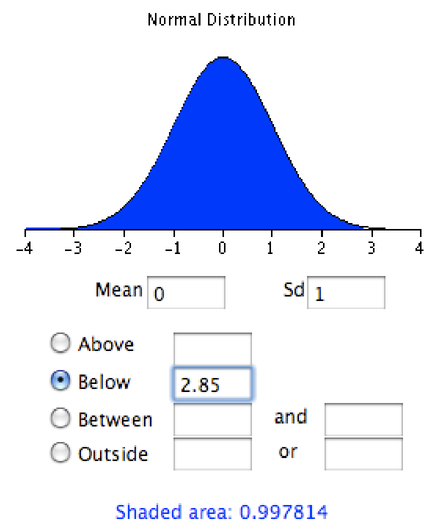
Unit 5 – The Normal Distribution
Practice Problems

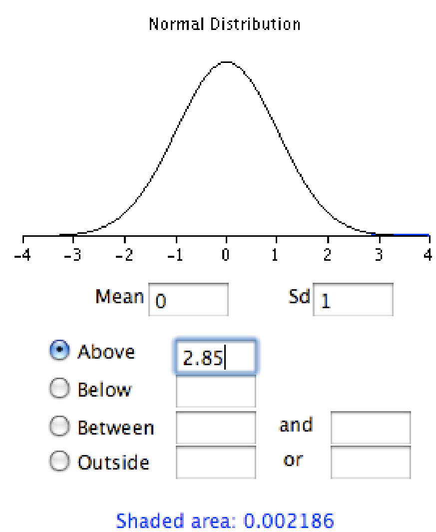
SOLUTIONS

Notes – To obtain the pictures below, I used the link available on the course website.
http://davidmlane.com/hyperstat/z_table.html

