Unit 2
Introduction to Probability
Practice Quiz

SOLUTIONS

1. Consider the experiment of dealing a card from a standard deck. The outcome noted is the denomination (ace, 2, 3, …, Jack, Queen, King) and suit (spade, heart, club, diamond). Let Q denote the event that the card is a queen and H the event that the card is a heart.

$Q = \{ Q_{spade}, Q_{heart}, Q_{club}, Q_{diamond} \}$
$H = \{ 2_{heart}, 3_{heart}, 4_{heart}, 5_{heart}, 6_{heart}, 7_{heart}, 8_{heart}, 9_{heart}, 10_{heart}, Jack_{heart}, Queen_{heart}, King_{heart}, Ace_{heart} \}$

(a) What is the union of Q and H?
Recalling **"union" is satisfied by a "Q" or a "H" or "both"**, the solution is:
$\{ 2_{heart}, 3_{heart}, 4_{heart}, 5_{heart}, 6_{heart}, 7_{heart}, 8_{heart}, 9_{heart}, 10_{heart}, Jack_{heart}, Queen_{heart}, King_{heart}, Ace_{heart},$ $Queen_{spade}, Queen_{club}, Queen_{diamond} \}$

(b) What is the intersection of Q and H?
Recalling that **"intersction" is satisfied when both "Q" and "H" are satisfied**, the solution is: $Queen_{heart}$

(c) What is the conditional probability Pr(Q|H)?
You can get this by either of two lines of reasoning:
(1) Restrict attention to the hearts and ask yourself, "among the restricted landscape of just hearts, what is the probability of a queen?" Answer:1/13
(2) Or, use the formula:

$$\Pr(Q \mid H) = \frac{\Pr(Q \text{ and } H)}{\Pr(H)} = \frac{1/52}{13/52} = \frac{1}{13}$$

(d) What is the conditional probability Pr(H|Q)? ¼

$$\Pr(H \mid Q) = \frac{\Pr(Q \text{ and } H)}{\Pr(Q)} = \frac{1/52}{4/52} = \frac{1}{4}$$

(e) Are Q and H independent? **YES**. The events Q and H are independent if the likelihood of occurrence of one event is not influenced by the occurrence of the other. That is, Q and H are independent if Prob[Q] = Prob [Q | H]. Note that Prob[Q] = 4/52 = 1/13. This is the same as Prob[Q|H] = 1/13

2. A diagnostic test for high intracranial pressure is based on observing retinal vein pulsation. The idea is that if the pressure is too high, the normal pulsation may not be observed. The following table shows the results of classifying 189 subjects according to their test results and their intracranial pressure status.

		True Status		
		High Pressure	Normal	Total
Test Result	+ = No Pulse	43	18	61
	- = Pulse	0	128	128
		43	146	189

Using these results, calculate

(a) Sensitivity Pr(+ | Disease) = 43/43 = 1.00
(b) Specificity Pr (- | No disease) = 128/146 = .87
(c) false positive rate Pr (+ | No disease) = 18/146 = .12
(d) false negative rate Pr (- | Disease) = 0/43 = 0
(e) positive predictive value Pr (Disease | +) = 43/61 = .70
(f) negative predictive value Pr (No disease | -) = 128/128 = 1.00

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3. Suppose that an experiment consists of flipping three coins – a penny, a nickel, and a dime – all at the same time. Assume that (1) all coins are “fair”, and (2) the coin flips are independent. Consider the following circumstances.

E = event that two or more coins land heads and

If the event E occurs, you get to keep all three coins
Otherwise, win = 0.

X = random variable representing the amount of money you get

(a) Construct the probability distribution of the random variable X.

Preliminary – If it's helpful, construct a table showing the 8 possible outcomes of tossing the 3 coins. Notice that any winning yields 16 cents because every win is "*keep all three coins*"

Penny Nickel Dime	Pr [Penny Nickel Dime]	Winning
H H H	1/8	16 cents
T H H	1/8	16 cents
H T H	1/8	16 cents
H H T	1/8	16 cents
H T T	1/8	0 cents
T H T	1/8	0 cents
T T H	1/8	0 cents
T T T	1/8	0 cents

Probability Distribution for X = Amount of Winning

Value of X, x =	Pr [X = x]
0 cents	½
16 cents	½
	1.00

(b) How much money would you expect to get from one run of the experiment?

Answer: 8 cents.

$$\text{Expected winning} = \sum_{\text{all possible winnings}} [\text{amount of winning}]\ \text{Probability}[\text{this amount is won}]$$

= [0 cents] Pr [0 cents won] + [16 cents] Pr [16 cents won]

= [0 cents] [0.50] + [16 cents] Pr [0.50]

= 8 cents