Micro-level Estimation of Optimal Consumption Choice with Intertemporal Nonseparability in Preferences and Measurement Errors
- Supplemental Appendix -

Wayne-Roy Gayle
and
Natalia Khorunzhina

January 31, 2016

Abstract

This appendix includes supplemental material to the paper. Section A contains the derivation of the main equation of interest. Section B contains the proof of identification of the structural parameters of the model and the nuisance parameters introduced by measurement errors. In section C, we investigate the finite sample performance of the estimator in recovering the parameters of interest, and analyze the performance of the estimator for the bounds for the conditional expectations of the intertemporal elasticity of substitution (IES) and relative risk aversion (RRA) by Monte Carlo methods. Section D presents derivations of the bounds for the conditional expectations for the IES and RRA defined as functions of observed consumption.

APPENDIX A: PROOF OF THEOREM 3.2

Proof. To obtain an expression in terms of observed consumption, we consider equation (3.2) piece by piece and express observed consumption in terms of true consumption and measurement errors, as stated above. We start with the first term:

$$
\begin{aligned}
& E\left[\beta(1+r_{it+1})\frac{\varphi_{it+1}}{c^o_{it+1}}\left(\frac{c^o_{it+1}}{c^{o\alpha}_{it}}\right)^{1-\gamma}|z^o_{it}\right]= \\
& E\left[\beta(1+r_{it+1})\frac{\varphi_{it+1}}{c_{it+1}}\left(\frac{c_{it+1}}{c^{\alpha}_{it}}\right)^{1-\gamma}\frac{1}{v_{it+1}}\left(\frac{v_{it+1}}{v^{\alpha}_{it}}\right)^{1-\gamma}|z^o_{it}\right]= \\
& E\left[\beta(1+r_{it+1})\frac{\varphi_{it+1}}{c_{it+1}}\left(\frac{c_{it+1}}{c^{\alpha}_{it}}\right)^{1-\gamma}|z^o_{it}\right]E\left[\frac{1}{v_{it+1}}\left(\frac{v_{it+1}}{v^{\alpha}_{it}}\right)^{1-\gamma}|z^o_{it}\right]= \\
& E\left[\beta(1+r_{it+1})\frac{\varphi_{it+1}}{c_{it+1}}\left(\frac{c_{it+1}}{c^{\alpha}_{it}}\right)^{1-\gamma}|z^o_{it}\right]E\left[\frac{1}{v_{it+1}}\left(\frac{v_{it+1}}{v^{\alpha}_{it}}\right)^{1-\gamma}\right]= \\
& E\left[\beta(1+r_{it+1})\frac{\varphi_{it+1}}{c_{it+1}}\left(\frac{c_{it+1}}{c^{\alpha}_{it}}\right)^{1-\gamma}|z^o_{it}\right]\mathcal{A}_1,
\end{aligned}
$$

where the third and fourth equalities are obtained from Assumption 3.1, and

$$
\mathcal{A}_1 = E\left[\frac{1}{v_{it+1}}\left(\frac{v_{it+1}}{v^{\alpha}_{it}}\right)^{1-\gamma}\right].
$$

Hence,

$$
E\left[\beta(1+r_{it+1})\frac{\varphi_{it+1}}{c_{it+1}}\left(\frac{c_{it+1}}{c^{\alpha}_{it}}\right)^{1-\gamma}|z^o_{it}\right]=E\left[\beta\mathcal{A}_1^{-1}(1+r_{it+1})\frac{\varphi_{it+1}}{c^o_{it+1}}\left(\frac{c^o_{it+1}}{c^{o\alpha}_{it}}\right)^{1-\gamma}|z^o_{it}\right].
$$

The second and the third terms are transformed similarly to get

$$
\begin{aligned}
E\left[\beta(1+r_{it+1})\frac{\varphi_{it+1}\varphi_{it+2}}{c_{it+1}}\left(\frac{c_{it+1}c_{it+2}}{(c_{it}c_{it+1})^{\alpha}}\right)^{1-\gamma}|z^o_{it}\right] &= E\left[\alpha\beta^2\mathcal{A}_2^{-1}(1+r_{it+1})\frac{\varphi_{it+1}\varphi_{it+2}}{c^o_{it+1}}\left(\frac{c^o_{it+1}c^o_{it+2}}{(c^o_{it}c^o_{it+1})^{\alpha}}\right)^{1-\gamma}|z^o_{it}\right] \\
E\left[\alpha\beta\varphi_{it+1}\left(\frac{c_{it+1}}{c^{\alpha}_{it}}\right)^{1-\gamma}|z^o_{it}\right] &= E\left[\alpha\beta\mathcal{A}_3^{-1}\varphi_{it+1}\left(\frac{c^o_{it+1}}{c^{o\alpha}_{it}}\right)^{1-\gamma}|z^o_{it}\right],
\end{aligned}
$$

where

$$
\begin{aligned}
\mathcal{A}_2 &= E\left[\frac{1}{v_{it+1}}\left(\frac{v_{it+1}v_{it+2}}{(v_{it}v_{it+1})^{\alpha}}\right)^{1-\gamma}\right], \quad \text{and} \\
\mathcal{A}_3 &= E\left[\left(\frac{v_{it+1}}{v^{\alpha}_{it}}\right)^{1-\gamma}\right].
\end{aligned}
$$

