

Standard 4D gravity on a brane in 6D flux compactification

Lorenzo Sorba

UMass



Amherst

Brane models and large extra dimensions

ADD, RS

Motivated by particle physics (hierarchy problem)...

...very interesting for gravity

(e.g. brane cosmology)

Gravity of codimension-1 brane models studied in great detail

But brane models can have more than just one extra dimension!

ADD with 2 extra dims, an especially interesting case...

...since codimension-1 is so well studied, the next step is codimension-2!

let us start considering just a codimension-2 0-brane: a point in 2+1 spacetime:

Gravity in 2+1 dimensions is nondynamical

$$R_{\alpha\beta}^{\mu\nu} = \epsilon^{\mu\nu\sigma} \epsilon_{\alpha\beta\lambda} G_{\sigma}^{\lambda}$$



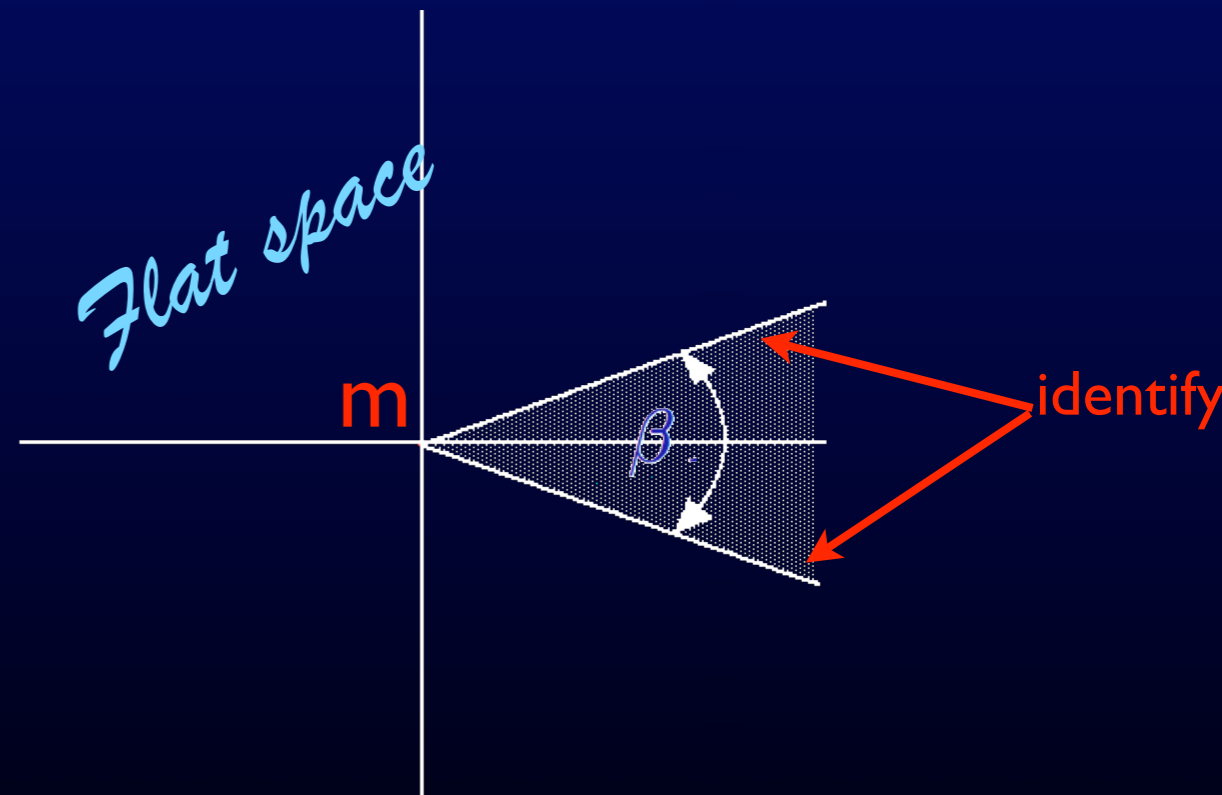
away from sources, Riemann=0

For a pointlike mass **m**

$$ds^2 = -dt^2 + d\rho^2 + \rho^2 d\theta^2$$

$$0 \leq \theta \leq 2\pi(1 - 4Gm)$$

deficit angle β



In analogous way, a 3-brane in a 6d bulk
does not curve the bulk, but generates just a deficit angle

The curvature singularity at the tip of the cone
matches the singular stress energy tensor
associated to the brane tension

