Unit 6 – Estimation
Self Evaluation Quiz
SOLUTIONS

Some studies of Alzheimer's disease (AD) have shown an increase in $_{14}CO^2$ production in patients with the disease. In one such study, the following $_{14}CO^2$ values were obtained from 16 neocortical biopsy samples from AD patients.

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1009  1280  1180  1255  1547  2352  1956  1080
1776  1767  1680  2050  1452  2857  3100  1621
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Assume that the population of such values is normally distributed with a standard deviation of $\sigma = 350$.

1. Construct a 95 percent confidence interval for μ.
   Answer: (1576.1, 1919.1)
   Solution:
   (i)   point estimate is $\overline{X} = 1747.6$
   (ii)   standard error of point estimate is $SE(X) = \sigma/\sqrt{n} = 350/4 = 87.5$
   (iii)   solution for confidence coefficient: Since $(1 - \alpha) = 0.95$, $(1-\alpha/2)=0.975$. Get 97.5^{th} percentile of Normal(0,1) = 1.96. Thus the required confidence interval is

   estimate $\pm$ { confidence coefficient} { se of estimate}
   $= 1747.6 \pm \{ 1.96 \} \{87.5 \}$
   $= (1576.1, 1919.1)$

2. If the true population mean is $\mu = 1800$ with $\sigma = 350$, what proportion of patient values would be greater than 1900?
   Answer: .3876
   Solution:
   Since $\mu = 1800$ and $\sigma = 350$, solve as

   Prob $(X \geq 1900) =$ Prob $(Z \geq \{ (1900 - 1800)/350 \})$
                  $=$ Prob $(Z \geq 0.2857)$
                  $= 0.3876$

3. If the true population mean is $\mu = 1800$ with $\sigma = 350$, what proportion of means of size 16 would be greater than 1900? What proportion of means from samples of size 25 would be greater than 1900?

   Answer:

Sample Size	Proportion of Means Greater than 1900
16	**.1265**
25	**.0765**

   Solution:

   $$\text{Prob} \left(\overline{X}_{n=16} \geq 1900 \right) = \text{Prob} \left(Z \geq \{ (1900 - 1800)/(350/4) \} \right)$$
   $$= \text{Prob} \left(Z \geq 1.143 \right)$$
   $$= 0.1265. \text{ Conclude that about 12.7\% of sample means of size 16 would be greater than 1900.}$$

   $$\text{Prob} \left(\overline{X}_{n=25} \geq 1900 \right) = \text{Prob} \left(Z \geq \{ (1900 - 1800)/(350/5) \} \right)$$
   $$= \text{Prob} \left(Z \geq 1.429 \right)$$
   $$= 0.0765. \text{ Conclude that about 7.7\% of sample means of size 25 would be greater than 1900.}$$

4. Considering the derivation of confidence interval estimates, comment on the role of sample size in the estimation of the unknown population mean parameter.

   Solution:

   Narrower, or more precise, confidence interval estimates are obtained when obtained from data sets of larger sample sizes.

   Recall that the width of a confidence interval for the mean parameter of a normal probability distribution with known variance is:

   $$(2) \ \{ \text{critical z} \} \ \{ \text{standard deviation} / \sqrt{n} \}$$

   where n is the number of observations in the sample.

   This means that, for fixed critical z and for fixed standard deviation, as n increases, the quantity $1/\sqrt{n}$ decreases. Consequently, the width of the confidence interval also decreases.

5. Now, assume that the population of such values is normally distributed with unknown mean and ***unknown*** variance. Construct a 95% confidence interval for the population mean. Compare this interval to the interval you got for question #1.

Answer: (1425.6, 2069.61)

Solution:

Confidence intervals constructed utilizing the Student's t-distribution for the determination of the confidence coefficient are larger than those using the Normal (0,1) distribution, all other things equal.

Calculate sample variance s^2 and from this obtain s = 604.65

(i)   point estimate is $\overline{X} = 1747.6$

(iv)   **sample** standard error of point estimate is $s/\sqrt{n}$ = 604.65/4 = 151.16

(v)   solution for confidence coefficient: Since $(1 - \alpha) = 0.95$, $(1-\alpha/2)=0.975$. Get 97.5^{th} percentile of student's t distribution with 15 degrees of freedom = 2.13. Thus the required confidence interval is

estimate $\pm$ { confidence coefficient from student's t} { standard error of estimate}
= 1747.6 $\pm$ { 2.13 } {151.16 }
= (1425.63, 2069.57)