

Unit 2 – Introduction to Probability
FAQ
Multiplication and Addition of Probabilities
When do I multiply and when do I add?

At first glance, the following two rules might seem confusing and difficult to use in practice.

Multiplication Rule

For two outcomes A and B, the probability of their joint occurrence is given by

$$\begin{aligned}\Pr[A \text{ and } B] &= \Pr[A] \times \Pr[B|A] \quad \text{note that this can also be written} \\ &= \Pr[B] \times \Pr[A|B]\end{aligned}$$

Addition Rule

For two outcomes A and B, the probability of the occurrence of either outcome or both outcomes occurring is given by

$$\Pr[A \text{ or } B] = \Pr[A] + \Pr[B] - \Pr[A \text{ and } B]$$

Special case :

When the outcomes A and B are *mutually exclusive*, the probability of the occurrence of either outcome or both outcomes is given by

$$\Pr[A \text{ or } B] = \Pr[A] + \Pr[B]$$

This is right since when A and B are mutually exclusive it is not possible for them both to occur ; that is $\Pr[A \text{ and } B] = 0$ when A and B are mutually exclusive.

Hint :

- When considering the joint occurrence of outcomes that occur in sequence **over time**, you want to use the ***multiplication rule***.
- When considering the occurrence of an event that represents a kind of “end of the day net” result, you want to use the ***addition rule***.

Consider the schematic on the next page.

As you follow the “tree” down the page, you will multiply probabilities.

At the bottom, as you read across the horizontal, you will add the probabilities.

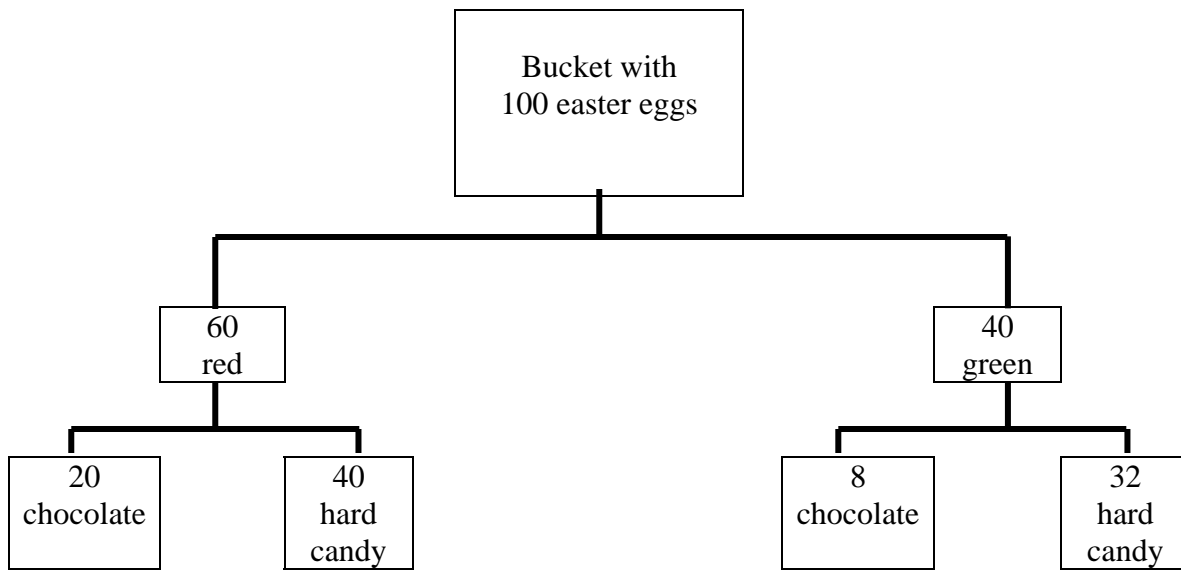


Illustration of Multiplication Rule:

As you travel **down** the “tree” you are traveling in time through series of outcomes. Multiply!

$$\text{Pr}[\text{red egg and chocolate}] = \text{Pr}[\text{red egg}] \times \text{Pr}[\text{chocolate} \mid \text{egg is red}] = [60/100] \times [20/60]$$

$$\text{Pr}[\text{red egg and hard}] = \text{Pr}[\text{red egg}] \times \text{Pr}[\text{hard candy} \mid \text{egg is red}] = [60/100] \times [40/60]$$

$$\text{Pr}[\text{green egg and chocolate}] = \text{Pr}[\text{green egg}] \times \text{Pr}[\text{chocolate} \mid \text{egg is green}] = [40/100] \times [8/40]$$

$$\text{Pr}[\text{green egg and hard}] = \text{Pr}[\text{green egg}] \times \text{Pr}[\text{hard} \mid \text{egg is green}] = [40/100] \times [32/40]$$

Illustration of Addition Rule

At the “end of the day” what is the probability of a chocolate?

Now you are moving **horizontally across the bottom** over the 4 mutually exclusive end results. Add!

The probability of a “net result” of getting a chocolate is the sum of the probabilities of the qualifying scenarios that are mutually exclusive.

$$\begin{aligned} \text{Pr}[\text{chocolate}] &= \text{Pr}[\text{red egg and chocolate}] + \text{Pr}[\text{green egg and chocolate}] \\ &= [60/100] \times [20/60] + [40/100] \times [8/40] \end{aligned}$$