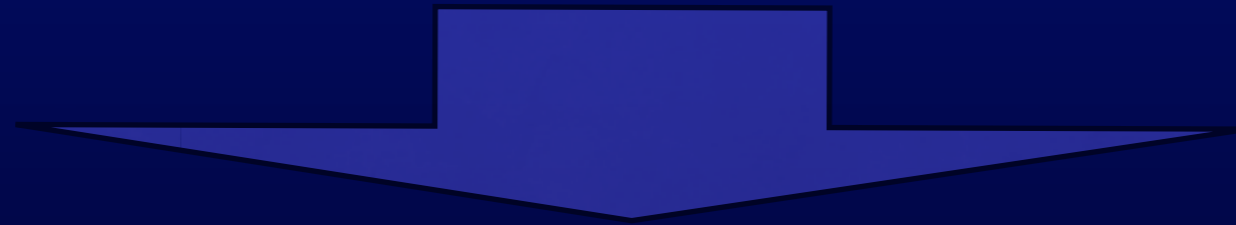


It is possible to find solutions with a flat brane
for whatever value of the brane tension!

Could this be used to help with the cosmological constant problem?

Yes, in the spirit of *self tuning*:
the theory may contain some fine-tunings,
but they do not involve the brane tension.

If matter lives only on the brane,
matter loops will contribute only to the brane tension



Can always find flat solutions
for any matter content of the brane

e.g. with broken susy on the brane

...but we need a consistent theory with 4d gravity on the brane!

(a flat infinite bulk cannot localize gravity, of course...)

A realistic setting: compactification on a 2-sphere

Freund and Rubin, 80

Matter content:

- Bulk cosmological constant Λ
- Bulk U(1) gauge field F_{MN}

$$S = \int d^6x \sqrt{-g_6} \left[M^4 R - \Lambda - \frac{1}{4} F_{AB} F^{AB} \right]$$

Gives Minkowski $\times S^2$ if

$$F_{ij} = B \sqrt{g_2} \epsilon_{ij} \quad , \quad \{i, j\} = \{\theta, \phi\}$$

with the constraint

$$\Lambda = \frac{B^2}{2}$$

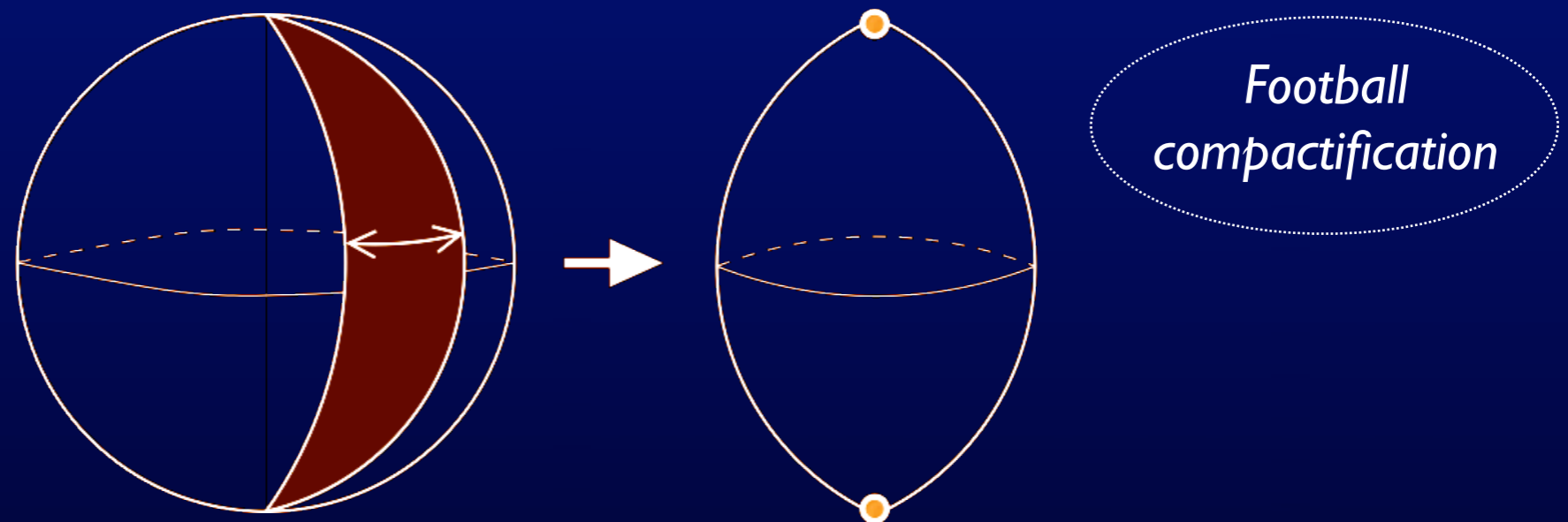
equivalent to fine-tuning the cosmological constant to zero

- radius of the sphere $R = \frac{M^2}{B}$

...then add two branes at the poles:

Carroll and Guica, 03

(Z_2 symmetry across the equator + azimuthal symmetry)



a deficit angle β is generated at the poles

$$1 - \beta = \frac{T}{2\pi M^4}$$

where T is the brane tension

changing the brane tension has the only effect
of changing the bulk deficit angle



can get flat 4D solutions
for any value of the brane tension

is the Cosmological Constant problem
alleviated this way?