The moment condition (3.2) for (unobserved) true consumption is therefore transformed into a moment condition for observed consumption:

$$E\left[\beta(1+r_{it+1})\frac{\varphi_{it+1}}{c^o_{it+1}}\left(\frac{c^o_{it+1}}{c^{o\alpha}_{it}}\right)^{1-\gamma}\left(\mathcal{A}_1^{-1}-\alpha\beta\mathcal{A}_2^{-1}\varphi_{it+2}\left(\frac{c^o_{it+2}}{c^{o\alpha}_{it+1}}\right)^{1-\gamma}\right)-\left(1-\alpha\beta\mathcal{A}_3^{-1}\varphi_{it+1}\left(\frac{c^o_{it+1}}{c^{o\alpha}_{it}}\right)^{1-\gamma}\right)|z^o_{it}\right] = 0.$$

□

APPENDIX B: PROOF OF THEOREM 4.2

Proof. Recall that

$$\rho(x^{o,t+2}_{it},\theta)=\beta(1+r_{it+1})\frac{\varphi_{it+1}}{c^o_{it+1}}\left(\frac{c^o_{it+1}}{c^{o,\alpha}_{it}}\right)^{1-\gamma}\left(\kappa_1-\alpha\beta\varphi_{it+2}\left(\frac{c^o_{it+2}}{c^{o,\alpha}_{it+1}}\right)^{1-\gamma}\right)-\left(\kappa_2-\alpha\beta\kappa_3\varphi_{it+1}\left(\frac{c^o_{it+1}}{c^{o,\alpha}_{it}}\right)^{1-\gamma}\right).$$

Equation (3.3) implies $E[\rho(x^{o,t+2}_{it},\theta_0)|z^o_{it}] = 0$. Suppose another set of parameters $\theta \in \Theta$ exists that satisfies $E[\rho(x^{o,t+2}_{it},\theta)|z^o_{it}] = 0$. Then $E[l(x^{o,t+2}_{it},\theta,\theta_0)|z^o_{it}] = 0$, where $l(x^{o,t+2}_{it},\theta,\theta_0) = \rho(x^{o,t+2}_{it},\theta) - \rho(x^{o,t+2}_{it},\theta_0)$. This equality and Assumption 4.1.i imply

$$l(x^{o,t+2}_{it},\theta,\theta_0) = \rho(x^{o,t+2}_{it},\theta) - \rho(x^{o,t+2}_{it},\theta_0) = 0 \tag{B.1}$$

almost surely. For the remainder of this proof, we drop the o superscript on consumption for ease of exposition.

First, suppose α_0=0. Then if $\alpha > 0$, holding $(r_{it+1},\Delta w_{it+1},\Delta w_{it+2},c_{it},c_{it+1})$ constant, $\rho(x^{o,t+2}_{it},\theta_0)$ is constant while $\rho(x^{o,t+2}_{it},\theta)$ varies monotonically with c_{it+2}, violating equation (B.1). Therefore, if $\alpha_0 = 0$, then $\alpha = 0$. Suppose $\alpha_0 > 0$. Then if $\alpha = 0$, holding $(r_{it+1},\Delta w_{it+1},\Delta w_{it+2},c_{it},c_{it+1})$ constant, $\rho(x^{o,t+2}_{it},\theta)$ is constant while $\rho(x^{o,t+2}_{it},\theta_0)$ varies monotonically with c_{it+2}, violating equation (B.1). Therefore, if $\alpha_0 > 0$, then $\alpha > 0$. If $\alpha = \alpha_0 = 0$, then equation (B.1) implies

$$\kappa_1\beta(1+r_{it+1})\varphi_{it+1}c^{-\gamma}_{it+1} - \kappa_2 = \kappa_{10}\beta_0(1+r_{it+1})\varphi_{0it+1}c^{-\gamma_0}_{it+1} - \kappa_{20}, \tag{B.2}$$

almost surely, where $\varphi_{0it+1} = \exp(\delta_0\Delta w_{it+1})$. Differentiating equation (B.2) with respect to r_{it+1}, taking logs, and gathering terms obtains

$$\ln(\kappa_1\beta/\kappa_{10}\beta_0)+\Delta w_{it+1}(\delta-\delta_0)-(\gamma-\gamma_0)\ln c_{it+1}=0 \tag{B.3}$$

almost surely. By differentiating equation (B.3) with respect to $\ln c_{it+1}$, we have $\gamma=\gamma_0$. This result and equation (B.3) imply

$$\ln(\kappa_1\beta/\kappa_{10}\beta_0)+\Delta w_{it+1}(\delta-\delta_0)=0, \tag{B.4}$$

almost surely. Under Assumption 4.1.ii, and the fact that every subset of a linearly independent set is also linearly independent, equation (B.4) implies $\kappa_1\beta=\kappa_{10}\beta_0$ and $\delta=\delta_0$. Substituting these equalities into equation (B.2) gives $\kappa_2=\kappa_{20}$. Recall that $\kappa_{20}=\mathcal{A}_{20}$ and $\kappa_{10}=\mathcal{A}_{20}/\mathcal{A}_{10}$. Then, from identification of $\mathcal{A}_{20}$ and $\kappa_{10}\beta_0$, $\mathcal{A}_{10}^{-1}\beta_0$ is identified. If Assumption 3.3 is satisfied, then σ_0 is identified from the identification of γ_0, α_0, and $\mathcal{A}_{20}$, along with the equality $\mathcal{A}_{20}=\exp\{\sigma_0^2\left(\alpha_0^2(1-\gamma_0)^2+\gamma_0^2+(1-\gamma_0)(1+\alpha_0)\right)\}$, which in turn implies identification of $\mathcal{A}_{10}=\exp\{\sigma_0^2\left(\alpha_0^2(1-\gamma_0)^2+\gamma_0^2+\alpha_0\gamma_0(1-\gamma_0)\right)\}$ and $\mathcal{A}_{30}=\exp\{\sigma_0^2\left((1+\alpha_0+\alpha_0^2)(1-\gamma_0)^2\right)\}$. Finally, identification of $\mathcal{A}_{10}$ and $\mathcal{A}_{10}^{-1}\beta_0$ imply identification of β_0.