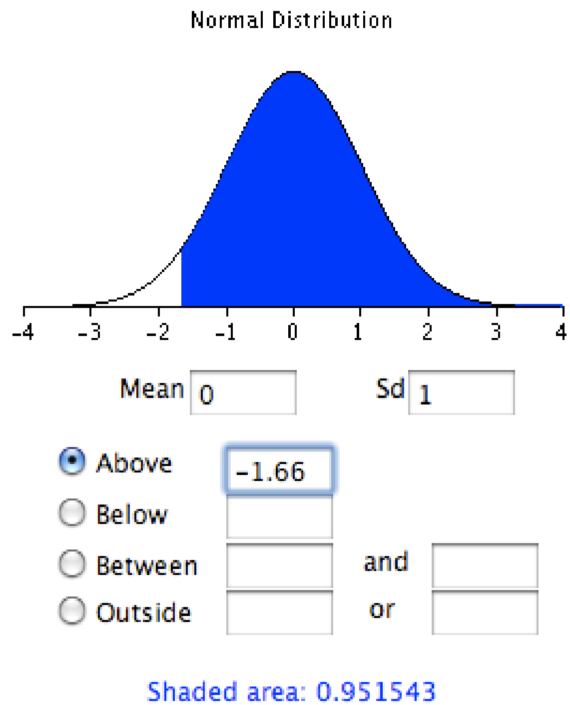
#1a. Pr (Z < 2.85) = .9978



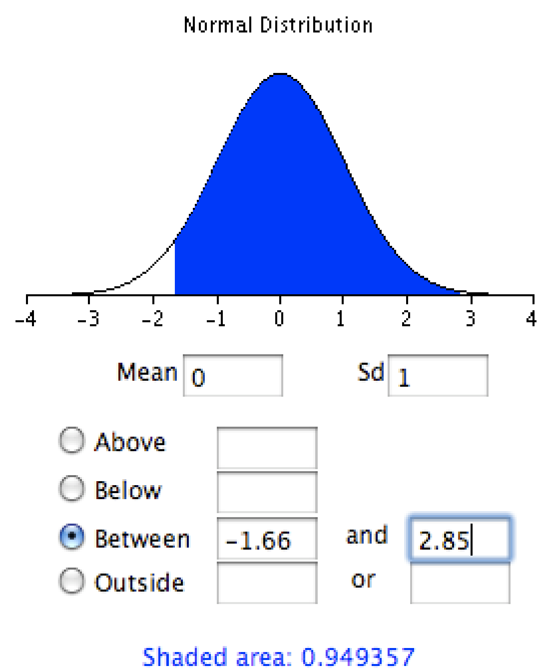
#1b. Pr (Z > 2.85) = .0022



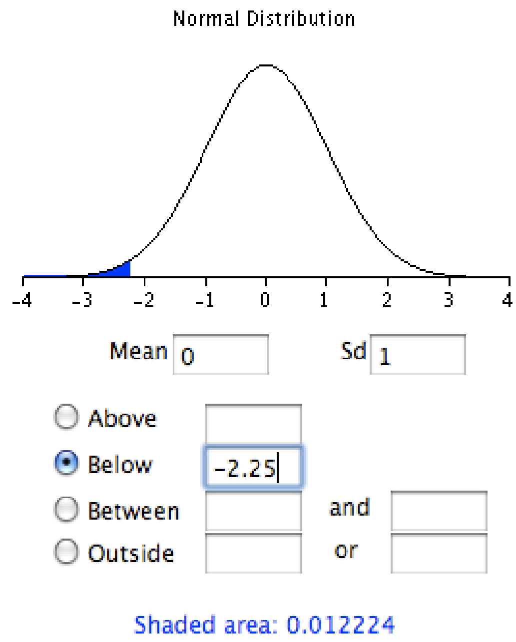
#1c. Pr (Z > -1.66) = .9515



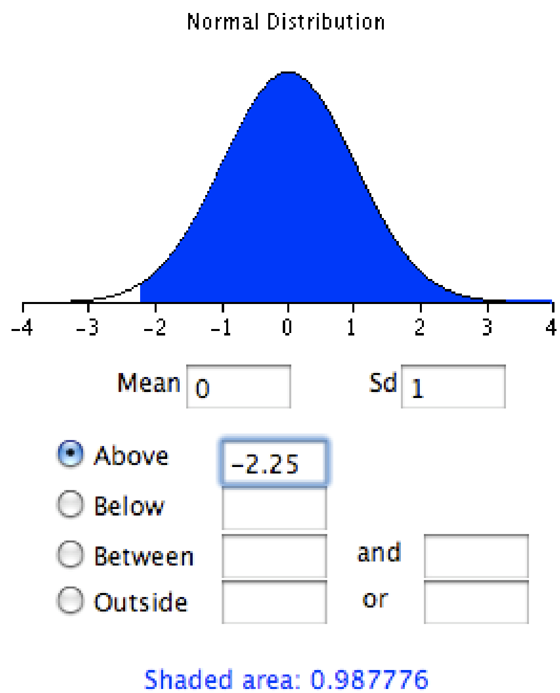
#1d. Pr (-1.66 < Z < 2.85) = .9494



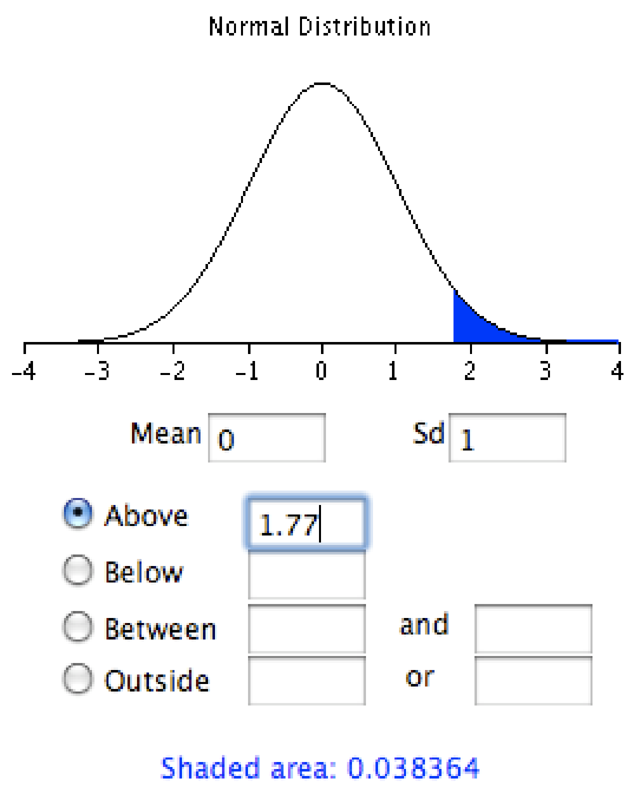
#1e. Pr (Z < -2.25) = .0122



