Unit 5 – Normal Distribution

Practice Quiz

SOLUTIONS

1. Suppose that the distribution of diastolic blood pressure in a population of hypertensive women is modeled well by a normal probability distribution with mean 100 mm Hg and standard deviation 14 mm Hg. Let X be the random variable representing this distribution. Find two symmetric values “a” and “b” such that probability \[ a < X < b \] = .99

Answer: a=63.95   b=136.05

Solution:
There is more than one approach for arriving at the same answer. Approach 1 is simpler. Approach 2 gives a better appreciation for the concepts involved

Approach 1 – Simpler

Step 1 –
Launch the David Lane Normal distribution calculator that can be found on the course website page for topic 5. The Normal Distribution

http://davidmlane.com/hyperstat/z_table.html

Step 2 –
Scroll down to the 2nd calculator that is provided. Enter 100 for the mean, 14 for the standard deviation, and 0.99 for the shaded area. Click on the button for “between”. The calculator returns the answer.

![Normal Distribution Calculator](http://davidmlane.com/hyperstat/z_table.html)
Approach 2 – Shows detail of the formula used

Step 1 – Identify symmetric values for the standard normal distribution such that the area enclosed is .99. Here, the idea is to recognize that the excluded area is .005 in each of the left and right tails. Thus, we want to find the 0.5th and the 99.5th percentiles.

Launch the David Lane Normal distribution calculator that can be found on the course website page for topic 5. *The Normal Distribution*

http://davidmlane.com/hyperstat/z_table.html

Step 2 –

Again, scroll down to the 2nd calculator that is provided. For the standard normal distribution, you should already see 0 for the mean and 1 for the standard deviation. Enter 0.99 for the shaded area. Click on the button for “between”. The calculator returns as the answer ± 2.5758.
Step 3 – Using the standardization formula as your starting point, solve backwards for the corresponding 0.5\textsuperscript{th} and 99.5\textsuperscript{th} percentiles of a normal distribution with mean 100 and standard deviation 14.

\[ z = \frac{x-\mu}{\sigma} \] says that \[ x = \sigma[z] + \mu \]

Thus a = 0.5\textsuperscript{th} percentile for \( X = 14[-2.57] + 100 = 63.95 \)

and b = 99.5\textsuperscript{th} percentile for \( X = 14[+2.57] + 100 = 136.05 \)

2. Suppose that the distribution of weights of New Zealand hamsters is distributed normal with mean 63.5 g and standard deviation 12.2 g. If there are 1000 weights in this population, how many of them are 78 g or greater?

Answer: 117

Solution:

\[ Pr [ \text{weight} > 78 \text{ g} ] = Pr [ \text{Normal} \mu=63.5 \sigma=12.2 > 78 ] \]

\[ = Pr [ \text{Standard normal} > \frac{78-\mu}{\sigma} ] \]

\[ = Pr [ \text{Standard normal} > \frac{78-63.5}{12.2} ] \]

\[ = Pr [ \text{Normal (0,1)} > 1.1885 ] \]

\[ = .117 \]

Therefore # Hamsters with weights > 78 g in a population of size 1000

\[ = (\text{Number of hamsters}) \times Pr[\text{weight} > 78 \text{ g}] \]

\[ = (1000)(.117) = 117 \]
3. Consider again the normal probability distribution of problem #2. What is the probability of selecting at random a sample of 10 hamsters that has a mean greater than 65 g?

**Answer:** .3483

**Solution:**

**Tip** – The solution to this problem requires noticing that the random variable is $\bar{X}$, so that the standardization to $Z$ must use the SE for this.

\[
\Pr [ \bar{X}_{n=10} > 65 \text{ g} ] = \Pr [ \text{Normal } \mu_{\bar{X}}=63.5 \text{ } \sigma_{\bar{X}}=\frac{12.2}{\sqrt{10}} > 65 ]
\]

\[
= \Pr [ \text{Standard normal } > \frac{65-\mu_{\bar{X}}}{\sigma_{\bar{X}}} ] = \Pr [ \text{Standard normal } > \frac{65-63.5}{12.2/\sqrt{10}} ]
\]

\[
= \Pr [ \text{Normal (0,1) } > 0.3888 ] = .3483
\]