

Unit 6 – Bernoulli and Binomial Distributions

Homework Solutions

For both: *Art of Stat Users and R Users*

*Note – I use R a lot, but for something like these exercises,
I would go straight to ArtofStat; it's so lovely for these sorts of things.*

1. Suppose that my 2022 BIOSTATS 540 class has just 10 students.
 - a. I wish to pair up students to work on homework together. How many pairs of 2 students could I form?

Answer: 45

Solution: *Tip: Think “peel an onion” here*

$$10 \text{ choose } 2 = \binom{10}{2} = \frac{10!}{2! 8!} = \frac{(10)(9)\cancel{8!}}{(2)(1)\cancel{8!}} = \frac{90}{2} = 45$$

- b. Next, I wish to form project groups of size 5. How many groups of 5 students could I form?

Answer: 252

Solution: $10 \text{ choose } 5 = \binom{10}{5} = \frac{10!}{5! 5!} = \frac{(10)(9)(8)(7)(6)\cancel{5!}}{(5)(4)(3)(2)(1)\cancel{5!}} = 252$

2. A die will be rolled six times. What are the chances that, over all six rolls, the die lands neither ace (one dot showing) nor deuce (two dots showing) exactly 2 times?

Answer = .08

“Translate” the wording of the question into the settings for n , $p=\pi$, and x .

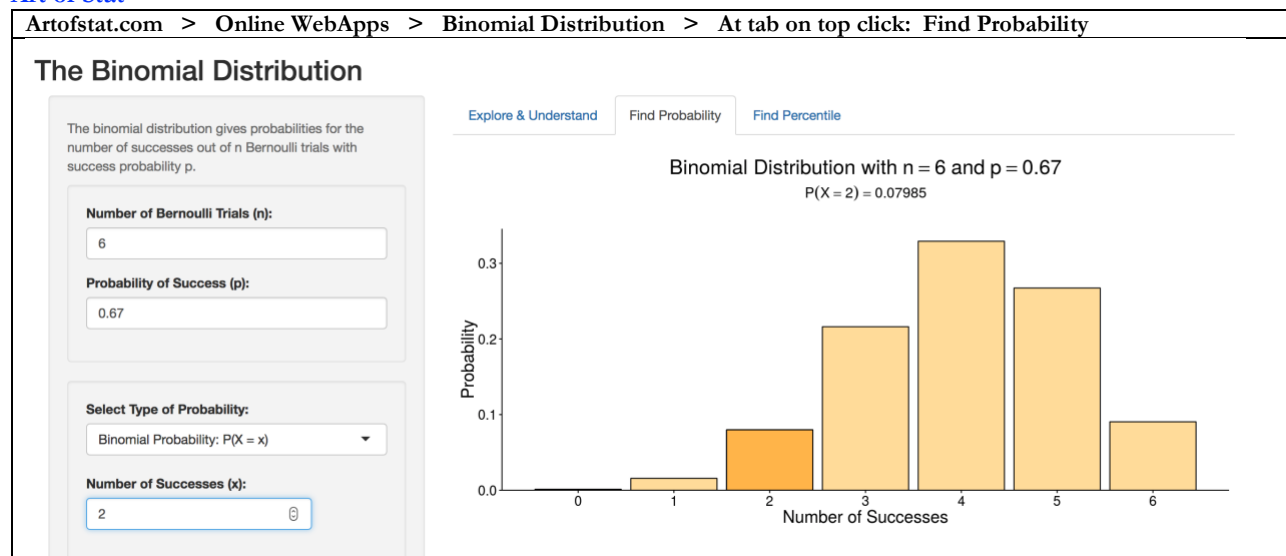
“over all six rolls” $\rightarrow n = 6$

“neither ace nor deuce” \rightarrow “lands on 3, 4, 5, or 6” $\rightarrow \pi = [4 \text{ winning faces}/6 \text{ faces total}] = .67$

“exactly 2 times” \rightarrow want $\Pr[X = 2]$

\rightarrow Settings are: $n=6$, $p=\pi=.67$, and $x=2$ and desired probability is: **$\Pr[X = 2]$**

Art of Stat



R using function dbinom()

```
# Question 2 - # Binomial(n, pi): Probability of exactly x events, Pr[X = x]
# Binomial(n=6, pi=.67), Prob[X=2] is:
# dbinom(x, ntrials, pi)

# Plain - No tags and order of inputs as R requires (X, ntrials, prob)
dbinom(2, 6, 0.67)
## [1] 0.07985399

# Fancy
paste("Pr [ Binom(6, .67) = 2 ] = ", round(dbinom(2, 6, 0.67), 4) )
## [1] "Pr [ Binom(6, .67) = 2 ] = 0.0799"
```

3. Suppose that, in the general population, there is a 2% chance that a child will be born with a genetic anomaly. What is the probability that no congenital anomaly will be found among four random births?

Answer = .92

“Translate” the wording of the question into the settings for n , $p=\pi$, and x .