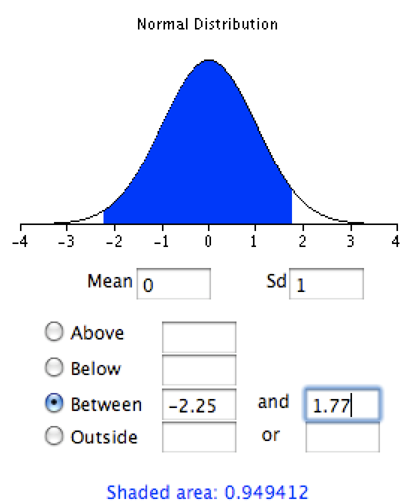
#1f. Pr (Z > -2.25) = .9878



#1g. Pr (Z > 1.77) = .0384



#1h. Pr (-2.25 < Z < 1.77) = .9494



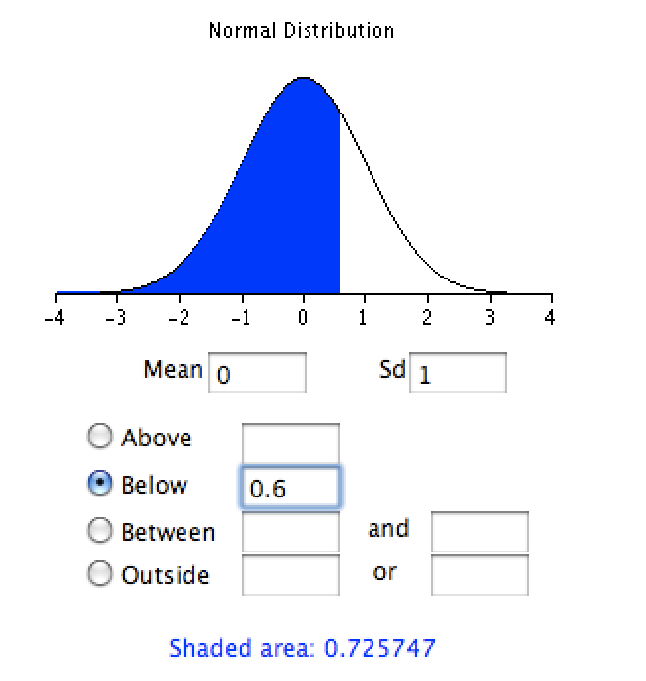
#2a.

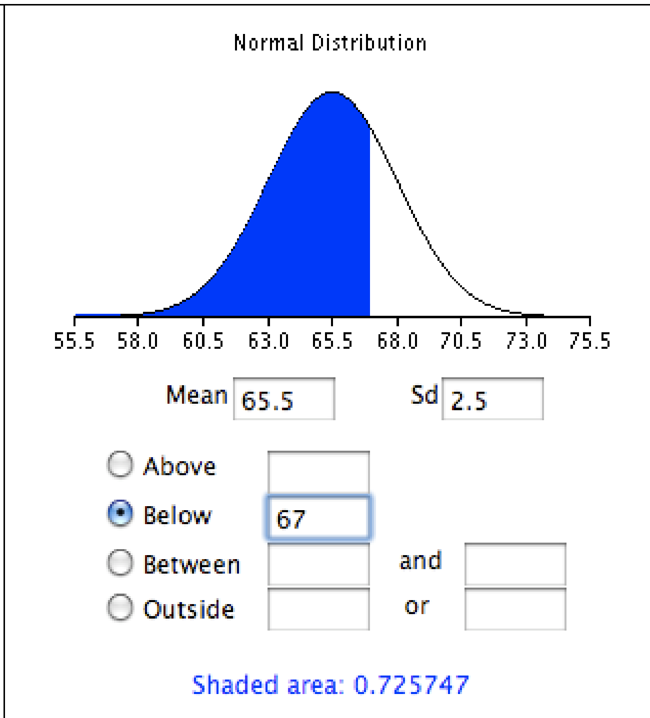
$$\text{pr}(X < 67) = \text{pr}\left[\left(\frac{X-\mu}{\sigma}\right) < \left(\frac{67-\mu}{\sigma}\right)\right]$$

