

Self Evaluation Quiz  
Topic 1 - Summarizing Data

***SOLUTIONS***

1. (b) **Discrete data with categories that do not follow a natural sequence.**

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$$2. \text{ new mean} = \frac{[\text{old mean} - 1000]}{5} \quad \text{new variance} = \frac{[\text{old variance}]}{5^2} \quad \text{new sd} = \frac{[\text{old sd}]}{5}$$

You could obtain this by brute force, that is by doing the calculations for the old data set and the new data set which is: 1, 2, 3, 4 and 5.

*For the interested reader ... An alternate solution works through the formulae as follows.*

$$\begin{aligned} \text{new mean} &= \frac{1}{n} \sum \left[ \frac{(x-1000)}{5} \right] \\ &= \frac{1}{5n} \left[ \sum (x-1000) \right] \\ &= \frac{1}{5n} \left[ \sum x - \sum 1000 \right] \\ &= \frac{1}{5n} \left[ \sum x - (1000)(n) \right] \\ &= \frac{\sum x}{5n} - \frac{(1000)(n)}{(5)(n)} \\ &= \frac{1}{5} \left[ \frac{\sum x}{n} - 1000 \right] \\ &= \frac{1}{5} [\text{old mean} - 1000] \end{aligned}$$

$$\begin{aligned}
\text{new variance} &= \left[ \frac{1}{n-1} \right] \sum (\text{new } x - \text{new mean})^2 \\
&= \left[ \frac{1}{n-1} \right] \sum \left( \left[ \frac{x-1000}{5} \right] - \text{new mean} \right)^2 \\
&= \left[ \frac{1}{n-1} \right] \sum \left( \left[ \frac{x-1000}{5} \right] - \left[ \frac{\text{old mean}-1000}{5} \right] \right)^2 \\
&= \left[ \frac{1}{n-1} \right] \left[ \frac{1}{5} \right]^2 \sum ([x-1000] - [\text{old mean} - 1000])^2 \\
&= \left[ \frac{1}{n-1} \right] \left[ \frac{1}{5} \right]^2 \sum (x-1000-\text{old mean} + 1000)^2 \\
&= \left[ \frac{1}{n-1} \right] \left[ \frac{1}{5} \right]^2 \sum (x - \text{old mean})^2 \\
&= \left[ \frac{1}{5} \right]^2 \left[ \frac{1}{n-1} \right] \sum (x - \text{old mean})^2 \\
&= \left[ \frac{1}{5} \right]^2 [\text{old variance}]
\end{aligned}$$

$$\begin{aligned}
\text{new standard deviation} &= \sqrt{\text{new variance}} \\
&= \sqrt{\left[ \frac{1}{5} \right]^2 (\text{old variance})} \\
&= \frac{1}{5} \sqrt{\text{old variance}} \\
&= \frac{1}{5} [\text{old standard deviation}]
\end{aligned}$$


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3. **New sample mean = 53.07**  
**New sample standard deviation = 9.137**

**Solution for new sample mean.**

Step 1 – Work with the original mean to obtain the sum of ages of the original 50

$$\text{IF } \bar{X} = \frac{\sum_{i=1}^{50} x_i}{50} \text{ THEN } \sum_{i=1}^{50} x_i = (50)(\bar{X}) = (50)(53.87) = 2,693.5$$

Step 2 - Use this to solve for the sum of the ages of the original 49 plus the new person

$$\sum_{i=1}^{49} x_i = (2,693.5 - 82) = 2611.5 \text{ and } \sum_{i=1}^{50} x_i = (2611.5 + 42) = 2653.5. \text{ Thus,}$$

$$\bar{X} = \frac{\sum_{i=1}^{50} x_i}{50} = \frac{2653.5}{50} = 53.07$$

**Solution for new sample standard deviation.**

$$\text{If } S^2 = \left[ \frac{1}{n-1} \right] \sum (x_i - \bar{x})^2 \text{ Then } S^2 = \left[ \frac{1}{n-1} \right] \left( \left[ \sum x_i^2 \right] - [n\bar{x}^2] \right)$$

Step 1 – Work with the original variance to obtain the sum of squared ages of the original 50

*Note – Save rounding until the last step (reporting)*

$$s = 9.87 \rightarrow$$

$$s^2 = 97.417 \rightarrow$$

$$(n-1)s^2 = \sum_{i=1}^{50} (x_i - \bar{x})^2 = (49)(97.417) = 4773.428 \rightarrow$$

$$4773.428 = \sum_{i=1}^{50} x_i^2 - (n)(\bar{x}^2) \rightarrow$$

$$4773.428 = \sum_{i=1}^{50} x_i^2 - (50)(53.87^2) \rightarrow$$

$$4773.428 = \sum_{i=1}^{50} x_i^2 - 145,098.845 \rightarrow$$

$$\sum_{i=1}^{50} x_i^2 = 149,872.273$$

Step 2 – Obtain the sum of squared ages of the new 50

$$\begin{aligned}\text{New } \sum_{i=1}^{50} x_i^2 &= \text{Old } \sum_{i=1}^{50} x_i^2 - 82^2 + 42^2 \\ &= 149,872.273 - 6724 + 1764 \\ &= 144,912.273\end{aligned}$$

Step 3 – Use this plus new mean to obtain new  $S^2$

$$\begin{aligned}\text{New } S^2 &= \left[ \frac{1}{n-1} \right] \left[ \sum_{i=1}^{50} x_i^2 - (n)(\bar{x}^2) \right] \rightarrow \\ S^2 &= \left[ \frac{1}{49} \right] \left[ 144,912.273 - (50)(53.07^2) \right] \\ &= 83.49\end{aligned}$$

Last Step – Use new  $S^2$  to obtain new S

$$\begin{aligned}\text{New } S &= \sqrt{\text{new } S^2} \\ &= \sqrt{83.49} \\ &= \mathbf{9.137}\end{aligned}$$

**4. False.**

Removal of an extreme value yields remaining data that are less variable.