Suppose $\alpha>0$ and $\alpha_0>0$. Taking the derivative of equation (B.1) with respect to c_{it+2} and c_{it} then obtains

$$\begin{aligned}&(\alpha\beta(1-\gamma))^2\psi_{it+2}c_{it}^{-\alpha(1-\gamma)-1}c_{it+1}^{-\gamma-\alpha(1-\gamma)}c_{it+2}^{-\gamma}\\&-(\alpha_0\beta_0(1-\gamma_0))^2\psi_{0it+2}c_{it}^{-\alpha_0(1-\gamma_0)-1}c_{it+1}^{-\gamma_0-\alpha_0(1-\gamma_0)}c_{it+2}^{-\gamma_0}=0\end{aligned} \tag{B.5}$$

almost surely, where $\psi_{it}=\exp((\Delta w_{it+1}+\Delta w_{it+2})\delta)$ and $\psi_{0it}=\exp((\Delta w_{it+1}+\Delta w_{it+2})\delta_0)$. Taking logs of equation (B.5) and collecting terms obtains

$$\begin{aligned}&\mu+(\Delta w_{it+1}+\Delta w_{it+2})(\delta-\delta_0)-[\alpha(1-\gamma)-\alpha_0(1-\gamma_0)]\ln c_{it}\\&-[\gamma+\alpha(1-\gamma)-\gamma_0-\alpha_0(1-\gamma_0)]\ln c_{it+1}-(\gamma-\gamma_0)\ln c_{it+2}=0\end{aligned} \tag{B.6}$$

almost surely, where $\mu=\ln([\alpha\beta(1-\gamma)/\alpha_0\beta_0(1-\gamma_0)]^2)$. Differentiating equation (B.6) with respect to $\ln c_{it+2}$ obtains $\gamma=\gamma_0$. Using this result and differentiating equation (B.6) with respect to $\ln c_{it}$ obtains $\alpha = \alpha_0$. These two results and equation (B.6) imply

$$\mu + (\Delta w_{it+1} + \Delta w_{it+2})(\delta - \delta_0) = 0 \tag{B.7}$$

almost surely. Equation (B.7) and Assumption 4.1.ii imply $\delta = \delta_0$, and $\mu = 0$. From $\gamma = \gamma_0$, $\alpha = \alpha_0$, and $\mu = 0$, we have that $\beta = \beta_0$. Substituting these results into equation (B.1) obtains

$$(\kappa_1 - \kappa_{10})\beta_0(1+r_{it+1})\frac{\varphi_{0it+1}}{c_{it+1}}\left(\frac{c_{it+1}}{c_{it}^{\alpha_0}}\right)^{1-\gamma_0} + (\kappa_3 - \kappa_{30})\alpha_0\beta_0\varphi_{0it+1}\left(\frac{c_{it+1}}{c_{it}^{\alpha_0}}\right)^{1-\gamma_0} + (\kappa_{20} - \kappa_2) = 0 \tag{B.8}$$

almost surely. Differentiating equation (B.8) with respect to $1 + r_{it+1}$ implies $\kappa_1 = \kappa_{10}$. Substituting this result into equation (B.8) and differentiating with respect to c_{it+1} obtain $\kappa_3 = \kappa_{30}$, which in turn implies $\kappa_2 = \kappa_{20}$. Identification of $(\mathcal{A}_{10}, \mathcal{A}_{20}, \mathcal{A}_{30})$, and therefore θ_0, is then obtained from $(\kappa_{10}, \kappa_{20}, \kappa_{30})$. Finally, if Assumption 3.3 is satisfied, then σ_0 is identified from the identification of θ_0 and the equality $\mathcal{A}_{20} = \exp\{\sigma_0^2\left(\alpha_0^2(1-\gamma_0)^2 + \gamma_0^2 + (1-\gamma_0)(1+\alpha_0)\right)\}$. $\square$

APPENDIX C: MONTE CARLO EXPERIMENT

In this section, we investigate the finite-sample performance of the estimator developed in section 3, as well as the performance of the approximated log-linearized habit-formation model. We do so by conducting a Monte Carlo simulation in which the life-cycle model presented in section 2 is solved under labor income and interest rate uncertainty. The details of the solution and simulation methods are standard for the intertemporal utility optimization framework and available from the authors upon request. The structural parameter values are set as follows: $\gamma = 5$, $\alpha = 0.85$, $\beta = 0.95$. The interest-rate series is a stationary AR(1) process with a mean of 0.05 and autoregressive coefficient of 0.6. We solve the model for 40 periods; however, in estimation, we only use the 13 middle periods in order to match the length of the artificial panel with the one used in the empirical analysis. Additionally, due to this trimming, starting and ending effects of the artificial consumption series are not an issue. Consumption paths are simulated to obtain 100 samples of 1,700 households observed over 13 periods. Next, the simulated consumption data are contaminated by measurement errors drawn independently over households and time from a log-normal distribution with variance equal to 75% of the variance in consumption.