$$= \text{pr}\left[Z < \left(\frac{67-65.5}{2.5}\right)\right]$$

$$= \text{pr}[Z < .6]$$

$$= .7257$$



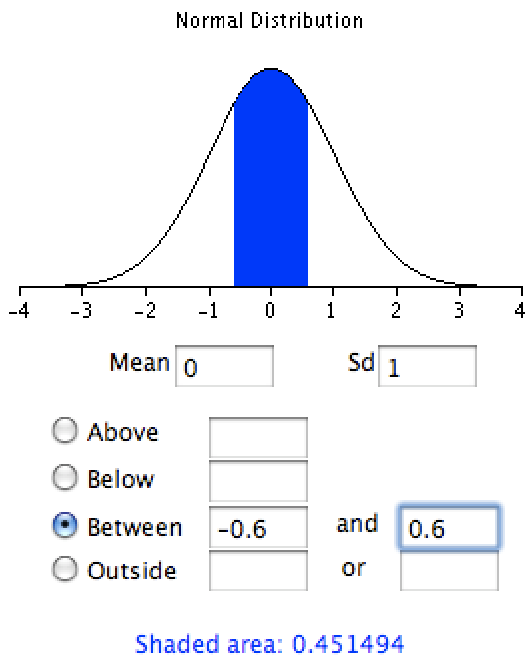


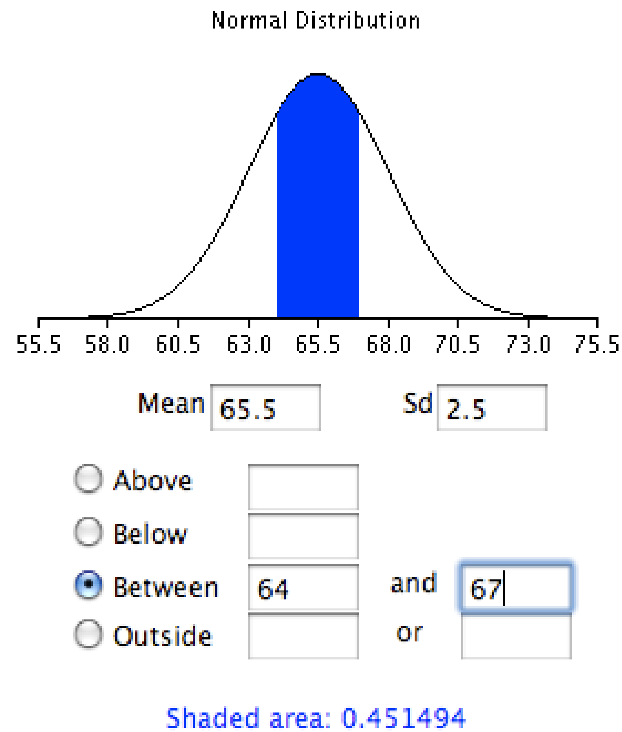
#2b.

$$pr(64 < X < 67) = pr\left[\left(\frac{64\text{-}65.5}{2.5}\right) < Z < \left(\frac{67\text{-}65.5}{2.5}\right)\right]$$

$$= pr[-0.6 < Z < +0.6]$$

$$=.4515$$

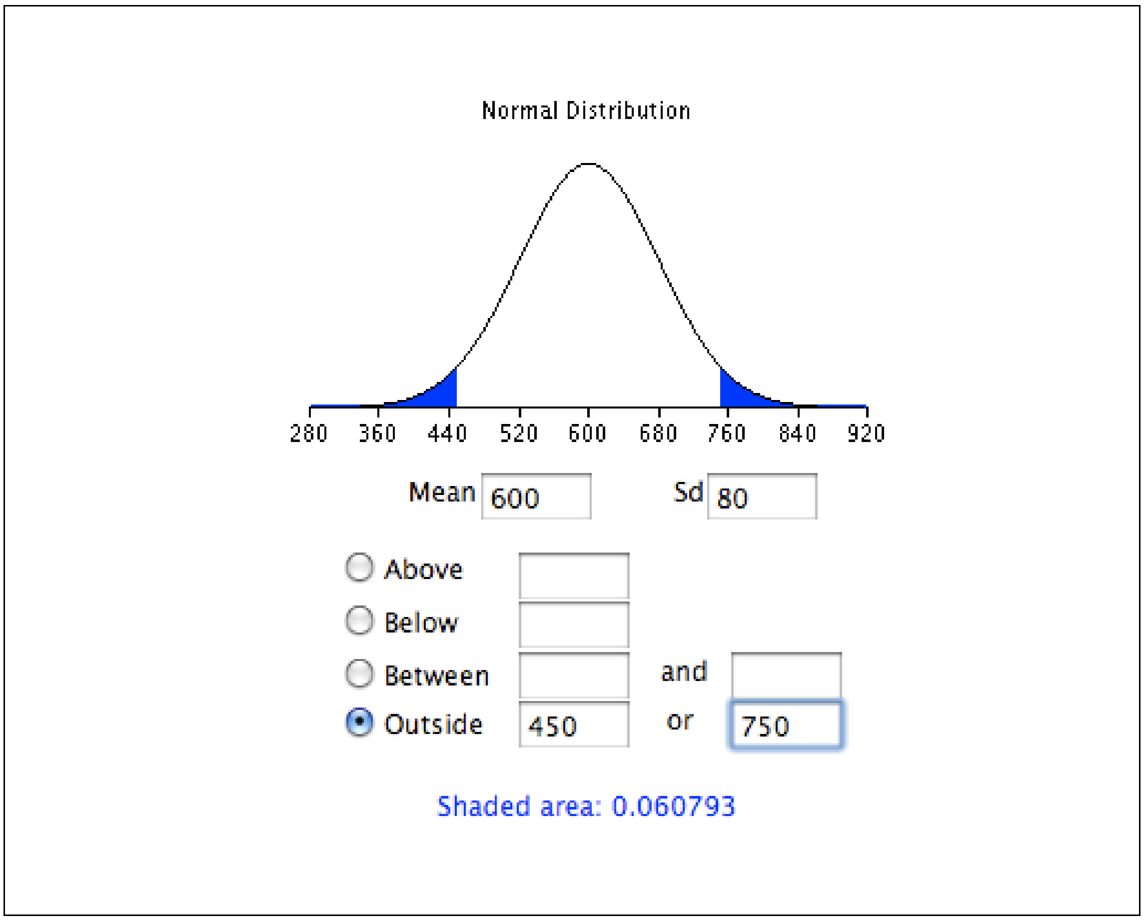




3. Suppose the distribution of GRE scores satisfies the assumptions of normality with a mean score of μ=600 and a standard deviation of σ=80.

