Unit 4 – Bernoulli and Binomial Distributions
Practice Problems

SOLUTIONS

1. ***This exercise gives you practice in calculating "number of ways to choose"***.
Suppose that my 2011 BE540 class that meets "in class" in Worcester, MA has just 10 students.

a. I wish to pair up students to work on homework together. How many pairs of 2 students could I form?
Answer: 45

Solution: 10 choose 2 = $\binom{10}{2} = \frac{10!}{2!\ 8!} = \frac{(10)(9)8!}{(2)(1)8!} = \frac{90}{2} = 45$

b. Next, I wish to form project groups of size 5. How many groups of 5 students could I form?
Answer: 252

Solution: 10 choose 5 = $\binom{10}{5} = \frac{10!}{5!\ 5!} = \frac{(10)(9)(8)(7)(6)5!}{(5)(4)(3)(2)(1)5!} = 252$

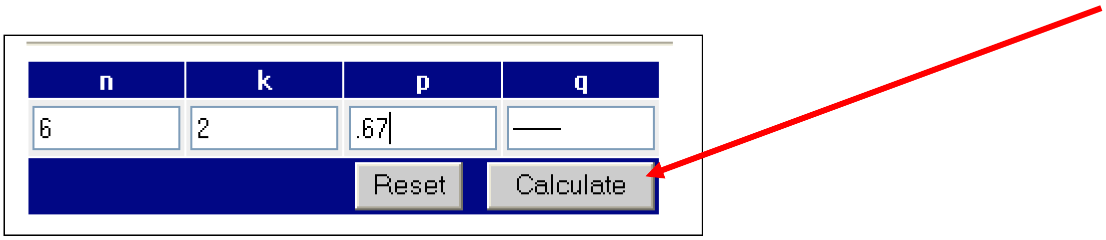
2. ***This exercise is a straightforward application of a binomial probability calculation.***
A die will be rolled six times. What are the chances that, over all six rolls, the die lands neither ace (one dot showing) nor deuce (two dots showing) exactly 2 times?
Answer: .08
Solution:
Success on roll of die occurs for event "neither ace nor deuce". This has probability 4/6=.67
Outcome of interest is "exactly 2 successes and 4 failures".
Define random variable X = # successes on six rolls of one die
Binomial number of trials, N = 6
Event probability π=.67
Want Pr [X = 2]

Pr [X=2 | N=6 and π=.67] = $\binom{6}{2}[.67]^2[1-.67]^4 = .07985$

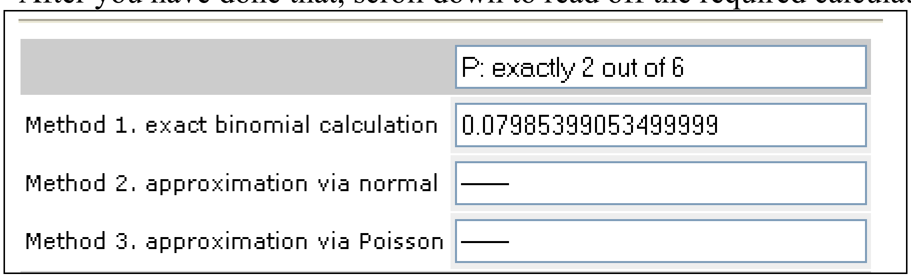
Probability[one scenario of 2 success and 4 failure] = $[4/6]^{2\text{ times}}\,[2/6]^{4\text{ times}} = .0055$

Number of scenarios yielding exactly 2 success and 4 failure] = $\binom{6}{2} = \frac{6!}{2!4!} = \frac{(6)(5)4!}{(2)(1)4!} = 15$

You can also get this using the binomial calculator on line. From the course website, click on the Bernoulli and Binomial unit from the left navigation bar. From there, scroll down and click on Vassar Statistics Binomial Calculator. Scroll down and fill in the following values. Calculate



After you have done that, scroll down to read off the required calculation.



3. ***This is also an application of a binomial probability calculation.***
Suppose that, in the general population, there is a 2% chance that a child will be born with a genetic anomaly. What is the probability that no congenital anomaly will be found among four random births?

Answer: .92

Solution:

The event of interest is "no congenital anomaly".
If "genetic anomaly" occurs with probability = .02
Then "No genetic anomaly" occurs with probability = 1 - .02 = .98
Therefore, Event probability π=.98
"four random births" is telling you that the Binomial "number of trials" n=4
"no genetic anomaly will be found" is telling you that X=4

Thus, we want to solve for Pr [X = 4 | n=4 and π=.98]

$$\Pr[X=4 \mid n=4, \pi=.98] = \binom{4}{4}[.98]^4[.02]^{4-4}$$

$$=1*[.98]^4*1 = [.98]^4 = .92$$

In the Vassar Stats calculator, enter n=4, k=4 (This calculator uses the notation "k" instead of the notation "x"), and p=.98 (this calculator uses the notation "p" instead of the notation π). Then click CALCULATE. From there, scroll down to find the desired solution.

n	k	p	q
4	4	.98	——
		Reset	Calculate

	P: exactly 4 out of 4
Method 1. exact binomial calculation	0.9223681599999999
Method 2. approximation via normal	——
Method 3. approximation via Poisson	——

4. ***This is a slightly harder application of a binomial probability calculation.*** Suppose it is known that, for a given couple, there is a 25% chance that a child of theirs will have a particular recessive disease. If they have three children, what are the chances that at least one of them will be affected?

