

Non-Conservation of Transverse Magnetization in Spin Diffusion in Trapped Boltzmann Gases

W. J. Mullin¹ and R. J. Ragan²

¹*Physics Department, Hasbrouck Laboratory, University of Massachusetts, Amherst, MA 01003 and*
²*Physics Department, University of Wisconsin at Lacrosse, La Crosse, WI 54601*

(Dated: April 22, 2005)

Experiments in a mixture of two hyperfine states of trapped Bose gases show behavior analogous to a spin-1/2 system, including transverse spin waves and other familiar Leggett-Rice-type effects. We have derived the kinetic equations applicable to these systems, including the spin dependence of interparticle interactions in the collision integral. We find that the hydrodynamic diffusive modes cease to exist because interactions with different scattering lengths for up-up, up-down, and down-down spins lead to a spin-spin relaxation that causes non-conservation of transverse magnetization. We give results for the quadrupole modes, the modes studied in experiments with equal scattering lengths. Instead of a q^2 , dependence on wave number q for the diffusive mode, we find a divergence at small wave number. No such effect occurs in Fermi gases.

Bose-Einstein condensation; ideal Bose gas; permutation cycles; trapped gas

Spin waves and spin diffusion in Fermi and Bose fluids since Leggett and Rice suggested unusual mean-field effects in 1970.[1]. Recent progress has been made in sorting out experimental[2] and theoretical[3] uncertainties in anisotropic spin diffusion. A mixture of two trapped hyperfine states of a gas can be viewed as a pseudo-spin system and analogous spin experiments can be performed. Indeed experimental striations in hyperfine state densities have been interpreted in terms of spin waves.[4] We have recently began examining anisotropic spin diffusion for these trapped gas systems.[5] In Ref. [5] we found that diffusion constants were anisotropic in Bose systems, but not in the Fermi case, even in the Boltzmann limit, if the scattering lengths for different hyperfine states were unequal. However that reference considered only dipole spin modes, while experiments looked at the quadrupole modes. Here we extend our computations to the quadrupole modes and emphasize an important difference in Bose and Fermi systems, namely, that transverse spin waves and spin diffusion are not good modes in the Bose gas, because the transverse magnetization is not conserved when the scattering lengths differ among the two hyperfine states. This feature is not present in the Fermi gas, where transverse quadrupolar diffusive modes are well-behaved.

We solve a kinetic equation for the non-equilibrium part δm_{p+} of the transverse polarization, which in the trapped gas case takes the form

$$\frac{\partial \delta m_{p+}}{\partial t} + i \frac{\eta t_{12}}{\hbar} \left(m_p^{(0)} \delta M_+ - M_0 \delta m_{p+} \right) - i \Omega_0 \delta m_{p+} + \sum_i \left[\frac{p_i}{m} \frac{\partial \delta m_{p+}}{\partial r_i} - \frac{\partial U}{\partial r_i} \frac{\partial \delta m_{p+}}{\partial p_i} \right] = 2(2|\hat{L}_p|1), \quad (1)$$

where the 2×2 density \hat{n}_p has been written as $\hat{n}_p = \frac{1}{2} (f_p \hat{I} + \mathbf{m}_p \cdot \hat{\sigma})$ where $\hat{\sigma}$ is a Pauli matrix, $\frac{1}{2}(f_p \pm m_{pz})$ give the diagonal components of the density $n_{pi} = n_{pii}$, while \mathbf{m}_p represents the polarization, which in equilib-

rium is along the axis \hat{z} . The transverse components are $m_{p\pm}(\mathbf{r}) = (m_{px} \pm i m_{py})$. The total magnetization is $\mathbf{M}(\mathbf{r}) = \int d\mathbf{p}/h^3 \mathbf{m}_p(\mathbf{r})$; with M_0 evaluated at the equilibrium polarization $m_p^{(0)}$. The t 's can be evaluated in terms of the measured scattering lengths $a_{\alpha\beta}$ by using $t_{\alpha\beta} = 4\pi\hbar a_{\alpha\beta}/m$; these are taken different for the two hyperfine states 1 and 2. Experiments to date using Rb[4] have had equal t values. U is the average external potential while Ω_0 is proportional to the difference. This latter quantity leads to an important coupling between modes, but we will neglect it in this brief report. The collision integral has been evaluated explicitly[5] and \hat{L}_p is the linearized form of it. The constant η is +1 (-1) one for bosons (fermions).