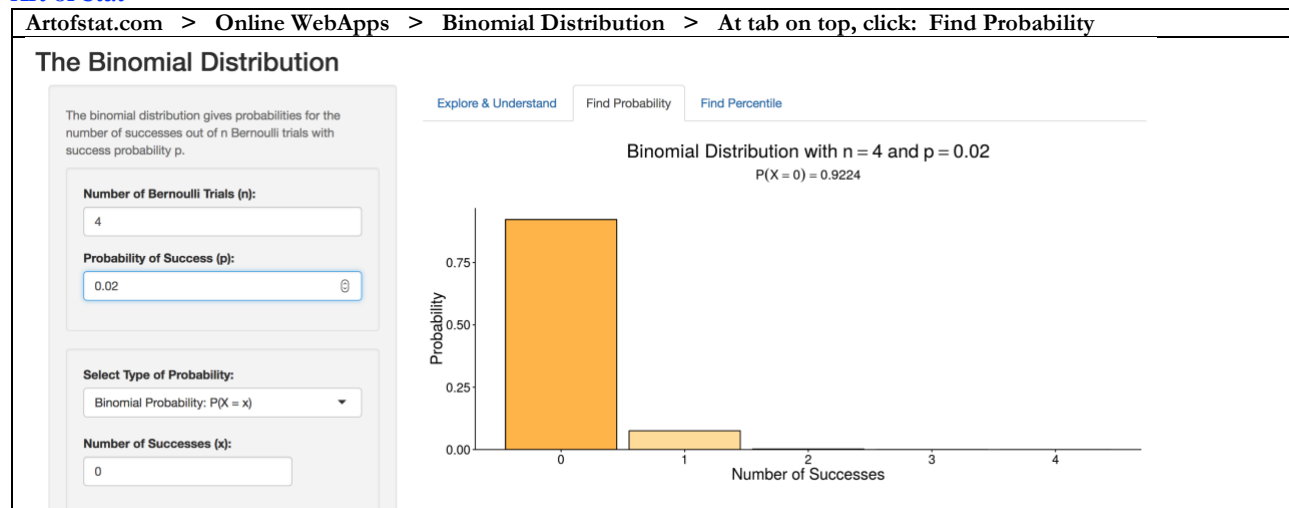
“four random births” $\rightarrow n = 4$

“2% chance of congenital anomaly” $\rightarrow \pi = .02$

“no congenital anomaly will be found” \rightarrow want $\Pr[X = 0]$

\rightarrow Settings are: $n=4$, $p=\pi=.02$, and $x=0$ and desired probability is: $\Pr[X = 0]$

Art of Stat



R using function dbinom()

```
# Question 3 - # Binomial(n, pi): Probability of exactly x events, Pr[X = x]
# Binomial(n=4, pi=.02), Prob[X=0] is:
# dbinom(x,ntrials,pi)

# Plain - No tags and order of inputs as R requires (X,ntrials,prob)
dbinom(0, 4, 0.02)
## [1] 0.9223682

# Fancy
paste("Pr [ Binom(4,.02) = 0 ] = ",round(dbinom(0,4,0.02),4) )
## [1] "Pr [ Binom(4,.02) = 0 ] = 0.9224"
```

4. Suppose it is known that, for a given couple, there is a 25% chance that a child of theirs will have a particular recessive disease. If they have three children, what are the chances that at least one of them will be affected?

Answer = .58

“Translate” the wording of the question into the settings for n , $p=\pi$, and x .

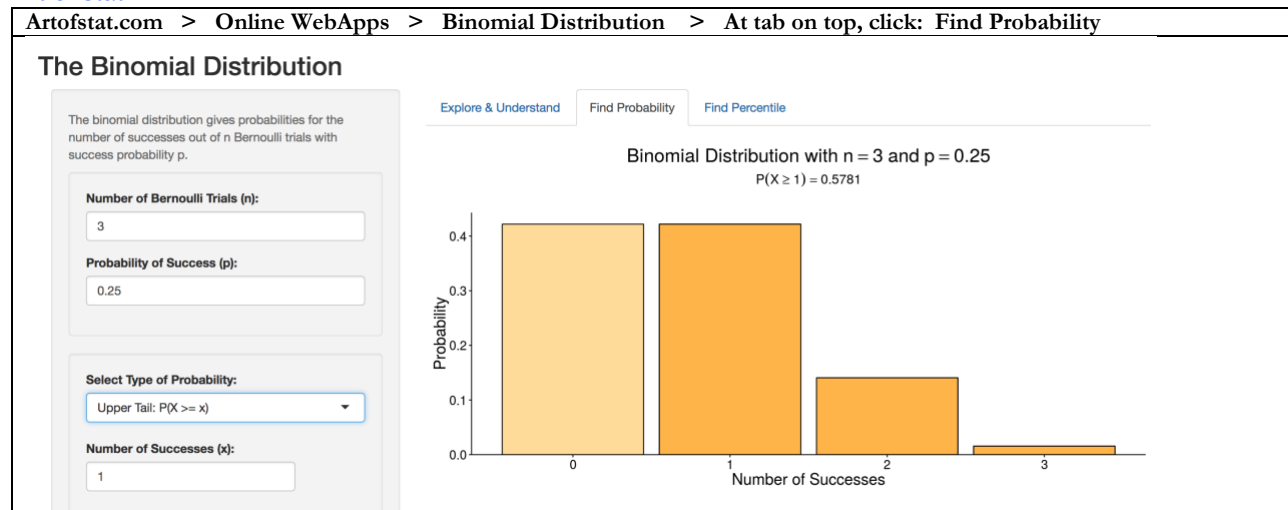
“if they have three children” $\rightarrow n = 3$