Answer: .58

Solution:

The event being investigated is that of a "particular recessive disease"

Event probability π=.25

The number of trials considered is N=3

There is more than one way to reason out the solution.

One approach:

"chances that at least one will be affected"

= 1 – "chances that zero will be affected"

= 1 – Probability [X=0] for X distributed Binomial (N=3, π=.25)

= 1 - .421875

= .58

	P: exactly 0 out of 3
Method 1. exact binomial calculation	0.421875
Method 2. approximation via normal	
Method 3. approximation via Poisson	

Another approach:

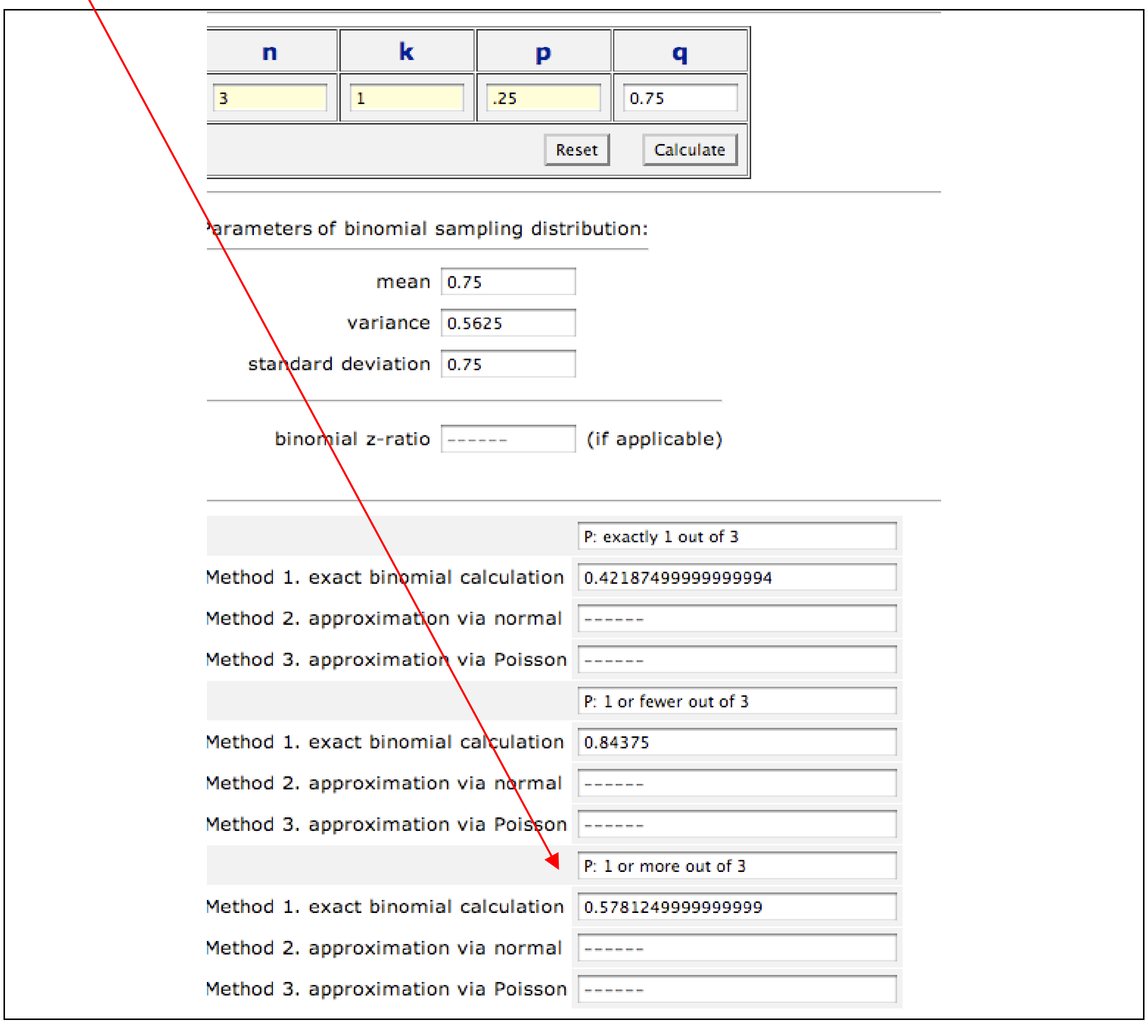
“chances that at least one will be affected”

= “chances that one or two or three will be affected”

= Probability [X=1] + Probability [X=2] + Probability [X=3]

for X distributed Binomial (N=3, π=.25)

= .58



Yet another approach:

"chances that at least one will be affected"

= 1 - "chances that zero are affected"

