Unit 6  
The Bernoulli and Binomial Distributions  
Practice Quiz  
SOLUTIONS

1. According to a recent poll, 41% of US citizens approved of the job the President was doing at the time. Assume that this proportion was actually true for the whole of the US population.

(a) Suppose a simple random sample of 10 citizens was selected. What is the probability that a majority (more than 5) approved of the job that the President was doing?

Answer: 0.18  
Solution:  
Define $X$ is the number of approve. We need to find $P(X > 5)$.  
Recall that binomial distribution is given by  
$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x},$$  
where $p$ is the population parameter for the probability of success, $n$ is the number of trials and $x$ is the number of successes.  
We have, $p=0.41$, $n=10$ and $x>5$.  

$$P(X > 5) = P(X=6) + P(X=7) + P(X=8) + P(X=9) + P(X=10)$$

$$= \binom{10}{6} (0.41)^6 (1-0.41)^{10-6} + \binom{10}{7} (0.41)^7 (1-0.41)^{10-7} + \binom{10}{8} (0.41)^8 (1-0.41)^{10-8} +$$

$$+ \binom{10}{9} (0.41)^9 (1-0.41)^{10-9} + \binom{10}{10} (0.41)^{10} (1-0.41)^{10-10}$$

$$= 0.1834$$  
Yikes. That was a lot of hand calculation. Fortunately, we can let the computer do the work.

(b) Suppose a simple random sample of 50 citizens was selected. What is the probability that a majority (more than 25) approved of the job that the President was doing?

Answer: 0.08  
Solution:  
$$P(X > 25) = P(X \geq 26) = 0.0761$$  
Note: $X > 25$ is the same as $X \geq 26$. This is handy on next page.

This result can be obtained as in part (a). A more practical approach would be to use a probability calculator applet on http://www.artofstat.com/webapps.html.
Click on the tab “Find Probability”, then choose as your inputs the following: 1) Number of Bernoulli trials = 50, 2) Probability of success, p = 0.41 3) Select type of probability Upper tail P(X >= x) and 4) Number of successes (x) = 26 You will then see:
2. The probability that a person suffering from a migraine headache will obtain relief from a particular drug is 0.9. Three randomly selected sufferers from migraine headache are given the drug. Find the probability that the number obtaining relief will be 2 or 3.

**Answer: .97**

**Solution:**

\[
P(X \geq 2) = P(x=2) + P(x=3) = 0.9720
\]

\[
P(X \geq 2) = \binom{3}{2}(0.9)^2(1-0.9)^{3-2} + \binom{3}{3}(0.9)^3(1-0.9)^{3-3} = 0.9720
\]
3. A home security system has a 80% reliability rate, meaning that it goes off 80% of the time when there is a burglary. Suppose that 12 homes equipped with this system experience an attempted burglary. What is the probability that more than 7 alarms go off?

**Answer:** .93

**Solution:**

\[
P(X>7) = P(X \geq 8) = P(X=8)+ P(X=9)+ P(X=10)+ P(X=11)+ P(X=12)
\]
4. *(Source: Virtual Lab)*. The common form of hemophilia is due to a defect on the X chromosome (one of the two chromosomes that determine gender). We will let “h” denote the defective gene, linked to hemophilia, and H the corresponding normal gene. Women have two X chromosomes, and “h” is recessive. Thus, a woman with gene type HH is normal, a woman with gene type “hH” or “Hh” is free of disease but is a carrier; and a woman with gene type “hh” has the disease. A man has only one X chromosome (the other sex chromosome, the Y chromosome, plays no role in the disease. A man with gene type h has hemophilia and a man with gene type H is healthy.

<table>
<thead>
<tr>
<th>Mother XX</th>
<th>Father X</th>
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<tbody>
<tr>
<td>HH – Normal</td>
<td>H - Normal</td>
</tr>
<tr>
<td>Hh – Carrier</td>
<td>h - Hemophilia</td>
</tr>
<tr>
<td>hh – Hemophilia</td>
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(a) Suppose that a mother is a carrier and the father is healthy. They have a son. What is the probability that the son will have hemophilia? Will be healthy?

**Answer:** \( \Pr[\text{son is hemophiliac}] = .5 \) \( \Pr[\text{son is healthy}] = .5 \)

**Solution:**

If child is a son, then father contributed a Y chromosome. The Y plays no role in disease. Thus son’s chances of H and h are both 0.5

(b) Suppose that a mother is a carrier and the father has hemophilia. They have a daughter. What is the probability that the daughter will have hemophilia? Will be a carrier?

**Answer:** \( \Pr[\text{daughter is hemophiliac}] = .5 \) \( \Pr[\text{daughter is carrier}] = .5 \)

**Solution:**

If child is a daughter, then father contributed an X chromosome. If it is known that the father is a hemophiliac, then we know he contributed an h with probability 1. If the mother is a carrier, then she is hH (or other way around) plays no role in disease. Thus daughter is hh or hH, each with probability .5