**5. Median = 121.5 IK**

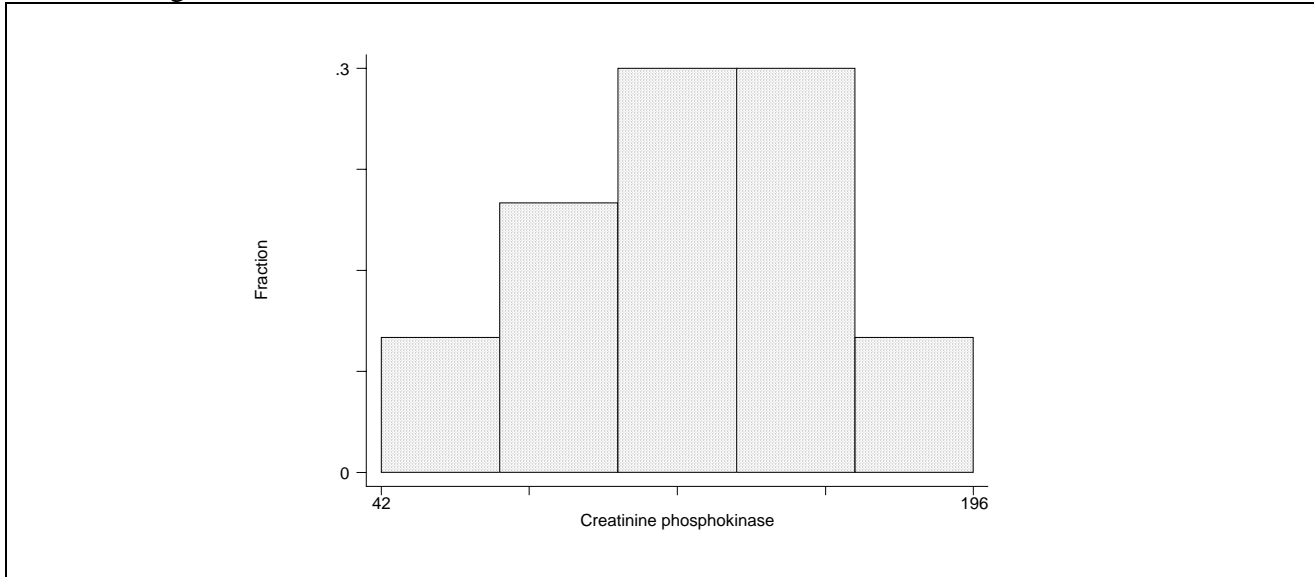
Ordering the data from smallest to largest yields

42 < 89 < 94 < 108 < **115 < 128** < 136 < 149 < 158 < 196

Thus,

$$\text{median} = \frac{1}{2} [115 + 128] = \mathbf{121.5 \text{ IK}}$$

6. Averaging the 10 values yields mean = 121.5  
 The similarity of the mean and median suggests that the data are **symmetric**.  
 A histogram confirms this.



*. graph cpk, histogram*

*source: quiz1num5.wmf*

7. A box weighs **16** ounces, give or take **0.2** ounces.

Not what you thought, is it! This question is a little more involved than you might have thought and, so, I don't expect that you will have gotten the correct answer the first time.

As the 4 sticks of butter are independent,

Mean of box = Sum of means of 4 sticks = (4) (4 ounces) = 16 ounces

Variance of box = Sum of variances of 4 sticks = (4) (0.1<sup>2</sup> ounces<sup>2</sup>) = 0.04 ounces<sup>2</sup>

Therefore, standard deviation of box =  $\sqrt{0.04 \text{ ounces}^2} = 0.2 \text{ ounces}$

8.  $\bar{X}=0.667$   
 $S=0.471$

$$\bar{X} = \left[ \frac{1}{n} \sum_{i=1}^{3000} X_i \right] = \left[ \frac{1}{3000} \right] [(1000)(0) + (2000)(1)] = 0.667$$

$$\begin{aligned}
S^2 &= \left[ \frac{1}{n-1} \right] \left[ \sum_{i=1}^{3000} X_i^2 - (n)(\bar{X}^2) \right] \\
&= \left[ \frac{1}{2999} \right] \left[ (1000)(0^2) + (2000)(1^2) - (3000)(.667^2) \right] \\
&= \left[ \frac{1}{2999} \right] \left[ (2000) - (1,333.33) \right] \\
&= 0.222 \quad \rightarrow \rightarrow \quad S = \sqrt{S^2} = \sqrt{0.222} = 0.471
\end{aligned}$$


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9.

(9.1) **50**(9.2) **25**(9.3) **40**10. **5 feet 7.8 inches***Preliminary**5 feet 8 inches = 68 inches**4 feet 11 inches = 59 inches*Old  $\bar{X} = 5$  feet 8 inches = 68 inches  $\rightarrow$ 

$$\sum_{i=1}^{49} X_i = (n)(\bar{X}) = (49)(68 \text{ inches}) = 3332 \text{ inches} \quad \text{Thus,}$$

$$\text{New } \bar{X} = \left[ \frac{1}{n} \right] \sum_{i=1}^{50} X_i = \left[ \frac{1}{50} \right] \left[ \sum_{i=1}^{49} X_i + 59 \text{ inches} \right] = \left[ \frac{1}{50} \right] [3332 + 59] = 67.82$$