It should not... : at distances larger than R the four dimensional effective theory is just gravity + a (finite) bunch of matter fields.

Weinberg's no-go theorem applies. So something must go wrong.

...and indeed, there is a first problem...

Aghababaie et al 03, Navarro 03

if there are fields charged under the bulk $U(1)$ group,
then the flux is quantized



the deficit angle is quantized

$$\beta = \frac{N}{2eM^2R}$$

$N=0,1,2,\dots$



the tension T can only take a discrete set of values

...and even if there is no matter that is charged under the $U(1)$,
the flux must be *conserved*:

Garriga and Porrati, 04

the product of magnetic field times internal volume cannot change

- suppose a phase transition on the brane changes the tension T
- the deficit angle should change
- the internal volume should change
- if the magnetic field does not change, then the magnetic flux must change

but this is **forbidden** by flux conservation!

so the only option is that the deficit angle stays constant and the brane
“curves” and starts expanding

...unless the phase transition is accompanied by emission of objects
(branes) charged under the magnetic field,
that change the magnetic field

but this is just Brown-Teitelboim with two extra dimensions!

...or a simplified version of Bousso-Polchinski

...so the football is probably not good
for self-tuning...

...still it is an interesting example of:

- 6D ADD scenario where the stabilization mechanism is consistently taken into account
- Brown-Teitelboim/Bousso-Polchinski brane model

One more reason to study codimension-2 braneworlds:

gravity of codimension-2 objects is special

(independently of phenomenological applications)

Geroch and Traschen, 88

The metric associated to a codimension-2 singular object does not
give well defined Einstein equations

(squares of Kronecker delta functions can appear)



a delta-like, codimension-2 brane can give different external metrics,
depending on the way the delta-like limit is taken.

(different from codimension-1 case)

...even worse than that:

There are constraints on the forms of matter that can be localized on a delta-like codimension-2 brane!

Cline et al., Bostock et al. 03

Let us study the Einstein equations locally, close to the brane:

$$ds^2 = g_{\mu\nu}(x, r) dx^\mu dx^\nu - L^2(x, r) d\theta^2 - dr^2$$

\nearrow \nearrow

$$\hat{g}_{\mu\nu}(x) + \mathcal{O}(r^2) \qquad \beta r + \mathcal{O}(r^2)$$

$$T_{MN} = \begin{pmatrix} \hat{T}_{\mu\nu}(x) & \frac{\delta(r)}{2\pi L} \\ 0 & 0 \end{pmatrix}$$

...then we match the singular parts of Einstein equations:

use:

$$\frac{L''}{L} = - (1 - \beta) \frac{\delta(r)}{L} + \text{regular}$$

to obtain

$$2\pi (1 - \beta) M^4 \hat{g}_{\mu\nu} = \hat{T}_{\mu\nu}$$



Only tension can be localized on the
delta-like defect!

One possible way out

Bostock et al. 03

Add higher curvature terms (Gauss-Bonnet) in the bulk

[consistent with String Theory]

$$\delta S = M^4 \int d^6 x \sqrt{-g} \alpha \left(R^2 - 4 R_{AB} R^{AB} + R_{ABCD} R^{ABCD} \right)$$

In 6D, only higher curvature term in the lagrangian that still leads to 2nd order differential equations
Lovelock 71

New equation for singular parts:

$$2\pi (1 - \beta) M^4 \left[\hat{g}_{\mu\nu} + 4\alpha \hat{G}_{\mu\nu} + \alpha \frac{\beta}{1 - \beta} \hat{W}_{\mu\nu} \right] = \hat{T}_{\mu\nu}$$

Weyl term from the bulk $W \sim g^{-1} \partial_r g \partial_r g$

This is certainly a solution, but...

- What does the bulk look like in this case?

[The higher curvature terms will deform the bulk solution,
need to re-study the global properties of the system]

- Does it mean that we have to introduce a Gauss-Bonnet term in our theory?

Seems to be too strong a conclusion...

