#1. Find the proportion of observations from a standard normal distribution that satisfies each of the following statements.

- a. \( Z < 2.85 \)
- b. \( Z > 2.85 \)
- c. \( Z > -1.66 \)
- d. \(-1.66 < Z < 2.85\)
- e. \( Z < -2.25 \)
- f. \( Z > -2.25 \)
- g. \( Z > 1.77 \)
- h. \(-2.25 < Z < 1.77\)

\[
\begin{align*}
\text{Pr}(Z < 2.85) & = 0.9978 \\
\text{Pr}(Z > 2.85) & = 0.0022 \\
\text{Pr}(Z > -1.66) & = 0.9515 \\
\text{Pr}(-1.66 < Z < 2.85) & = 0.9494 \\
\text{Pr}(Z < -2.25) & = 0.0122 \\
\text{Pr}(Z > -2.25) & = 0.9878 \\
\text{Pr}(Z > 1.77) & = 0.0384 \\
\text{Pr}(-2.25 < Z < 1.77) & = 0.9494
\end{align*}
\]

**Art of Stat Solution for #1a ONLY** (solutions for #1b-#1h are similar)

\#1a. \( \text{Pr}(Z < 2.85) = 0.9978 \)
R Solution

# 1a)  Pr[Normal(mean=0, sd=1) <= 2.85]
  pnorm(2.85)
  ## [1] 0.997814

# 1b)  Pr[Normal(mean=0, sd=1) > 2.85]
  pnorm(2.85, lower.tail=FALSE)
  ## [1] 0.002185961

# 1c)  Pr[Normal(mean=0, sd=1) > -1.66]
  pnorm(-1.66, lower.tail=FALSE)
  ## [1] 0.9515428

# 1d)  Pr[-1.66 <= Normal(mean=0, sd=1) <= 2.85]
  pnorm(2.85) - pnorm(-1.66)
  ## [1] 0.9493568

# 1e)  Pr[Normal(mean=0, sd=1) <= -2.25]
  pnorm(-2.25)
  ## [1] 0.01222447

# 1f)  Pr[Normal(mean=0, sd=1) > -2.25]
  1 - pnorm(-2.25)
  ## [1] 0.9877755

# 1g)  Pr[Normal(mean=0, sd=1) > 1.77]
  pnorm(1.77, lower.tail=FALSE)
  ## [1] 0.03836357

# 1h)  Pr[-2.25 <= Normal(mean=0, sd=1) <= 1.77]
  pnorm(1.77) - pnorm(-2.25)
  ## [1] 0.949412
#2. The height, $X$, of young American women is distributed normal with mean $\mu=65.5$ and standard deviation $\sigma=2.5$ inches. Find the probability of each of the following events

a. $X < 67$

**Art of Stat Solution**

#2a. .7257

$$\text{pr}(X < 67) = \text{pr}\left(\frac{X-\mu}{\sigma} < \left(\frac{67-\mu}{\sigma}\right)\right)$$

$$= \text{pr}\left[Z < \left(\frac{67-65.5}{2.5}\right)\right]$$

$$= \text{pr}[Z < .6]$$

$$= .7257$$

**R Solution**

```r
# Pr[Normal(mean=65.5, sd=2.5) < 67]
pnorm(67, mean=65.5, sd=2.5)
## [1] 0.7257469
```
b. $64 < X < 67$

**Art of Stat Solution**

#2b. .4515

\[
\Pr(64 < X < 67) = \Pr\left(\frac{64-65.5}{2.5} < Z < \frac{67-65.5}{2.5}\right)
\]

= \Pr[-0.6 < Z < +0.6]

= .4515

Using $X \sim \text{Normal}(\text{mean}=65.5, \text{sd}=2.5)$

Using Standardization to $Z \sim \text{Normal}(\text{mean}=0, \text{sd}=1)$

**R Solution**

```r
# Pr[64 < Normal(mean=65.5, sd=2.5) < 67]
pnorm(67, mean=65.5, sd=2.5) - pnorm(64, mean=65.5, sd=2.5)
## [1] 0.4514938
```
#3. Suppose that, in a certain population, the distribution of GRE scores is normal with mean $\mu=600$ and standard deviation $\sigma=80$.

a. What is the probability of a score less than 450 or greater than 750?