Table 1: Estimation of the Euler equation with habit formation, using simulated data

Parameters	Truth (1)	No ME (2)	Log-normal ME (3)	Nonparametric ME (4)	Ignoring ME (5)
γ	5.00	4.93	5.73	5.16	13.22
		[4.89]	[4.98]	[5.00]	[13.32]
		(0.33)	(2.65)	(1.03)	(2.03)
β	0.95	0.95	0.94	0.95	0.74
		[0.95]	[0.95]	[0.95]	[0.74]
		(0.01)	(0.07)	(0.01)	(0.05)
α	0.85	0.85	0.85	0.85	0.55
		[0.85]	[0.85]	[0.85]	[0.55]
		(0.01)	(0.03)	(0.01)	(0.05)
σ^2	0.04		0.04		
			[0.03]		
			(0.04)		

NOTE: In estimation, we reduce the time dimension of the artificial data panel to 13 years. The instrument set includes current and past interest rates and current income. Standard errors are in parentheses. Medians are in square brackets.

Table 1 presents results from the Monte Carlo investigation of the estimator developed in section 3. Column (1) gives the true values of the preference parameters that we aim to recover using the proposed estimator. Column (2) shows the estimator performs well in the absence of consumption measurement errors. The results also show the estimator performs well when the distribution of measurement errors is known to be log-normal (column 3) and when the distribution of measurement errors is unknown (column 4). Column (5) shows that not accounting for measurement errors results in upward bias in γ and downward bias in α and β.

With the simulated data in hand, we investigate the performance of estimation of the linear approximation models developed in Hayashi (1985), Muellbauer (1988), and Dynan (2000). The derivation of the estimator for the additive habit-formation model assumes (i) interest rates do not vary across individuals or over time, (ii) individuals live for an infinite period, and (iii)

Table 2: Estimation of equation (C.1) using the simulated data

Parameters	No ME		Nonparametric ME	
	(1)	(2)	(3)	(4)
β_0 (Constant)	0.042	-0.005	0.175	0.094
	[0.041]	[-0.006]	[0.175]	[0.094]
	(0.040)	(0.039)	(0.047)	(0.048)
α ($\Delta \ln C^o_{it-1}$)	0.170	0.197	0.106	0.163
	[0.171]	[0.197]	[0.105]	[0.161]
	(0.015)	(0.015)	(0.020)	(0.021)
β_1 (Age)	0.0001	-0.0004	-0.013	-0.012
	[0.0001]	[-0.0003]	[-0.013]	[-0.011]
	(0.004)	(0.004)	(0.004)	(0.005)
β_2 (Age^2/1000)	-0.091	-0.077	0.204	0.194
	[-0.097]	[0.082]	[0.192]	[0.180]
	(0.101)	(0.099)	(0.121)	(0.120)
β_3 ($\ln(1+r_t)$)	—	1.121	—	1.540
		[1.112]		[1.525]
		(0.149)		(0.243)

NOTE: The instrument set includes the first two lags of income growth and lagged interest rate. In columns (2) and (4), $\ln(1+r_t)$ is treated as endogenous. Standard errors are in parentheses.

$\Delta \ln(C_t - \alpha C_{t-1}) \approx \Delta \ln C_t - \alpha \Delta \ln C_{t-1}$. We derive a comparable estimator for which the first two assumptions are maintained. As Muellbauer (1988) shows, the third assumption requires that consumption does not vary significantly over time. In the multiplicative habit model, this assumption and the first two assumptions imply instrumental variable estimation of

$$\Delta \ln C^o_{it} = \beta_0 + \alpha \Delta \ln C^o_{it-1} + \beta_1 age_t + \beta_2 age^2 + \varepsilon_t \tag{C.1}$$

should yield $\alpha = 0.85$. The instruments for $\Delta \ln C^o_{it-1}$ are the first two lags of income growth and the lagged interest rate.

Table 2 reports two sets of results from the Monte Carlo investigation of this estimation method. The first two columns are the results for the case of the absence of consumption measurement errors, and the last two are for the case when consumption is measured with errors. The results show significant downward bias in the estimate of α even without consumption measurement errors. These results suggest the assumptions made to obtain equation (C.1) are substantial. The bias is more severe when consumption is measured with errors.

Lastly, we perform a Monte Carlo exercise to investigate the accuracy of the estimated confidence interval for the mean IES, presented in equation (8.5), and the analogously estimated confidence interval for the mean RRA. Specifically, we use the simulated consumption data without measurement errors to compute the simulated mean IES and RRA. The simulation exercise obtains the mean IES of 0.072 and the mean RRA of 13.05. Next, we use the simulated data samples of consumption contaminated with measurement errors to compute their 95% confidence intervals by implementing the bootstrap approach presented in section 8. We find the simulated mean IES falls within the estimated confidence intervals in 96% of the replications, whereas the simulated mean RRA falls within the estimated confidence intervals in 92% of the replications.