a. What is the probability of a score less than 450 or greater than 750? **Answer**: **.0608**
Solution: Probability { score < 450 OR score > 750 }

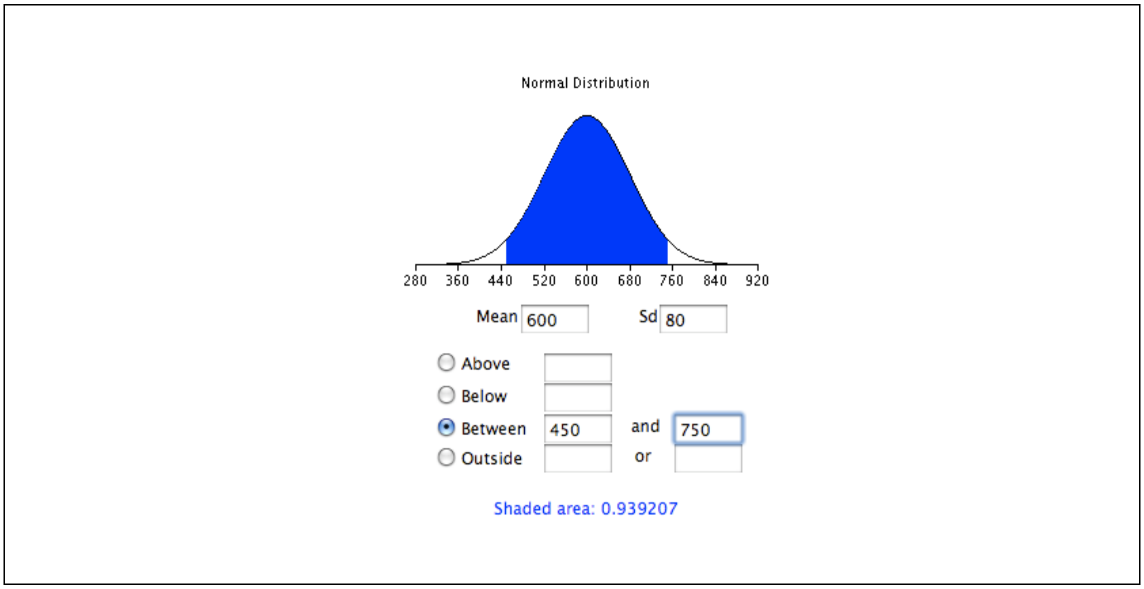
$$= \text{pr}[X < 450] + \text{pr}[X > 750]$$
$$= .0608$$



url used: http://davidmlane.com/hyperstat/z_table.html

b. What proportion of students have scores between 450 and 750? **Answer**: **.9392**
Solution: Proportion of students with scores between 450 and 750

$$= \text{pr}\left[450 < X < 750\right]$$
$$= .9392$$



url used: http://davidmlane.com/hyperstat/z_table.html

c. What score is equal to the 95th percentile? **Answer**: **731.2**
Solution: For Z~Normal(0,1)

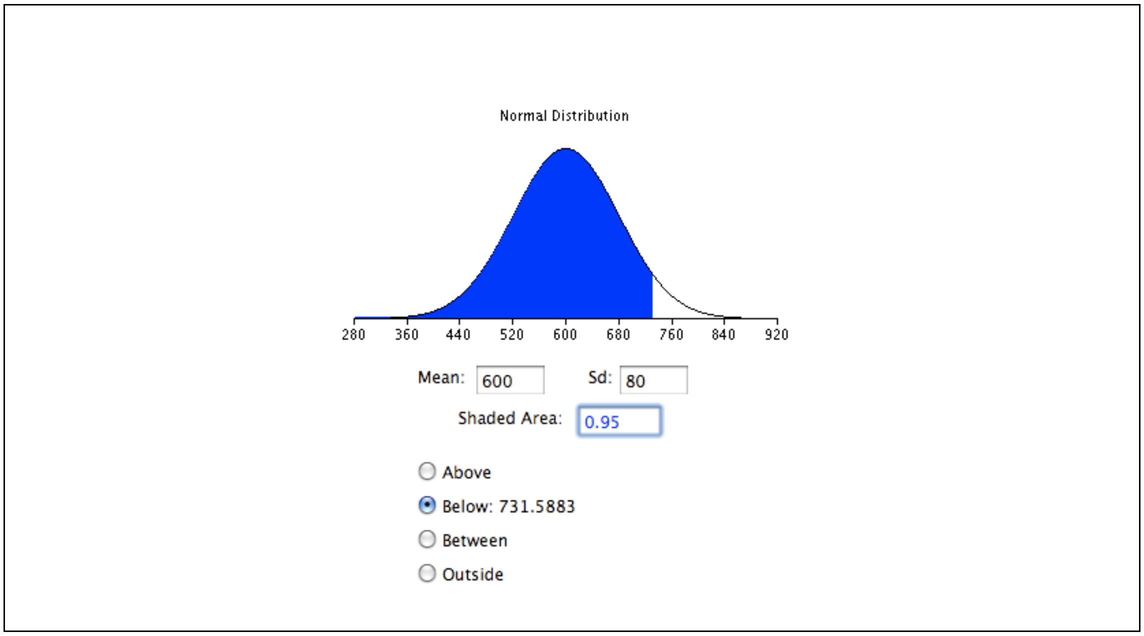
$$\text{pr}\left[Z_{.95} < 1.645\right] = .95$$

