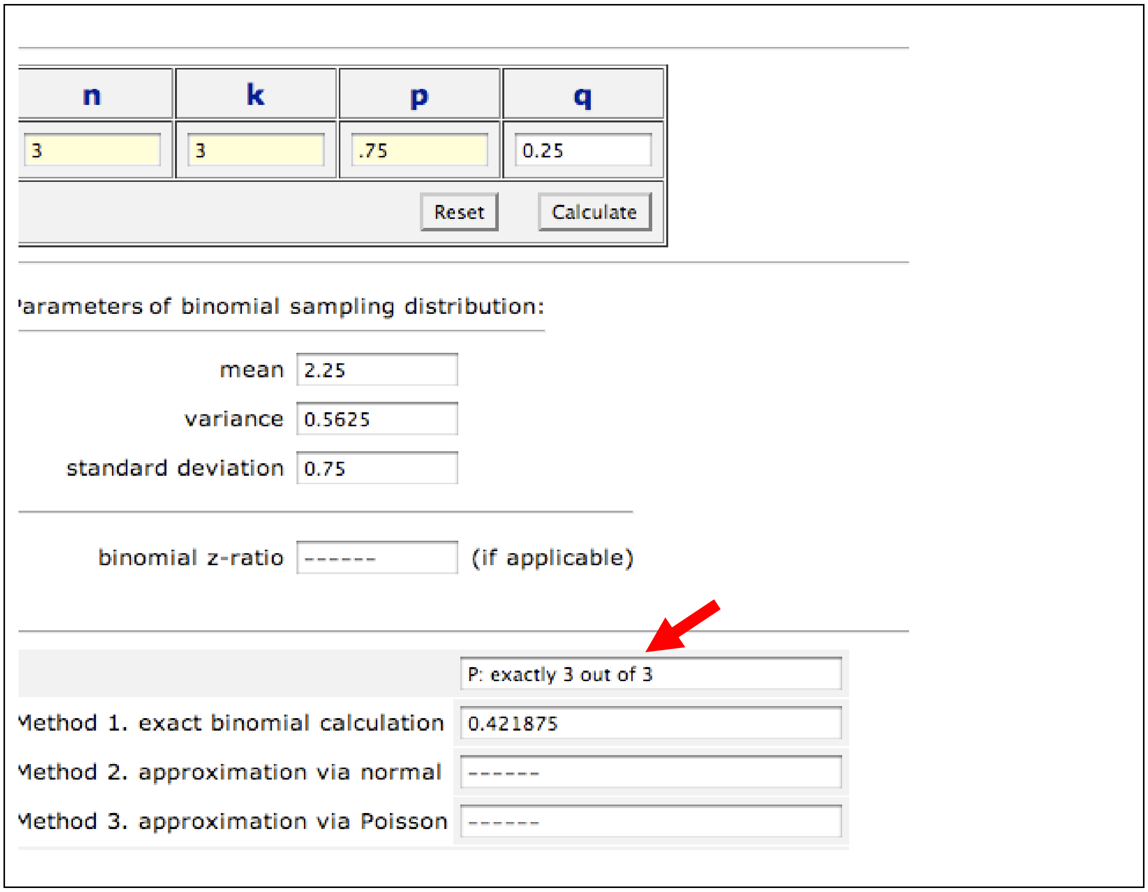
= 1 – "chances that ALL 3 are DISEASE FREE"

… so we consider the event of being disease free which occurs with probability = .75

= 1 – Probability [X=3] for X distributed Binomial (N=3, π=.75)

= 1 - .421875

= .58



5. *This exercise is the most involved.*
Suppose a quiz contains 20 true/false questions. You know the correct answer to the first 10 questions. You have no idea of the correct answer to questions 11 through 20 and decide to answer each using the coin toss method. Calculate the probability of obtaining a total quiz score of at least 85%.

Answer: .17

Solution:
As there are 20 questions, each is worth 5 points.
Having answered the first 10 questions correctly, you already have 50 points.
Remaining to get are 35 points or greater.
This corresponds to 7 or more correct answers among the last 10 questions.

Define random variable X = # correct answers among questions #11-20
Binomial number of trials, N = 10
Event probability π=.50 because you are using the coin toss method.
Want Pr [$X \geq 7$]

n	k	p	q
10	7	.5	——
		Reset	Calculate

	P: 7 or more out of 10
Method 1. exact binomial calculation	0.171875
Method 2. approximation via normal	0.171056
Method 3. approximation via Poisson	——