Before you begin:
This is a “take-home” exam. You are welcome to use any reference materials you wish. You are welcome to use the computer as you wish, too. However, you MUST work this exam by yourself and you may not consult with anyone.

Instructions and Checklist:
__1. Start each problem on a new page.
__2. Write your name on every page.
__3. Make a photo-copy of your exam for safekeeping prior to submission
__4. Complete the signature page

How to submit your exam:

<table>
<thead>
<tr>
<th>Worcester “In-Class” Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hand deliver to me, or mail your completed exam to me with post mark December 13, 2013 to my address below.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ONLINE Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>You may submit your exam in EITHER of 2 ways:</td>
</tr>
<tr>
<td>__1. Upload your completed exam to the ASSIGNMENT tab no later than 11:59 pm on Friday December 13, 2013. This must be a single pdf and be named using the convention lastname_exam4.pdf.</td>
</tr>
<tr>
<td>__2. Mail your completed exam to me with post mark no later than December 13, 2013 to my address below.</td>
</tr>
</tbody>
</table>

Address and telephone number for mailing

Carol Bigelow
School of Public Health/402 Arnold House
University of Massachusetts/Amherst
715 North Pleasant Street
Amherst, MA 01003-9304
Tel. 413-545-1319
Signature

This is to confirm that in completing this exam, I worked independently and did not consult with anyone.

Signature: __________________________________________________________

Printed Name: _______________________________________________________

Date: __________________________
Dear class,

This test has 100 points total, plus 10 points of bonus point questions. The bonus point questions are on page 13. Points earned on these will be added to your exam score, up to a maximum exam score of 100.

1. (10 points total)

In the movie “When Harry Met Sally”, there is a scene where Meg Ryan is shopping for a Christmas tree. The film shows a beautiful snowy night in New York city and Meg Ryan dragging her tree back to her apartment. Suppose the mean height of a sample of n=15 Christmas trees was $\bar{X} = 143$ cm. Assume that the population of tree heights is normal with known standard deviation $\sigma = 18$ cm.

1a. (5 points)

Construct a 90% confidence interval estimate of the unknown mean $\mu$.

1b. (5 points)

Have a look at the CI that you constructed in answering question #1a. On the basis of its lower bound, is it reasonable to say that the average tree height in the population is at least 130 cm? Explain.
2. (10 points total)

A laboratory is doing a quality assurance study of its performance. In a test of its measurement of serum cholesterol, 15 samples containing the same known amount of cholesterol (190 mg/dL) of serum cholesterol are submitted for measurement as part of a larger batch of samples, one sample each day over a three week period. Suppose that following daily values in mg/DL for serum cholesterol were obtained for these 15 samples

<table>
<thead>
<tr>
<th>Day</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>mg/dL</td>
<td>180</td>
<td>190</td>
<td>197</td>
<td>199</td>
<td>210</td>
<td>187</td>
<td>192</td>
<td>199</td>
<td>214</td>
<td>237</td>
<td>188</td>
<td>197</td>
<td>208</td>
<td>220</td>
<td>239</td>
</tr>
</tbody>
</table>

2a. (5 points)

Assume that the variance $\sigma^2$ for the measurement of serum cholesterol is supposed to be no larger than 100. Construct a 95% confidence interval estimate of the unknown true variance. You may assume that the 15 measurements are a simple random sample from a normal distribution.

2b. (5 points)

Have a look at the CI that you constructed in answering question #2a. Does it include the value of 100? In 1-2 sentences, interpret the confidence interval you obtained.
3. (10 points total)

My house is in the woods and there are lots of neighboring woods. These woods are preferred areas for local hunters and December is hunting season in New England (what’s more - each week has its own designated gun type). During these weeks, joggers are advised to wear the same “hunting” orange, even if they are careful and jog only on the roads.

A study of hunting injuries and the wearing of “hunting” orange found that, among 123 actual injured hunters who were mistaken for game, 6 were wearing “hunting” orange at the time.

Construct a 95% confidence interval for the population proportion \( \pi \) of injured hunters who are wearing “hunting” orange.
4. (10 points total)

4a. (2 points) True or False.
A hypothesis test for which the type I error occurs with probability $\alpha$ has probability of type II error equal to $(1 - \alpha)$.