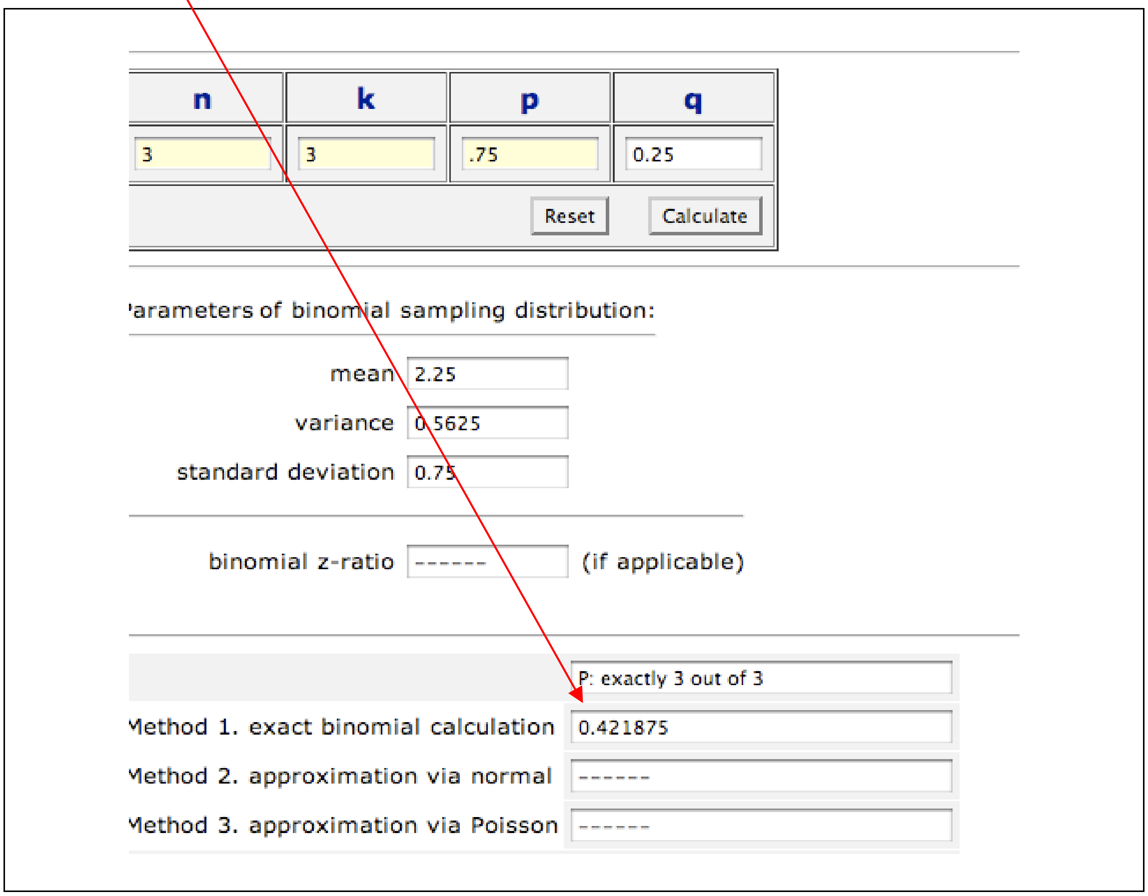
= 1 – "chances that ALL 3 are DISEASE FREE"

… so we consider the event of being disease free which occurs with probability = .75

= 1 – Probability [X=3] for X distributed Binomial (N=3, π=.75)

= 1 - .421875

= .58



5. ***This exercise is the most involved.***
Suppose a quiz contains 20 true/false questions. You know the correct answer to the first 10 questions. You have no idea of the correct answer to questions 11 through 20 and decide to answer each using the coin toss method. Calculate the probability of obtaining a total quiz score of at least 85%.

Answer: .17

Solution:

The goal is a total quiz score of at least 85%
85% of 20 true/false questions is 20*.85 = 17
So we need to get 17 OR MORE questions correct
We already have 10 correct
The remaining number we need to get is therefore 17-10 = 7 OR MORE

"questions 11 through 20" remain → Binomial number of trials N = 10
"true/false using coin toss method" → Event probability π=.50
Since we need to get 7 or MORE
Want Pr [X ≥ 7]

$$\Pr[X^{3}7 \mid n=10, \pi=.5] = \sum_{x=7}^{10} \binom{10}{x} [.50]^{x} [1-.50]^{10-x}$$

$$= \sum_{x=7}^{10} \binom{10}{x} [.50]^{10}$$

$$= \binom{10}{7}[.50]^{10} + \binom{10}{8}[.50]^{10} + \binom{10}{9}[.50]^{10} + \binom{10}{10}[.50]^{10}$$

$$= [.50]^{10} * \left\{ \binom{10}{7} + \binom{10}{8} + \binom{10}{9} + \binom{10}{10} \right\}$$

n	k	p	q
10	7	.5	——
		Reset	Calculate

	P: 7 or more out of 10
Method 1. exact binomial calculation	0.171875
Method 2. approximation via normal	0.171056
Method 3. approximation via Poisson	——