In our moments approach in the Boltzmann limit,[6],[5] we consider a trial form $\delta m_{p+} = (a_0 + a_1 z^2 + a_2 z p_z + a_3 p_z^2) m_p^{(0)}$. We integrate over position and momentum arriving at a set of coupled equations for the parameters a_i , which we then solve. Our transverse results depend on two relaxation rates arising from our expression for the collision integral: The first, $1/\tau_{\perp}$, adds corrections to the longitudinal rate proportional to $(1 + \eta)$ times factors depending on $t_{\sigma\sigma} - t_{12}$. Thus it shows an anisotropy between longitudinal and transverse modes for bosons, but none for fermions even in the Boltzmann limit. The second relaxation rate γ_T , depends simply on $(1 + \eta)$ times factors depending on $t_{\sigma\sigma} - t_{12}$. This rate, which, vanishes for Fermions and for equal t values is analogous to a $1/T_2$ relaxation rate in NMR. For bosons this relaxation rate occurs in the expression for the monopole mode, which means the transverse magnetization is not conserved and also in the expressions for the quadrupole modes, which drastically affects their diffusive lifetime.

In Fig. 1 we show the imaginary part ω_I of the quadrupole diffusive mode frequency as a function of $\omega_z \tau_{\perp}$, where ω_z is the frequency in the long axis of the trap, both with and without the γ_T rate. With $\gamma_T = 0$, as for fermions, one can show that the quadrupole rate

in the hydrodynamic limit $\omega_z \tau_\perp \rightarrow 0$ is

$$\omega = 4\omega_z^2 \tau_2 (i - \omega_M \tau_2)$$

whereas for bosons one finds that $\omega_I \sim \gamma_T$ in the small $\omega_z \tau_\perp$ limit as one can see from the figure. In this there is a mean-field of frequency $\omega_M \sim \eta t_{12}$, which gives rise to the spin waves.

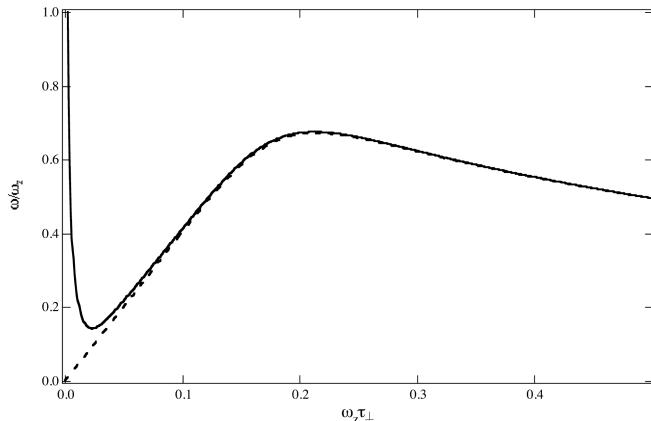


FIG. 1: Quadrupole diffusive mode. In the dashed curve the non-conserving relaxation is neglected. The solid curve shows the rapid decay when that rate is $0.002/\tau_{perp}$

Our result suggests that in Bose systems with un-

equal scattering lengths spin waves and diffusive behavior will not be possible because of the short lifetime of this mode—with the magnetization not being conserved, while for Fermi systems the magnetization is conserved and spin waves and spin diffusion can exist.

-
- [1] A. J. Leggett, *J. Phys. C* **3**, 448 (1970); A. J. Leggett and M. J. Rice, *Phys. Rev. Lett.* **20**, 586; **21**, 506(1968).
 - [2] H. Akimoto, D. Candela, J. S. Xia, W. J. Mullin, E. D. Adams, and N. S. Sullivan, *Phys. Rev. Lett.* **90**, 105301 (2003). This paper has a rather complete list of theoretical and experimental references for homogeneous real-spin Fermi fluids.
 - [3] W. J. Mullin and R. J. Ragan, *Jour. Low Temp. Phys.*, **138**, 73 (2005)
 - [4] H. J. Lewandowski, D. M. Harber, D. L. Whitaker, and E. A. Cornell, *Phys. Rev. Lett.* **88**, 070403 (2002); J. M. McGuirk, H. L. Lewandowski, D. M. Harper, T. Nikuni, J. E. Williams, and E. A. Cornell, *Phys. Rev. Lett.* **89**, 090402 (2002); M. O. Oktel and L. S. Levitov, *Phys. Rev. Lett.* **88**, 230403 (2002); J. N. Fuchs, D. M. Gangardt, and F. Laloë, *Phys. Rev. Lett.* **88**, 230404 (2002); J. E. Williams, T. Nikuni, and C. W. Clark, *Phys. Rev. Lett.* **88**, 230405 (2002).
 - [5] W. J. Mullin and R. J. Ragan, *Jour. Low Temp. Phys.*, **138**, 711 (2005).
 - [6] T. Nikuni, J. E. Williams, and C. W. Clark, *Phys. Rev. A* **66**, 043411.