From $Z = \dfrac{X - \mu}{\sigma}$ substitute

$$1.645 = \frac{X_{.95} - 600}{80}$$

Thus, $X_{.95} = \sigma Z_{.95} + \mu$

$$= (80)\left[1.645\right] + 600$$
$$= 731.6$$



url used: http://davidmlane.com/hyperstat/z_table.html

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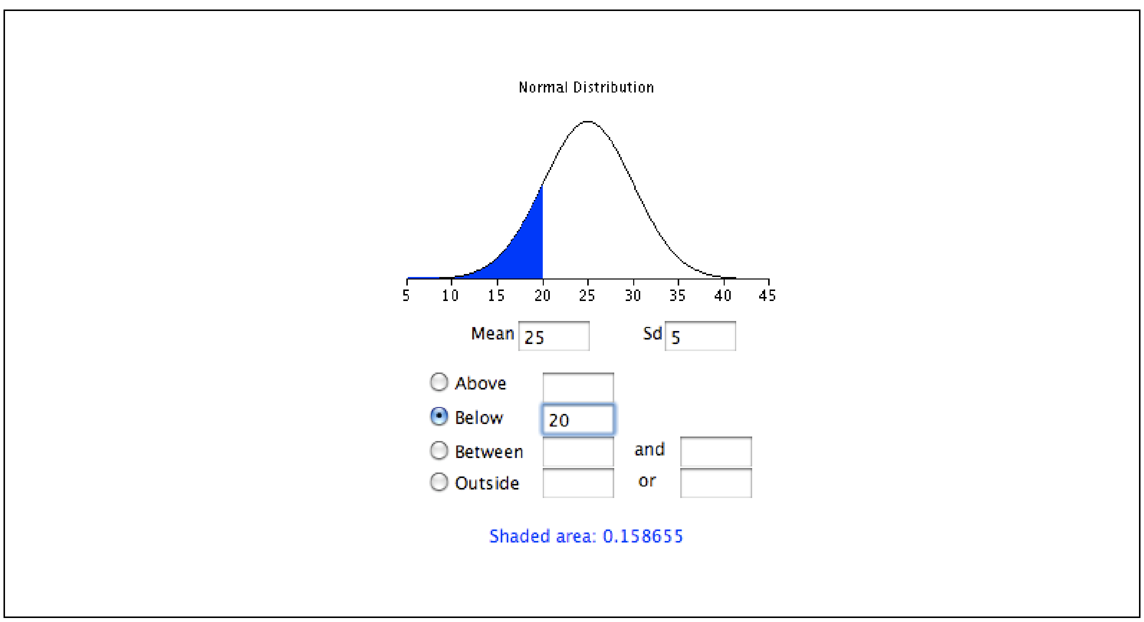
4. The Chapin Social Insight Test evaluates how accurately the subject appraises other people. In the reference population used to develop the test, scores is normally distributed with mean μ=25 and standard deviation σ=5. The range of possible scores is 0 to 41.

a. What proportion of the population has scores below 20 on the Chapin test?