APPENDIX D: PROOF OF THEOREM 8.1

Proof. The individual-specific intertemporal elasticity of substitution can be found from:

$$\frac{1}{IES_{it}} = \left| \frac{\partial \ln \frac{MU_{it}}{MU_{it+1}}}{\partial \ln \frac{C_{it+1}}{C_{it}}} \right|, \tag{D.1}$$

where

$$\begin{aligned} \frac{MU_{it}}{MU_{it+1}} &= \frac{\frac{\phi_{it}}{C_{it}}\left(\frac{C_{it}}{C_{it-1}^{\alpha}}\right)^{1-\gamma} - \alpha\beta\frac{\phi_{it+1}}{C_{it}}\left(\frac{C_{it+1}}{C_{it}^{\alpha}}\right)^{1-\gamma}}{\frac{\phi_{it+1}}{C_{it+1}}\left(\frac{C_{it+1}}{C_{it}^{\alpha}}\right)^{1-\gamma} - \alpha\beta\frac{\phi_{it+2}}{C_{it+1}}\left(\frac{C_{it+2}}{C_{it+1}^{\alpha}}\right)^{1-\gamma}} \\ &= \frac{\left(1-\alpha\beta\varphi_{it+1}\left(\frac{c_{it+1}}{c_{it}^{\alpha}}\right)^{1-\gamma}\right)}{\frac{\varphi_{it+1}}{c_{it+1}}\left(\frac{c_{it+1}}{c_{it}^{\alpha}}\right)^{1-\gamma}\left(1-\alpha\beta\varphi_{it+2}\left(\frac{c_{it+2}}{c_{it+1}^{\alpha}}\right)^{1-\gamma}\right)}. \end{aligned} \tag{D.2}$$

Taking logs of (D.2) and partial derivatives with respect to $\ln c_{it+1} = \ln(C_{it+1}/C_{it})$, we obtain

$$\frac{1}{IES_{it}(\theta)} = \gamma - \frac{\alpha\beta(1-\gamma)\varphi_{it+1}\left(\frac{c_{it+1}}{c_{it}^{\alpha}}\right)^{1-\gamma}}{1-\alpha\beta\varphi_{it+1}\left(\frac{c_{it+1}}{c_{it}^{\alpha}}\right)^{1-\gamma}} - \frac{\alpha^2\beta(1-\gamma)\varphi_{it+2}\left(\frac{c_{it+2}}{c_{it+1}^{\alpha}}\right)^{1-\gamma}}{1-\alpha\beta\varphi_{it+2}\left(\frac{c_{it+2}}{c_{it+1}^{\alpha}}\right)^{1-\gamma}}. \tag{D.3}$$

Recall that under Assumption 3.1,

$$E\left[\alpha\beta\varphi_{it+1}\left(\frac{c_{it+1}}{c_{it}^{\alpha}}\right)^{1-\gamma}|z_{it}^{o}\right] = E\left[\alpha\beta\mathcal{A}_3^{-1}\varphi_{it+1}\left(\frac{c_{it+1}^{o}}{c_{it}^{o\alpha}}\right)^{1-\gamma}|z_{it}^{o}\right]. \tag{D.4}$$

Note also that

$$\begin{aligned} MU_{it}(\theta_0) &= \frac{\phi_{0it}}{C_{it}}\left(\frac{C_{it}}{C_{it-1}^{\alpha_0}}\right)^{1-\gamma_0} - \alpha_0\beta_0\frac{\phi_{0it+1}}{C_{it}}\left(\frac{C_{it+1}}{C_{it}^{\alpha_0}}\right)^{1-\gamma_0} \\ &= \frac{\phi_{0it}}{C_{it}}\left(\frac{C_{it}}{C_{it-1}^{\alpha_0}}\right)^{1-\gamma_0}\left(1-\alpha_0\beta_0\varphi_{0it+1}\left(\frac{c_{it+1}}{c_{it}^{\alpha_0}}\right)^{1-\gamma_0}\right). \end{aligned}$$

Because $\alpha_0\beta_0\varphi_{0it+1}(c_{it+1}/c_{it}^{\alpha_0})^{1-\gamma_0} < 1$, the inverse IES is convex in $\alpha_0\beta_0\varphi_{0it+1}(c_{it+1}/c_{it}^{\alpha_0})^{1-\gamma_0}$ and $\alpha_0\beta_0\varphi_{0it+2}(c_{it+2}/c_{it+1}^{\alpha_0})^{1-\gamma_0}$. Therefore, Jensen's inequality, along with equations (D.3) and (D.4), obtain

$$\begin{aligned} E\left[\frac{1}{IES_{it}(\theta_0)}\middle|z_{it}^{o}\right] &\geq \gamma_0-(1-\gamma_0)\frac{\alpha_0\beta_0 E\left[\varphi_{0it+1}\left(\frac{c_{it+1}}{c_{it}^{\alpha_0}}\right)^{1-\gamma_0}\middle|z_{it}^{o}\right]}{1-\alpha_0\beta_0 E\left[\varphi_{0it+1}\left(\frac{c_{it+1}}{c_{it}^{\alpha_0}}\right)^{1-\gamma_0}\middle|z_{it}^{o}\right]} - \alpha_0(1-\gamma_0)\frac{\alpha_0\beta_0 E\left[\varphi_{0it+2}\left(\frac{c_{it+2}}{c_{it+1}^{\alpha_0}}\right)^{1-\gamma_0}\middle|z_{it}^{o}\right]}{1-\alpha_0\beta_0 E\left[\varphi_{0it+2}\left(\frac{c_{it+2}}{c_{it+1}^{\alpha_0}}\right)^{1-\gamma_0}\middle|z_{it}^{o}\right]} \\ &= \gamma_0-(1-\gamma_0)\frac{\mathcal{A}_{30}^{-1}\alpha_0\beta_0 E\left[\varphi_{0it+1}\left(\frac{c_{it+1}^{o}}{c_{it}^{o\alpha_0}}\right)^{1-\gamma_0}\middle|z_{it}^{o}\right]}{1-\mathcal{A}_{30}^{-1}\alpha_0\beta_0 E\left[\varphi_{0it+1}\left(\frac{c_{it+1}^{o}}{c_{it}^{o\alpha_0}}\right)^{1-\gamma_0}\middle|z_{it}^{o}\right]} - \alpha_0(1-\gamma_0)\frac{\mathcal{A}_{30}^{-1}\alpha_0\beta_0 E\left[\varphi_{0it+2}\left(\frac{c_{it+2}^{o}}{c_{it+1}^{o\alpha_0}}\right)^{1-\gamma_0}\middle|z_{it}^{o}\right]}{1-\mathcal{A}_{30}^{-1}\alpha_0\beta_0 E\left[\varphi_{0it+2}\left(\frac{c_{it+2}^{o}}{c_{it+1}^{o\alpha_0}}\right)^{1-\gamma_0}\middle|z_{it}^{o}\right]}. \end{aligned}$$