4b. (2 points) True or False.
If a one sided test indicates that the null hypothesis can be rejected at the 5% level, then a two sided test performed on the same set of data is necessarily significant at the 5% level.

4c. (2 points) True or False.
For a given sample variance $s^2$ and sample mean $\bar{X}$, a 90% confidence interval for an unknown mean $\mu$ is narrower than a 99% confidence interval.

4d. (2 points) True or False.
An investigator is performing a t-test for which the assumptions are satisfied could, in the absence of a student’s t-distribution tables, use a Normal(0,1) probability table provided the degrees of freedom is sufficiently large.

4e. (2 points) Choose ONE.
The meaning of a p-value is

i. the power of the test
ii. the probability of getting a result as extreme or more extreme than the one observed if the null hypothesis is false.
iii. the probability that the null hypothesis is true
iv. the probability of making a type II error
v. the probability of getting a result as extreme or more extreme than the one observed if the null hypothesis is true.
5. (10 points total)
In (a) – (e) below, you may assume that the data are a simple random sample (or samples) from a normal distribution (or distributions).

5a. (2 points)
For a single sample size of \( n = 15 \) and the estimation of the population mean \( \mu \) when the variance is unknown using a 90% confidence interval, what are the formulae and values of the confidence coefficients?

5b. (2 points)
For a single sample size \( n = 35 \) and the estimation of a variance parameter \( \sigma^2 \) using a 95% confidence interval, what are the formulae and values of the confidence coefficients?

5c. (2 points)
For a single sample size of \( n = 25 \) and the estimation of the population mean \( \mu \) when the variance is known using a 80% confidence interval, what are the formulae and values of the confidence coefficients?

5d. (2 points)
Next, consider the setting of two independent samples, one with sample size \( n_1 = 13 \) and the other with sample size \( n_2 = 22 \), and estimation of the difference between the two population means, \( [\mu_1 - \mu_2] \). Assume that the two population variances are both unknown and unequal. **Suppose further that** \( S_1^2 = 2.20 \) and \( S_2^2 = 0.70 \). If it is of interest to construct a 95% confidence interval estimate of \( [\mu_1 - \mu_2] \), what are the formulae and values of the confidence coefficients?

5e. (2 points)
Finally, consider again the setting of two independent samples, one with sample size \( n_1 = 13 \) and the other with sample size \( n_2 = 22 \). Here, however, it is of interest to construct a 90% confidence interval estimate of the ratio of the two population variances, \( [\sigma_1^2/\sigma_2^2] \). What are the formulae and values of the confidence coefficients?
6. (10 points total)

In (a) and (b) below, you are asked to state the null (H₀) and alternative (Hₐ) hypotheses.

6a. (3 points)

A drug company claims that a single dose of a new drug relieves symptoms for a longer period of time than the current formulation. The current formulation is known to relieve symptoms for eight hours on average. State the null (H₀) and alternative (Hₐ) hypotheses that would be defined in a statistical hypothesis test of this claim. Be sure to define all terms.

6b. (3 points)

Americans gain an average of one pound per year as they age between the ages of 25 and 45. A public health campaign aims to decrease the amount of weight gained during this interval. State the null (H₀) and alternative (Hₐ) hypotheses that would be defined in a statistical hypothesis test of this claim. Be sure to define all terms.

6c. (2 points)

What is wrong with the following statement?

H₀: \( \bar{X} = 100 \)
Hₐ: \( \bar{X} \neq 100 \)

6d. (2 points)

What is wrong with the following statement?

H₀: \( \mu = 100 \)
Hₐ: \( \mu \neq 110 \)
7. (10 points total)

7a. (2 points)

Consider the construction of a 98% (note – I said 98%, not 95%) confidence interval of the mean for the setting of a simple random sample from a Normal distribution where the variance parameter $\sigma^2$ is known. What is the value of $\frac{1}{2}z_{1-\alpha/2}$ that is used in this confidence interval construction?

Questions #7b, #7c, #7d, and #7e all pertain to the following:
Suppose next that you are told that $\text{(262.09, 374.11)}$ is the result of calculating a 95% confidence interval for the mean of a Normal distribution for the setting of a simple random sample of size $n=80$ from a normal distribution where the variance parameter $\sigma^2$ is known.