Answer: **.1587**

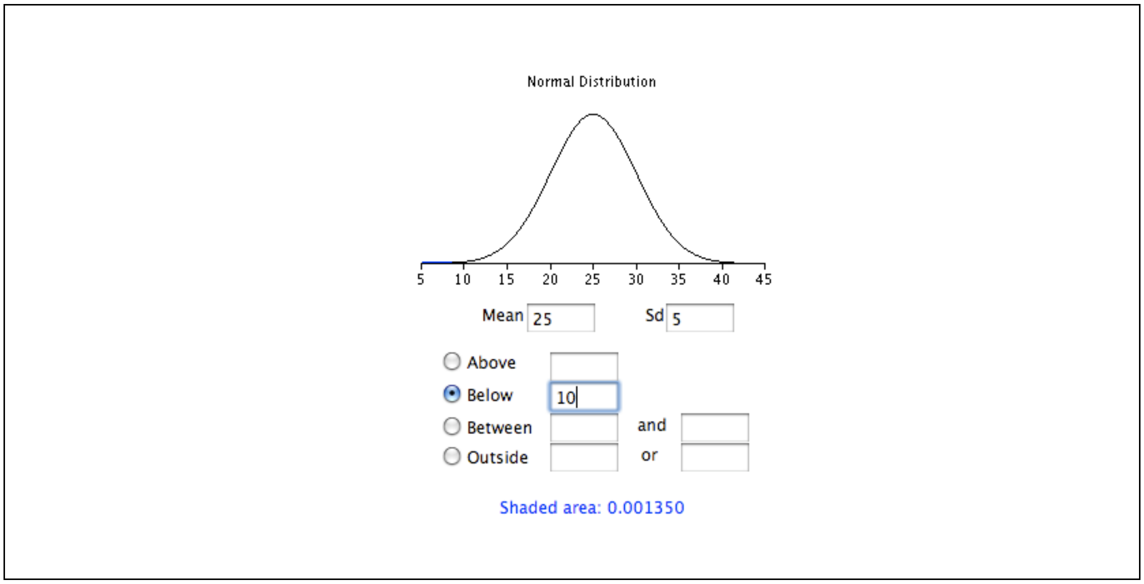
Solution: $\text{pr}(X < 20) = .1587$



b. What proportion has scores below 10?

Answer: **.0014**

Solution: $pr(X < 10) = .0014$



url used: http://davidmlane.com/hyperstat/z_table.html

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c. How high a score must you have in order to be in the top quarter of the population in social insight?