Also, because the IES is concave in $\alpha_0\beta_0\varphi_{0it+1}(c_{it+1}/c_{it}^{\alpha_0})^{1-\gamma_0}$ and $\alpha_0\beta_0\varphi_{0it+2}(c_{it+2}/c_{it+1}^{\alpha_0})^{1-\gamma_0}$, Jensen's inequality, along with equations (D.3) and (D.4), obtains

$$E\left[IES_{it}(\theta_0)|z_{it}^o\right] \leq \left(\gamma_0 - (1-\gamma_0)\frac{\mathcal{A}_{30}^{-1}\alpha_0\beta_0 E\left[\varphi_{0it+1}\left(\frac{c_{it+1}^o}{c_{it}^{o\alpha_0}}\right)^{1-\gamma_0}\middle| z_{it}^o\right]}{1-\mathcal{A}_{30}^{-1}\alpha_0\beta_0 E\left[\varphi_{0it+1}\left(\frac{c_{it+1}^o}{c_{it}^{o\alpha_0}}\right)^{1-\gamma_0}\middle| z_{it}^o\right]} - \alpha_0(1-\gamma_0)\frac{\mathcal{A}_{30}^{-1}\alpha_0\beta_0 E\left[\varphi_{0it+2}\left(\frac{c_{it+2}^o}{c_{it+1}^{o\alpha_0}}\right)^{1-\gamma_0}\middle| z_{it}^o\right]}{1-\mathcal{A}_{30}^{-1}\alpha_0\beta_0 E\left[\varphi_{0it+2}\left(\frac{c_{it+2}^o}{c_{it+1}^{o\alpha_0}}\right)^{1-\gamma_0}\middle| z_{it}^o\right]} \right)^{-1}.$$

To derive an upper bound for the inverse IES, note that equation (D.3) has the following representation:

$$\begin{aligned}
\frac{1}{IES_{it}(\theta_0)} = \;& \gamma_0 - \alpha_0\beta_0(1-\gamma_0)\varphi_{0it+1}\left(\frac{c_{it+1}}{c_{it}^{\alpha_0}}\right)^{1-\gamma_0}\sum_{j=0}^{\infty}\left(\alpha_0\beta_0\varphi_{0it+1}\left(\frac{c_{it+1}}{c_{it}^{\alpha_0}}\right)^{1-\gamma_0}\right)^j \\
& -\alpha_0^2\beta_0(1-\gamma_0)\varphi_{0it+2}\left(\frac{c_{it+2}}{c_{it+1}^{\alpha_0}}\right)^{1-\gamma_0}\sum_{j=0}^{\infty}\left(\alpha_0\beta_0\varphi_{0it+2}\left(\frac{c_{it+2}}{c_{it+1}^{\alpha_0}}\right)^{1-\gamma_0}\right)^j \\
= \;& \gamma_0 - (1-\gamma_0)\sum_{j=1}^{\infty}\left(\alpha_0\beta_0\varphi_{0it+1}\left(\frac{c_{it+1}}{c_{it}^{\alpha_0}}\right)^{1-\gamma_0}\right)^j - \alpha_0(1-\gamma_0)\sum_{j=1}^{\infty}\left(\alpha_0\beta_0\varphi_{0it+2}\left(\frac{c_{it+2}}{c_{it+1}^{\alpha_0}}\right)^{1-\gamma_0}\right)^j,
\end{aligned}$$

which is a valid representation because the assumption of positive marginal utility implies each term in the infinite sum is between 0 and 1. For the same reason, the dominated convergence theorem applies and we find

$$\begin{aligned}
E\left[\frac{1}{IES_{it}(\theta_0)}|z_{it}^o\right] = \;& \gamma_0 - (1-\gamma_0)\sum_{j=1}^{\infty} E\left[\left(\alpha_0\beta_0\varphi_{0it+1}\left(\frac{c_{it+1}}{c_{it}^{\alpha_0}}\right)^{1-\gamma_0}\right)^j |z_{it}^o\right] \\
& -\alpha_0(1-\gamma_0)\sum_{j=1}^{\infty} E\left[\left(\alpha_0\beta_0\varphi_{0it+2}\left(\frac{c_{it+2}}{c_{it+1}^{\alpha_0}}\right)^{1-\gamma_0}\right)^j |z_{it}^o\right].
\end{aligned} \tag{D.5}$$