**Art of Stat Solution**

**Answer:** .0608

**Solution:**
Define the random variable $X = \text{GRE score}$.
Thus, $X$ is distributed normal with mean $\mu=600$ and standard deviation $\sigma=80$.
We write this more compactly as $X \sim \text{Normal (} \mu=600, \sigma=80\text{)}$.

Probability \{ \text{score} < 450 \text{ OR } \text{score} > 750 \} = \text{pr}[X < 450] + \text{pr}[X > 750]

\begin{align*}
&= 1 - \text{pr} [ 450 < X < 750 ] \\
&= 1 - .9392 \\
&= .0608
\end{align*}

**R Solution**

```r
# Pr[ Normal(mean=600, sd=80) < 450 ] + Pr[ Normal(mean=600, sd=80) > 750] 
pnorm(450, mean=600, sd=80) + pnorm(750, mean=600, sd=80, lower.tail=FALSE)
## [1] 0.06079272
```
b. What proportion of students has scores between 450 and 750?

**Art of Stat Solution**

**Answer:** .9392

**Solution:** “Proportion” of students with scores between 450 and 750 → we want:

\[
\text{Pr}[450 < X < 750]
\]

\[= .9392\]

**R Solution**

```r
# Pr[ 450 < Normal(mean=600, sd=80) < 750 ]
pnorm(750, mean=600, sd=80) - pnorm(450, mean=600, sd=80)
## [1] 0.9392073
```
#3 – Continued. The Chapin Social Insight Test evaluates how accurately the subject appraises other people. In the reference population used to develop the test, Chapin Social Insight Test scores are distributed normal with mean $\mu=25$ and standard deviation $\sigma=5$.

c. What proportion of the population has scores below 20 on the Chapin test?

**Art of Stat Solution**

**Answer:** .1587

**Solution:**
The solution for the “proportion of the population” is a probability calculation.

Define the random variable $X =$ Chapin Social Insight Test Score.

$X$ is distributed Normal ($\mu=25$, $\sigma=5$).

Want: $\Pr(X < 20) = .1587$

---

![Normal Distribution](image)

---

**R Solution**

```r
# Pr[ Normal(mean=25, sd=5) < 20 ]
pnorm(20, mean=25, sd=5)
## [1] 0.1586553
```
d. What proportion has scores below 10?

**Art of Stat Solution**

**Answer:** .0013

**Solution:**

This is similar to “a”.

The solution for the “proportion of the population” is a probability calculation.

$X$ is distributed Normal ($\mu=25$, $\sigma=5$).

**Want:** $\Pr(X < 10) = .0013$

**R Solution**

```
# Pr[ Normal(mean=25, sd=5) < 10 ]
pnorm(10, mean=25, sd=5)
## [1] 0.001349898
```
#4. Consider again the setting in questions #3a and #3b: in a certain population, the distribution of GRE scores is normal with mean $\mu=600$ and standard deviation $\sigma=80$.

a. What score is equal to the 95th percentile?

**Art of Stat Solution**

**Answer: 731.6**

There are at least two solutions to this question:

**Solution I** – Simple “plug in” variety

**Solution II** – 2 step solution that re-enforces the concepts..

Step 1: Obtain the 95th percentile for $Z \sim \text{Normal}(0,1)$. Call this $Z_{.95}$

Step 2: Use $Z_{.95}$ and the formula on page 26 of the course notes to obtain $X_{.95}$