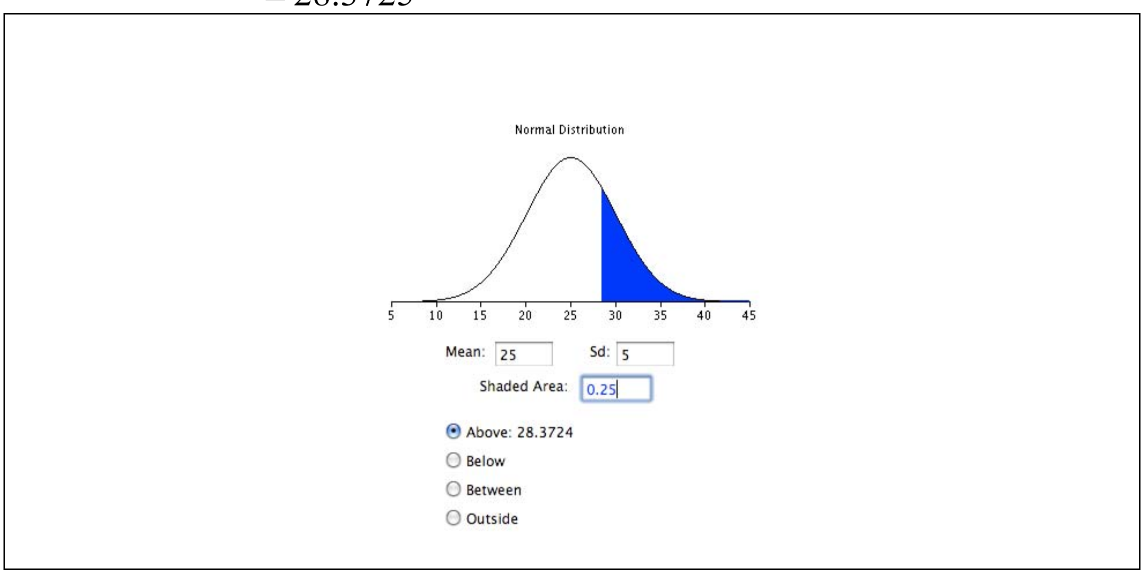
Answer: **28.35**

Solution: $pr(Z > 0.6745) = .25$

Thus, $X = \sigma Z + \mu$

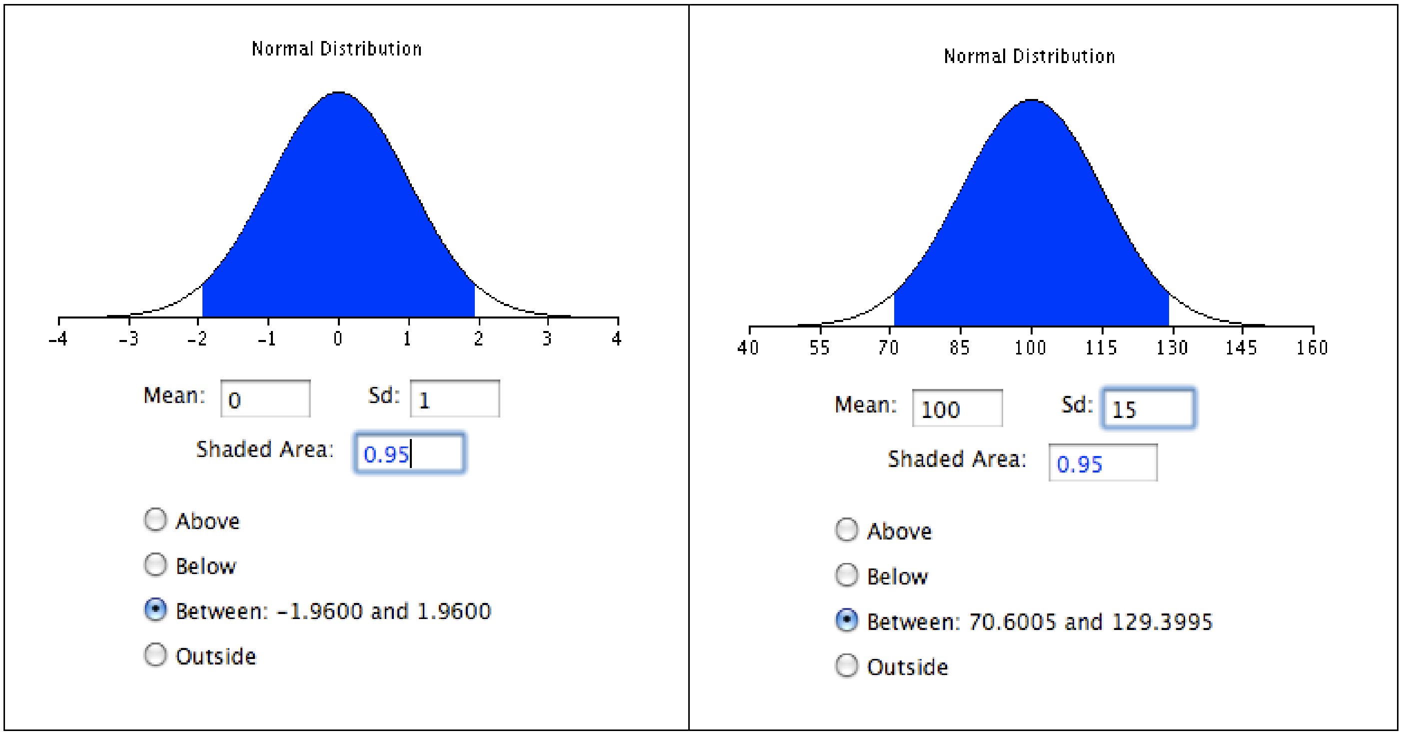
$$= (5)[0.6745] + 25$$

$$= 28.3725$$



5. A normal distribution has mean μ=100 and standard deviation σ=15 (for example, IQ). Give limits, symmetric about the mean, within which 95% of the population would lie:

Solution:



First obtain an interval for Z~Normal(0,1)

$$\text{pr}\left[\, -1.96 < Z < +1.96 \,\right] = .95$$

Next, recall that the standard error, SE, of $\bar{X}$, is related to σ via $\text{se}[\bar{X}] = \sigma / \sqrt{n}$

And the mean of $\bar{X}$ is $E[\bar{X}] = \mu$

Thus, the standardization formula can be manipulated to yield a formula for $\bar{X}$ in terms of Z.

From $Z = \dfrac{\bar{X} - E[\bar{X}]}{SE[\bar{X}]}$, solve for $\bar{X}$.

$$\bar{X} = \left\{\text{se}(\bar{X})\right\} Z + \mu$$

$$= \left(\sigma / \sqrt{n} \right) Z + \mu$$

Now we can make a little table

	Mean	SE	Lower limit	Upper limit
Z	0	1	-1.96	+1.96
X	100	15	$(15)(-1.96)+100=70.6$	$(15)(+1.96)+100=129.4$
$\overline{X}_{n=4}$	100	15/2	$(15/2)(-1.96)+100=85.3$	$(15/2)(+1.96)+100=114.7$
$\overline{X}_{n=16}$	100	15/4	$(15/4)(-1.96)+100=92.65$	$(15/4)(+1.96)+100=107.35$
$\overline{X}_{n=100}$	100	15/10	$(15/10)(-1.96)+100=97.06$	$(15/10)(+1.96)+100=102.94$

a. Individual observations. **Answer**: **70.6, 129.4**

b. Means of 4 observations. **Answer**: **85.3, 114.7**

c. Means of 16 observations. **Answer**: **92.65, 107.35**

d. Means of 100 observations. **Answer**: **97.06, 102.94**

e. Write down an expression for the width of the limits symmetric about the mean, within which 95% of the population of means of samples of size n would lie.

Solution:

Width of limits symmetric about the mean is therefore

= (Upper endpoint) - (Lower endpoint)

$$= \left[\frac{\sigma}{\sqrt{n}}(Z)+\mu \right] - \left[\frac{\sigma}{\sqrt{n}}(-Z)+\mu \right]$$

$$= \frac{2\sigma Z}{\sqrt{n}}$$

Thus, the width gets smaller as the sample size n gets larger. On to confidence intervals in unit 6!