Types of Events

- Two events \( A \) and \( B \) are \textit{mutually exclusive} if they cannot happen at the same time\( P(A \cap B) = 0 \)

- Two events \( A \) and \( B \) are \textit{independent} if the probability of the first one happening is the same no matter how the second one turns out

Properties of Independent Events

That \( A \) and \( B \) are independent means:

- \( P(A|B) = P(A) \)

- \( P(B|A) = P(B) \)

- \( P(A \cap B) = P(A) \cdot P(B) \)

Diagnostic Testing

- Experiment: performing a diagnostic test for some disease
- Data: information on a random sample of \( n \) people who were tested and whose disease status is known

<table>
<thead>
<tr>
<th>Test ( \pm )</th>
<th>Disease (D+)</th>
<th>No disease (D-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test + (T+)</td>
<td>( a )</td>
<td>( b )</td>
</tr>
<tr>
<td>Test – (T-)</td>
<td>( c )</td>
<td>( d )</td>
</tr>
<tr>
<td></td>
<td>( a+c )</td>
<td>( b+d )</td>
</tr>
</tbody>
</table>

- Want to learn: how good the test is
- There are four measures of how good a test is:

1. Sensitivity = \( P(T+|D+) = \frac{a}{a+c} \)
2. Specificity = $P(T^-|D^-) = \frac{d}{b+d}$

3. Predictive value of a positive test = $PV^+ = P(D^+|T^+) = \frac{a}{a+b}$

4. Predictive value of a negative test = $PV^- = P(D^-|T^-) = \frac{d}{c+d}$

Problem for Rare Diseases

- Need very large number of people in the study to have enough people with the disease

- Solution: instead of choosing $n$ people at random choose $n$ people with disease and $n$ people without disease