**Solution I:** Set mean=600 and standard deviation = 80

**Solution II Step I:** Set mean=0 and standard deviation = 1 and then solve for the percentile of $X$
Solution II  **Step 2:**
Use the formula on page 26 of the unit 7 notes with the following inputs:  
1. $Z_{.95} = 1.645$  
2. $\mu = 600$ and $\sigma = 80$

$$X_{.95} = \sigma Z_{.95} + \mu$$

$$= (80)(1.645) + 600$$

$$= 731.6$$

---

**R Solution**

```r
# 95th percentile of a Normal(mean=600, sd=80). I added the round() so as to get just 2 digits
round(qnorm(.95, mean=600, sd=80), digits=2)
## [1] 731.59
```

---

#4 – Continued. Next, consider again the setting of questions #3c and #3d:  
The Chapin Social Insight Test evaluates how accurately the subject appraises other people. In the reference population used to develop the test, Chapin Social Insight Test scores are distributed normal with mean $\mu=25$ and standard deviation $\sigma=5$.

b. How high a score must you have in order to be in the top quarter of the population in social insight?

**Art of Stat Solution**

**Answer:** 28.37

**Solution:**

Hone your translation skills here. To be in the “top quarter” your score must be $\geq 75^{th}$ percentile

![Normal Distribution with $\mu = 25$ and $\sigma = 5$](image)

Of course you can always just do the brute force right tail probability $= .25$ to get the same answer!!
5. A normal distribution has mean μ=100 and standard deviation σ=15 (for example, IQ).

Give limits, symmetric about the mean, within which 95% of the population would lie:

Solution:

This exercise is asking you to work with the following characteristic of the Normal distribution:

If $X_1, X_2, \ldots, X_n$ are a simple random sample, each distributed $\text{Normal}(\mu, \sigma^2)$

Then the sample mean of $n$ observations is distributed $\text{Normal}(\mu, \sigma^2/n)$

*Tip!*

It is necessary to input the value of $\sqrt{\sigma^2/n}$ in the box “standard deviation”
a) Individual observations

Answer: 70.6, 129.4

"Individual observations" →
“mean” = \( \mu = 100 \)
“standard deviation” = \( \sigma = 15 \).

R Solution

```r
# Distribution of Individual Observations: 2.5th and 97.5th percentiles of Normal(mean=100, sd=15)
paste(round(qnorm(.025, mean=100, sd=15), digits=2), " and ", round(qnorm(.975, mean=100, sd=15), digits=2))
```

# [1] "70.6 and 129.4"

b) Means of 4 observations

Answer: 85.3, 114.7

"Means of 4 observations" →
“mean” = \( \mu = 100 \)
“standard deviation” = \( SE = \sqrt{(\sigma^2/n)} = \sigma/\sqrt{4} = 15/2 = 7.5 \)
R Solution

```
# Means of n=4 Observations: 2.5th and 97.5th percentiles of Normal(mean=100, sd=7.5)
paste(round(qnorm(.025, mean=100, sd=7.5), digits=2), " and ", round(qnorm(.975, mean=100, sd=7.5), digits=2))
## [1] "85.3 and 114.7"
```

c) Means of 16 observations

**Answer:** 92.65, 107.35

“Means of 16 observations” →

“mean” = μ = 100

“standard deviation = SE = √(σ²/n) = σ/√16 = 15/4 = 3.75

R Solution

```
# Means of n=16 Observations: 2.5th and 97.5th percentiles of Normal(mean=100, sd=3.75)
paste(round(qnorm(.025, mean=100, sd=3.75), digits=2), " and ", round(qnorm(.975, mean=100, sd=3.75), digits=2))
## [1] "92.65 and 107.35"
```
d) Means of 100 observations

**Answer: 97.06, 102.94**

“Means of 100 observations” →

“mean” = μ = 100

“standard deviation” = SE = \sqrt{(\sigma^2 / n)} = \sigma / \sqrt{100} = 15/10 = 1.5

---

**R Solution**

```R
# Means of n=100 Observations: 2.5th and 97.5th percentiles of Normal(mean=100, sd=1.5)
paste(round(qnorm(.025, mean=100, sd=1.5), digits=2), " and ", round(qnorm(.975, mean=100, sd=1.5), digits=2))
## [1] "97.06 and 102.94"
```