“25% chance of recessive disease” $\rightarrow \pi = .25$

“at least one of them is affected” \rightarrow want $\Pr[X \geq 1]$

\rightarrow Settings are: $n=3$, $p=\pi=.25$, and $x=1$ and desired probability is: $\Pr[X \geq 1]$

Art of Stat



R using function dbinom()

```
# Question 4 - # Binomial(n, pi): Probability of x or more events, Pr[X >= x]
# Binomial(n=3, pi=.25), Prob[ X >= 1 ] = 1 - Prob [ X = 0 ] is:
# 1 - dbinom(x,ntrials,pi)

# Plain - No tags and order of inputs as R requires (X,ntrials,prob)
1 - dbinom(0, 3, 0.25)
## [1] 0.578125

# Fancy
paste("Pr [ Binom(3,.25) >= 1 ] = ",round(1 - dbinom(0,3,0.25),4) )
## [1] "Pr [ Binom(3,.25) >= 1 ] = 0.5781"
```

5. Suppose a quiz contains 20 true/false questions. You know the correct answer to the first 10 questions. You have no idea of the correct answer to questions 11 through 20 and decide to answer each using the coin toss method. Calculate the probability of obtaining a total quiz score of at least 85%.

Answer = .17

“Translate” the wording of the question into the settings for n , $p=\pi$, and x .

“score of at least 85%” → # correct must be at least 17 because 85% of 20 = 17

“if they have three children” → $n = 3$

“you know the correct answer to the first 10 questions” → consider only last 10 questions

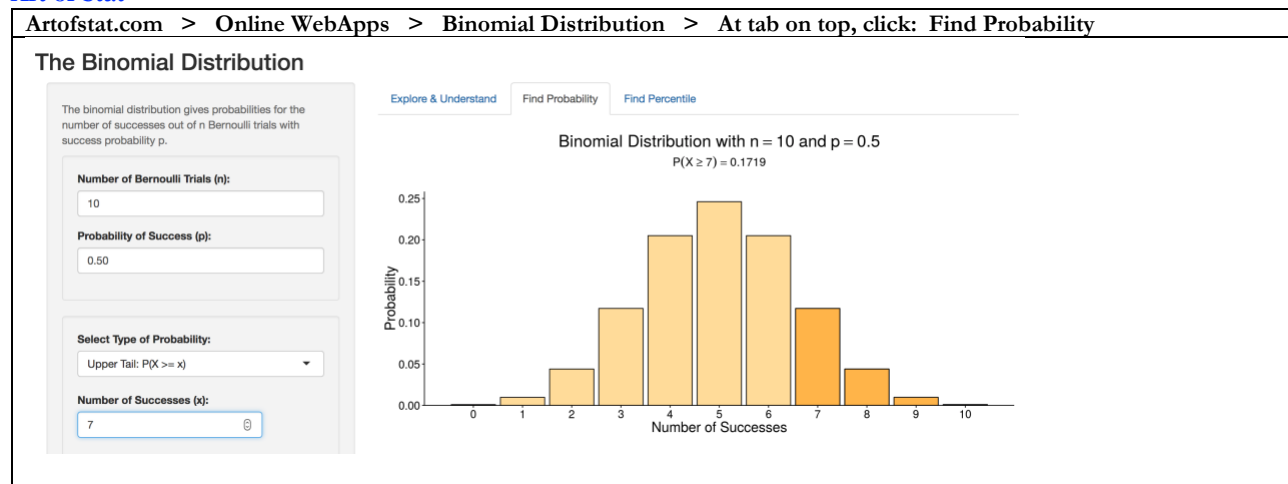
“score of at least 85%” → need just 7 additional correct answers among last 10 question

“true/false questions” → $p=\pi = .50$

“at least 85%” → want $\Pr[X \geq 7]$

→ Settings are: $n=10$, $p=\pi=.50$, and $x=7$ and desired probability is: $\Pr[X \geq 7]$

Art of Stat



R using function pbinom()

```
# Question 5 - # Binomial(n, pi): Probability of x or more events, Pr[X >= x]
# Binomial(n=10, pi=.50), Prob[ X >= 7 ] = 1 - Prob [ X <= 6 ] is:
# 1 - pbinom(x, ntrials, pi)

# Plain - No tags and order of inputs as R requires (q, ntrials, prob)
1 - pbinom(6, 10, 0.50)
## [1] 0.171875

# Fancy
paste("Pr [ Binom(3,.25) >= 7 ] = ", round(1 - pbinom(6, 10, 0.50), 4) )
## [1] "Pr [ Binom(3,.25) >= 7 ] = 0.1719"
```