Unit 7 – Normal Distribution
Self Evaluation Quiz
SOLUTIONS

1. Suppose that the distribution of diastolic blood pressure in a population of hypertensive women is modeled well by a normal probability distribution with mean 100 mm Hg and standard deviation 14 mm Hg. Let X be the random variable representing this distribution. Find two symmetric values "a" and "b" such that probability [a < X < b] = .99

Answer: a=63.95 b=136.05

Solution:

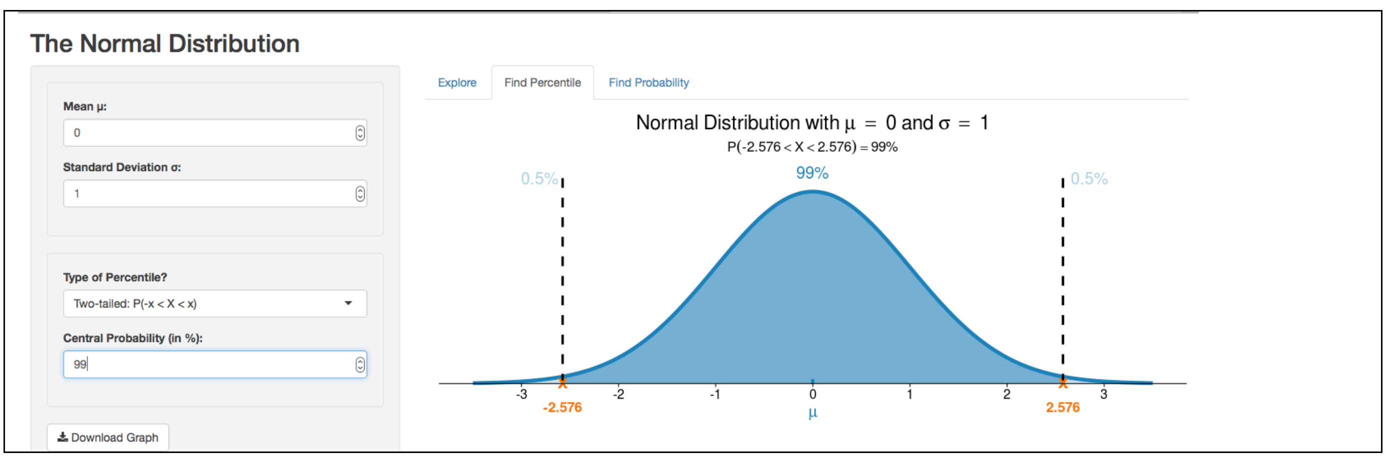
There is more than one approach for arriving at the same answer. I am showing you a detailed one that is more revealing of the concepts involved.

Step 1 – Identify symmetric values for the standard normal distribution such that the area enclosed is .99. Here, the idea is to recognize that the excluded area is .005 in each of the left and right tails. Thus, we want to find the 0.5^{th} and the 99.5^{th} percentiles.

Launch your favorite normal distribution applet or R or Stata. I happen to like the following:

https://istats.shinyapps.io/NormalDist/

Click on the tab "Find Percentile", set mean=**0**, standard deviation=**1**, choose "Type of Percentile" **Two tailed Pr [-x < X < +x]**Tand for "Central Probability (in %) enter 99. The calculator will return the 0.5^{th} and 99.5^{th} percentile values **± 2.57**



Tip - Notice that the 0.5^{th} and 99.5^{th} percentiles are -2.57 and +2.57, symmetric about zero. So, really, we only needed to solve for one of them.

Step 2 – Using the standardization formula as your starting point, solve backwards for the corresponding 0.5^{th} and 99.5^{th} percentiles of a normal distribution with mean 100 and standard deviation 14.

$$z = \frac{x-\mu}{\sigma} \text{ says that } \quad x = \sigma[z] + \mu$$

Thus "a" = 0.5^{th} percentile for X = 14[-2.57] + 100 = 63.95

and "b" = 99.5^{th} percentile for X = 14[+2.57] + 100 = 136.05

2. Suppose that the distribution of weights of New Zealand hamsters is distributed normal with mean 63.5 g and standard deviation 12.2 g. If there are 1000 weights in this population, how many of them are 78 g or greater?

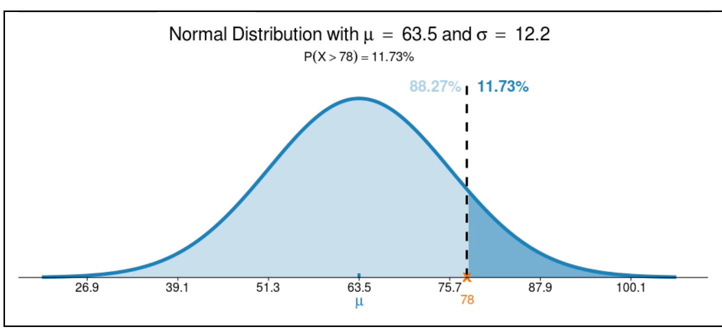
Answer: 117

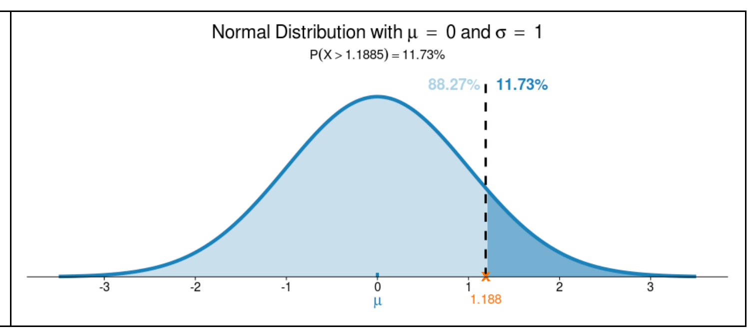
Solution:

$$\Pr[\text{ weight} > 78 \text{ g }] = \Pr[\text{ Normal } \mu{=}63.5 \ \ \sigma{=}12.2 > 78 \ ]$$

$$= \Pr\left[\text{ Standard normal } > \frac{78-\mu}{\sigma}\ \right] = \Pr\left[\text{ Standard normal } > \frac{78-63.5}{12.2}\ \right] = \Pr[\text{ Normal }(0,1) > 1.1885\ ] = .117$$

Therefore # Hamsters with weights > 78 g in a population of size 1000 = (1000)(.117) = 117





3. Consider again the normal probability distribution of problem #2. What is the probability of selecting at random a sample of 10 hamsters that has a mean greater than 65 g?

Answer: .3463 or .3487 (If you can sort out why the 2 answers don't match, you get a prize!)
Solution:

Tip – The solution to this problem requires noticing that the random variable is $\bar{X}$, so that the standardization to Z must use the SE for this.

$$\Pr[\bar{X}_{n=10} > 65 \text{ g}] = \Pr[\text{Normal } \mu_{\bar{X}}=63.5 \;\; \sigma_{\bar{X}}=\frac{12.2}{\sqrt{10}} > 65] = \Pr[\text{Normal } \mu_{\bar{X}}=63.5 \;\; \sigma_{\bar{X}}=3.7947 > 65]$$

$$= \Pr[\text{Standard normal} > \frac{65-\mu_{\bar{X}}}{\sigma_{\bar{X}}}] = \Pr[\text{Standard normal} > \frac{65-63.5}{12.2/\sqrt{10}}]$$

$$= \Pr[\text{Normal }(0,1) > 0.3888] = .3483$$