7b. (2 points)

What is the point estimate of the population mean $\mu$?

7c. (2 points)

What is the value of the standard error of the mean?

7d. (2 points)

What is the value of the population variance?

7e. (2 points)

In 1-2 sentences at most, write a clear interpretation of the confidence interval.
8. (10 points total)

For each of the following single sample hypothesis test situations, what is the critical region? In developing your answer you will be providing the range of values (called the critical region) of the sample average $\bar{X}$ that will prompt rejection of the null hypothesis.  Tip – Do not give me the critical region for the test statistic (Z-test or a T-test or a Chi square or an F statistic). I am asking you to solve for the critical region for $\bar{X}$

**8a (2 points)** Critical Region for $\bar{X}$ is: _________________________________

$H_0$: $\mu = 220$

$H_A$: $\mu \neq 220$

$\alpha = 0.05$, $n = 20$, $\sigma = 14$, known

**8b (2 points)** Critical Region for $\bar{X}$ is: _________________________________

$H_0$: $\mu \leq 15$

$H_A$: $\mu > 15$

$\alpha = 0.01$, $n = 35$, $\sigma = 5$, known

**8c (2 points)** Critical Region for $\bar{X}$ is: _________________________________

$H_0$: $\mu = 120$

$H_A$: $\mu \neq 120$

$\alpha = 0.05$, $n = 25$, sample sd = 11

**8d (2 points)** Critical Region for $\bar{X}$ is: _________________________________

$H_0$: $\mu \geq 100$

$H_A$: $\mu < 100$

$\alpha = 0.01$, $n = 16$, sample sd = 9

**8e (2 points)** Critical Region for $\bar{X}$ is: _________________________________

$H_0$: $\mu \leq 55$

$H_A$: $\mu > 55$

$\alpha = 0.05$, $n = 857$, sample sd = 7.3
9. (10 points total)

Yellowstone National Park has really big teddy bears, some more “bear” than “teddy”. Suppose a random sample of n=54 bears has a mean weight of $\bar{X} = 282.9$ lbs. Suppose further that the population standard deviation is known and is $\sigma = 121.8$ lbs. Construct an alpha (type I error) = 0.10 critical region test (two sided) of the claim that the population mean weight is $\mu = 250$ lbs.

Include in your answer the following:

9a (2 points)
The null and alternative hypotheses.

9b (5 points)
The critical region.

9c (3 points)
The final conclusion.
10. (10 points total)

Sometimes drugs are recalled because of failure to meet “content uniformity” requirements. Suppose that “content uniformity” is expressed in terms of variance. For example, suppose a “content uniformity” requirement specifies that the variance in the amount of Phenobarbital in tablets is supposed to be less than or equal to 0.015 grams$^2$.

The satisfaction of this requirement is being evaluated in a simple random sample of 30 tablets. For this sample, the sample standard deviation was found to be $s=0.14$ grams. Carry out the appropriate statistical hypothesis test to assess compliance with the content uniformity specification of variance less than or equal to 0.015 grams$^2$. In developing your answer, address the following:

10a  (2 points)
What is the null hypothesis?

10b  (2 points)
What is the alternative hypothesis?

10c  (2 points)
What is the appropriate test statistic and what is its value for these data?

10d  (2 points)
What is the p-value?

10e  (2 points)
In 1-2 sentences at most, what is your interpretation and conclusion?
Santa is testing a new chimney descent device. He will switch to the new device only if there is evidence that the new device yields faster descent times to your living room floor than the old system. In order to make a decision, a sample of 7 houses was selected. Santa descended into each house using both devices. Descent times, in seconds, are shown below. Under the assumption of normality, is there statistically significant evidence that the new device yields faster descent times?

<table>
<thead>
<tr>
<th>House</th>
<th>Decent time, Old</th>
<th>Descent time, New</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
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<tr>
<td>7</td>
<td>12</td>
<td>9</td>
</tr>
</tbody>
</table>

Include in your answer the following:

a. (2 points)
The null and alternative hypothesis.

b. (2 points)
The formula for the correct test statistic. Be sure to define all terms.

c. (2 points)
The value of the test statistic you defined in part (b).

d (2 points)
The value of the p-value calculation.

e (2 points)
The final conclusion