Unit 5 – Populations and Samples
Self Evaluation Quiz

SOLUTIONS

1a.

(i)   population mean $= \mu = \dfrac{\displaystyle\sum_{i=1}^{N=5} X_i}{N=5} = 184.0$

(ii)   population variance $= \sigma^2 = \dfrac{\displaystyle\sum_{i=1}^{N=5}\left(X_i - \mu\right)^2}{\text{size of population}} = \dfrac{\displaystyle\sum_{i=1}^{N=5}\left(X_i - 184\right)^2}{N=5} = 810.8$

(iii)   population standard deviation $= \sqrt{\text{population variance}} = \sigma = 28.47$

(iv)   population median $= P_{50} = 192$

1b. For population size N=5 and sample size n=2, there are $5^2 = N^n = 25$ "with replacement" samples.

1c. Here they are!

1	2	3	4	5	6	7	8	9	10
130	130	130	130	130	192	192	192	192	192
130	192	201	185	212	130	192	201	185	212

11	12	13	14	15	16	17	18	19	20
201	201	201	201	201	185	185	185	185	185
130	192	201	185	212	130	192	201	185	212

21	22	23	24	25
212	212	212	212	212
130	192	201	185	212

1d

1	2	3	4	5
130	130	130	130	130
130	192	201	185	212
$\overline{X}=130$	$\overline{X}=161$	$\overline{X}=165.5$	$\overline{X}=157.5$	$\overline{X}=171$
$S^2 = 0$	$S^2 = 1922$	$S^2 = 2520.5$	$S^2 = 1512.5$	$S^2 = 3362$
$S = 0$	$S = 43.841$	$S = 50.205$	$S = 38.891$	$S = 57.983$
$\tilde{X} = 130$	$\tilde{X} = 161$	$\tilde{X} = 165.5$	$\tilde{X} = 157.5$	$\tilde{X} = 171$

6	7	8	9	10
192	192	192	192	192
130	192	201	185	212
$\overline{X}=161$	$\overline{X}=192$	$\overline{X}=196.5$	$\overline{X}=188.5$	$\overline{X}=202$
$S^2 = 1922$	$S^2 = 0$	$S^2 = 40.5$	$S^2 = 24.5$	$S^2 = 200$
$S = 43.841$	$S = 0$	$S = 6.364$	$S = 4.9497$	$S = 14.142$
$\tilde{X} = 161$	$\tilde{X} = 192$	$\tilde{X} = 196.5$	$\tilde{X} = 188.5$	$\tilde{X} = 202$

11	12	13	14	15
201	201	201	201	201
130	192	201	185	212
$\overline{X}=165.5$	$\overline{X}=196.5$	$\overline{X}=201$	$\overline{X}=193$	$\overline{X}=206.5$
$S^2 = 2520.5$	$S^2 = 40.5$	$S^2 = 0$	$S^2 = 128$	$S^2 = 60.5$
$S = 50.205$	$S = 6.364$	$S = 0$	$S = 11.314$	$S = 7.7782$
$\tilde{X} = 165.5$	$\tilde{X} = 196.5$	$\tilde{X} = 201$	$\tilde{X} = 193$	$\tilde{X} = 206.5$

16	17	18	19	20
185	185	185	185	185
130	192	201	185	212
$\overline{X}=157.5$	$\overline{X}=188.5$	$\overline{X}=193$	$\overline{X}=185$	$\overline{X}=198.5$
$S^2 = 1512.5$	$S^2 = 24.5$	$S^2 = 128$	$S^2 = 0$	$S^2 = 364.5$
$S = 38.891$	$S = 4.9497$	$S = 11.314$	$S = 0$	$S = 19.092$
$\tilde{X} = 157.5$	$\tilde{X} = 188.5$	$\tilde{X} = 193$	$\tilde{X} = 185$	$\tilde{X} = 198.5$

21	22	23	24	25
212	212	212	212	212
130	192	201	185	212

$\bar{X}$=171	$\bar{X}$=202	$\bar{X}$=206.5	$\bar{X}$=198.5	$\bar{X}$=212
$S^2 = 3362$	$S^2 = 200$	$S^2 = 60.5$	$S^2 = 364.5$	$S^2 = 0$
$S = 57.983$	$S = 14.142$	$S = 7.7782$	$S = 19.092$	$S = 0$
$\tilde{X} = 171$	$\tilde{X} = 202$	$\tilde{X} = 206.5$	$\tilde{X} = 198.5$	$\tilde{X} = 212$

Thus, one sampling distribution has 25 values of $\bar{X}$, another sampling distribution has 25 values of S^2, another sampling distribution has 25 values of S, and the last sampling distribution has 25 values of $\tilde{X}$.

This exercise asks you to compute the mean of each of these four sampling distributions.

Sampling Distribution of Statistic =		Has its own Mean ($\mu_{\text{of statistic}}$)=
	$\bar{X}$	$\mu_{\bar{X}} = 184$
	S^2	$\mu_{S^2} = 810.8$
	S	$\mu_S = 20.365$
	$\tilde{X}$	$\mu_{\tilde{X}} = 184.0$

1e.

Unbiased: mean, sample variance
   Biased: median, standard deviation

1f.

The variance of the sampling distribution of $\bar{X}$ is

$$\sigma_{\bar{X}}^2 = \frac{\overset{\text{All 25 samples in sampling distribution}}{\underset{j=1}{\sum}} \left(\bar{X}_j - \mu_{\bar{X}}\right)^2}{25} = 405.4$$

We compare this to the variance of the original population of 5 observations, which is

$$\sigma^2_{\text{original individual X}} = \frac{\sum_{i=1}^{N \text{ of population} = 5} \left(X_i - \mu_X\right)^2}{5} = 810.8$$

Thus, we have confirmation of something noted in the lecture notes, namely:

$$\sigma^2_{\bar{X}} = \frac{\sigma^2_{\text{in population of individual X}}}{\text{size of sample drawn}} = \frac{\sigma^2_X}{n}$$

In this exercise, we have

$$405.4 = \sigma^2_{\bar{X}} = \frac{\sigma^2_X}{n=2} = \frac{810.8}{2}$$