Next, for each j, we have

$$\begin{aligned}
E\left[\left(\alpha_0\beta_0\varphi_{0it+1}\left(\frac{c_{it+1}^o}{c_{it}^{o\alpha_0}}\right)^{1-\gamma_0}\right)^j |z_{it}^o\right] &= E\left[\left(\alpha_0\beta_0\varphi_{0it+1}\left(\frac{c_{it+1}}{c_{it}^{\alpha_0}}\right)^{1-\gamma_0}\right)^j \left(\left(\frac{v_{it+1}}{v_{it}^{\alpha_0}}\right)^{1-\gamma_0}\right)^j |z_{it}^o\right] \\
&= E\left[\left(\alpha_0\beta_0\varphi_{0it+1}\left(\frac{c_{it+1}}{c_{it}^{\alpha_0}}\right)^{1-\gamma_0}\right)^j |z_{it}^o\right] E\left[\left(\left(\frac{v_{it+1}}{v_{it}^{\alpha_0}}\right)^{1-\gamma_0}\right)^j |z_{it}^o\right].
\end{aligned}$$

Because $j \geq 1$, Jensen's inequality implies

$$E\left[\left(\left(\frac{v_{it+1}}{v_{it}^{\alpha_0}}\right)^{1-\gamma_0}\right)^j |z_{it}^o\right] \geq \left(E\left[\left(\frac{v_{it+1}}{v_{it}^{\alpha_0}}\right)^{1-\gamma_0} |z_{it}^o\right]\right)^j = \mathcal{A}_{30}^j. \tag{D.6}$$

Therefore, equations (D.5) and (D.6) obtain

$$E\left[\left(\mathcal{A}_{30}^{-1}\alpha_0\beta_0\varphi_{0it+1}\left(\frac{c_{it+1}^o}{c_{it}^{o\alpha_0}}\right)^{1-\gamma_0}\right)^j |z_{it}^o\right] \quad \geq \quad E\left[\left(\alpha_0\beta_0\varphi_{0it+1}\left(\frac{c_{it+1}}{c_{it}^{\alpha_0}}\right)^{1-\gamma_0}\right)^j |z_{it}^o\right]. \tag{D.7}$$

Substituting equation (D.7) into equation (D.5) obtains

$$\begin{aligned} E\left[\frac{1}{IES_{it}(\theta_0)}|z_{it}^o\right] \leq \quad & \gamma_0 - (1-\gamma_0)\textstyle\sum_{j=1}^{\infty} E\left[\left(\mathcal{A}_{30}^{-1}\alpha_0\beta_0\varphi_{0it+1}\left(\frac{c_{it+1}^o}{c_{it}^{o\alpha_0}}\right)^{1-\gamma_0}\right)^j |z_{it}^o\right] \\ & -\alpha_0(1-\gamma_0)\textstyle\sum_{j=1}^{\infty} E\left[\left(\mathcal{A}_{30}^{-1}\alpha_0\beta_0\varphi_{0it+2}\left(\frac{c_{it+2}^o}{c_{it+1}^{o\alpha_0}}\right)^{1-\gamma_0}\right)^j |z_{it}^o\right], \end{aligned} \tag{D.8}$$

which is finite by assumption of the theorem. Therefore, under the conditions of the theorem and by the dominated convergence theorem, the inequality in equation (D.8) is also given by

$$E\left[\frac{1}{IES_{it}(\theta_0)}|z_{it}^o\right] \leq \gamma_0 - (1-\gamma_0)E\left[\frac{\mathcal{A}_{30}^{-1}\alpha_0\beta_0\varphi_{0it+1}\left(\frac{c_{it+1}^o}{c_{it}^{o\alpha_0}}\right)^{1-\gamma_0}}{1-\mathcal{A}_{30}^{-1}\alpha_0\beta_0\varphi_{0it+1}\left(\frac{c_{it+1}^o}{c_{it}^{o\alpha_0}}\right)^{1-\gamma_0}}\middle| z_{it}^o\right] - \alpha_0(1-\gamma_0)E\left[\frac{\mathcal{A}_{30}^{-1}\alpha_0\beta_0\varphi_{0it+2}\left(\frac{c_{it+2}^o}{c_{it+1}^{o\alpha_0}}\right)^{1-\gamma_0}}{1-\mathcal{A}_{30}^{-1}\alpha_0\beta_0\varphi_{0it+2}\left(\frac{c_{it+2}^o}{c_{it+1}^{o\alpha_0}}\right)^{1-\gamma_0}}\middle| z_{it}^o\right].$$

Again, by Jensen's inequality, we have $(E[1/IES_{it}(\theta_0)|z_{it}^o])^{-1} \leq E[IES_{it}(\theta_0)|z_{it}^o]$ so that

$$E\left[IES_{it}(\theta_0)|z_{it}^o\right] \geq \left(\gamma_0 - (1-\gamma_0)E\left[\frac{\mathcal{A}_{30}^{-1}\alpha_0\beta_0\varphi_{0it+1}\left(\frac{c_{it+1}^o}{c_{it}^{o\alpha_0}}\right)^{1-\gamma_0}}{1-\mathcal{A}_{30}^{-1}\alpha_0\beta_0\varphi_{0it+1}\left(\frac{c_{it+1}^o}{c_{it}^{o\alpha_0}}\right)^{1-\gamma_0}}\middle| z_{it}^o\right] - \alpha_0(1-\gamma_0)E\left[\frac{\mathcal{A}_{30}^{-1}\alpha_0\beta_0\varphi_{0it+2}\left(\frac{c_{it+2}^o}{c_{it+1}^{o\alpha_0}}\right)^{1-\gamma_0}}{1-\mathcal{A}_{30}^{-1}\alpha_0\beta_0\varphi_{0it+2}\left(\frac{c_{it+2}^o}{c_{it+1}^{o\alpha_0}}\right)^{1-\gamma_0}}\middle| z_{it}^o\right]\right)^{-1}.$$

Turning to the RRA, the individual-specific relative risk aversion is defined as

$$RRA_{it} = -C_{it}\frac{\partial MU_{it}/\partial C_{it}}{MU_{it}}. \tag{D.9}$$

Consequently, the risk-aversion parameters implied by our model are given by

$$RRA_{it}(\theta_0) = \frac{\gamma - (1+\alpha(1-\gamma))\alpha\beta\varphi_{it+1}\left(\frac{c_{it+1}}{c_{it}^{\alpha}}\right)^{1-\gamma}}{1-\alpha\beta\varphi_{it+1}\left(\frac{c_{it+1}}{c_{it}^{\alpha}}\right)^{1-\gamma}}. \tag{D.10}$$

Because RRA_{it} is convex in $\alpha_0\beta_0\varphi_{0it+1}(c_{it+1}/c_{it}^{\alpha_0})^{1-\gamma_0}$,

$$\begin{aligned} E[RRA_{it}(\theta_0)|z_{it}^o] &\geq \frac{\gamma_0 - (1+\alpha_0(1-\gamma_0))\alpha_0\beta_0 E\left[\varphi_{0it+1}\left(\frac{c_{it+1}}{c_{it}^{\alpha_0}}\right)^{1-\gamma_0}\middle| z_{it}^o\right]}{1-\alpha_0\beta_0 E\left[\varphi_{0it+1}\left(\frac{c_{it+1}}{c_{it}^{\alpha_0}}\right)^{1-\gamma_0}\middle| z_{it}^o\right]} \\ &= \frac{\gamma_0 - (1+\alpha_0(1-\gamma_0))\mathcal{A}_{30}^{-1}\alpha_0\beta_0 E\left[\varphi_{0it+1}\left(\frac{c_{it+1}^o}{c_{it}^{o\alpha_0}}\right)^{1-\gamma_0}\middle| z_{it}^o\right]}{1-\alpha_0\beta_0\mathcal{A}_{30}^{-1} E\left[\varphi_{0it+1}\left(\frac{c_{it+1}^o}{c_{it}^{o\alpha_0}}\right)^{1-\gamma_0}\middle| z_{it}^o\right]}. \end{aligned} \tag{D.11}$$

On the other hand, using the same method and conditions as used to compute the inverse IES,

$$\begin{aligned} E[RRA_{it}(\theta_0)|z_{it}^o] &= \gamma_0 \sum_{j=0}^{\infty} E\left[\left(\alpha_0\beta_0\varphi_{0it+1}\left(\frac{c_{it+1}}{c_{it}^{\alpha_0}}\right)^{1-\gamma_0}\right)^j |z_{it}^o\right] \\ &- (1+\alpha_0(1-\gamma_0))\sum_{j=1}^{\infty} E\left[\left(\alpha_0\beta_0\varphi_{0it+1}\left(\frac{c_{it+1}}{c_{it}^{\alpha_0}}\right)^{1-\gamma_0}\right)^j |z_{it}^o\right] \\ &\leq \gamma_0 \sum_{j=0}^{\infty} E\left[\left(\mathcal{A}_{30}^{-1}\alpha_0\beta_0\varphi_{it+1}\left(\frac{c_{it+1}^o}{c_{it}^{o\alpha_0}}\right)^{1-\gamma_0}\right)^j |z_{it}^o\right] \\ &- (1+\alpha_0(1-\gamma_0))\sum_{j=1}^{\infty} E\left[\left(\mathcal{A}_{30}^{-1}\alpha_0\beta_0\varphi_{0it+1}\left(\frac{c_{it+1}^o}{c_{it}^{o\alpha_0}}\right)^{1-\gamma_0}\right)^j |z_{it}^o\right] \\ &= E\left[\frac{\gamma_0 - (1+\alpha_0(1-\gamma_0))\mathcal{A}_{30}^{-1}\alpha_0\beta_0\varphi_{0it+1}\left(\frac{c_{it+1}^o}{c_{it}^{o\alpha_0}}\right)^{1-\gamma_0}}{1-\mathcal{A}_{30}^{-1}\alpha_0\beta_0\varphi_{0it+1}\left(\frac{c_{it+1}^o}{c_{it}^{o\alpha_0}}\right)^{1-\gamma_0}}\middle| z_{it}^o\right]. \end{aligned} \tag{D.12}$$

□

References

Dynan, K.E. (2000), "Habit Formation in Consumer Preferences: Evidence from Panel Data," *American Economic Review*, 90(3), 391-406.

Hayashi, F. (1985), "The Permanent Income Hypothesis and Consumption Durability: Analysis Based on Japanese Panel Data," *Quarterly Journal of Economics*, 100(4), 1083-1113.

Muellbauer, J. (1988), "Habits, Rationality and Myopia in the Life Cycle Consumption Function," *Annales d' Economie et de Statistique*, 9, 47-70.