Our perspective

Peloso, LS, Tasinato 06

- Keep Einstein gravity in the bulk
- In the spirit of Geroch and Traschen, consider a thick defect
 - [We do not consider this as a regularization,
but as a physical structure of the defect]
- In order to be able to use our knowledge of codimension-1 branes, use a 4-brane wrapped around an axis of the system

What are we looking for:

- Possibility to put matter with arbitrary equation of state on the brane
- Behavior of gravity in the football compactification
- Physical understanding of the origin of the constraint on the equation of state for matter localized on a delta-like brane

Our system

“Inside” sphere

$$ds_2^2 = R_i^2 (d\theta^2 + \cos^2 \theta d\phi^2)$$

$$F_{\theta\phi} = M^2 R_i \cos \theta$$

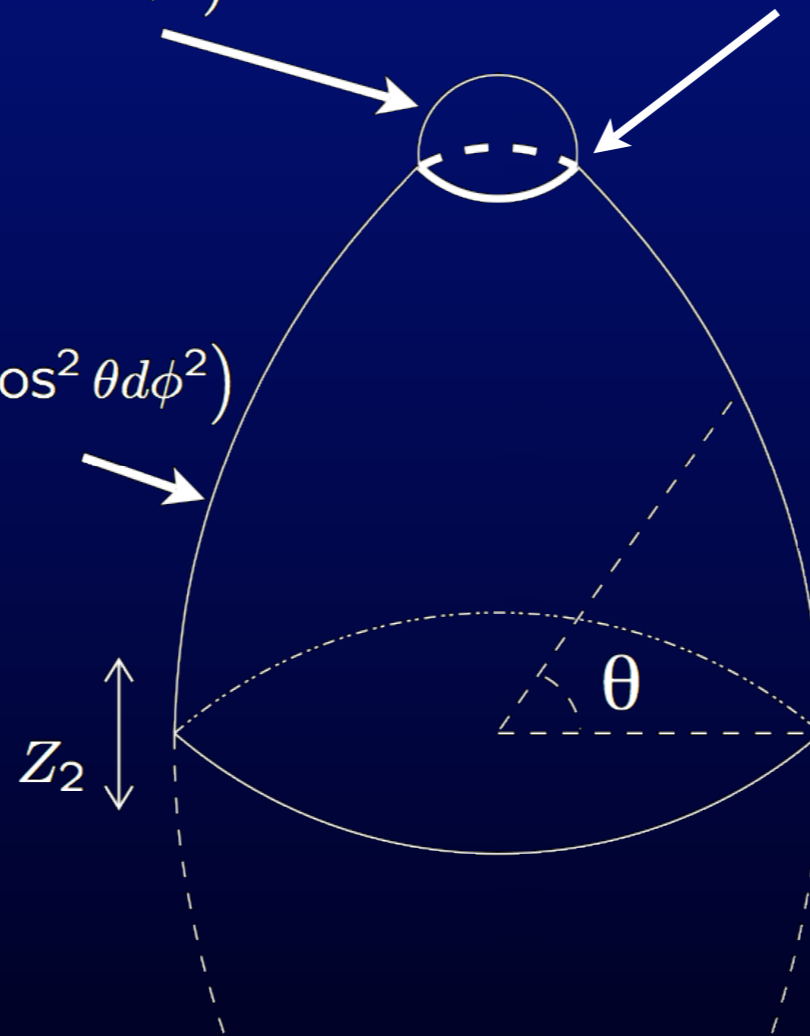
“Outside” ball

$$ds_2^2 = R_o^2 (d\theta^2 + \beta^2 \cos^2 \theta d\phi^2)$$

$$F_{\theta\phi} = M^2 \beta R_o \cos \theta$$

$$R_i = \beta R_o$$

4-brane located at $\theta = \bar{\theta}$



Let us put some numbers, to fix ideas...

This is a good model for a 5D Standard Model
in a 6D bulk (ADD)

$$\beta=O(1), R\sim\text{mm}$$

so that the fundamental Planck scale is $M\sim\text{TeV}$

The radius of the brane is

$$r_{\text{brane}} = R \cos \bar{\theta}$$

and we can set it to be $\sim\text{TeV}^{-1}$

Before adding matter..
what do we put on the brane?

Cannot put pure tension!

1- Junction conditions want $T_{\mu\nu} \propto g_{\mu\nu}$ while $T_{\phi\phi} = 0$

Chacko, Nelson 99

2- The brane must carry magnetic charge



Need to include brane matter charged under the bulk U(1)

The brane action

$$S = - \int d^5x \sqrt{-\gamma} \left[\lambda_s + \frac{v^2}{2} (\partial_M \sigma - e A_M) (\partial^M \sigma - e A^M) \right]$$

Charged field $|\phi| = v$
(explicitly) breaks $U(1)$

Phase of the field ϕ

- Maxwell equations determine $e^2 v^2$ in terms bulk quantities
- Brane position determined by $\tan^2 \bar{\theta} = \frac{2\lambda_s}{M^4 e^2 v^2}$
- After setting the brane fields to their vev, the brane stress energy tensor is $T_{\mu\nu} = T_4 g_{\mu\nu}$, $T_{\phi\phi} = 0$, as required by the junction conditions
- Deficit angle $1 - \beta = \frac{T_4}{2M^4 \pi \sin \bar{\theta}}$ gives the correct limit as $\bar{\theta} \rightarrow \pi/2$
(limit of delta-like brane)

And now let us add matter to the brane

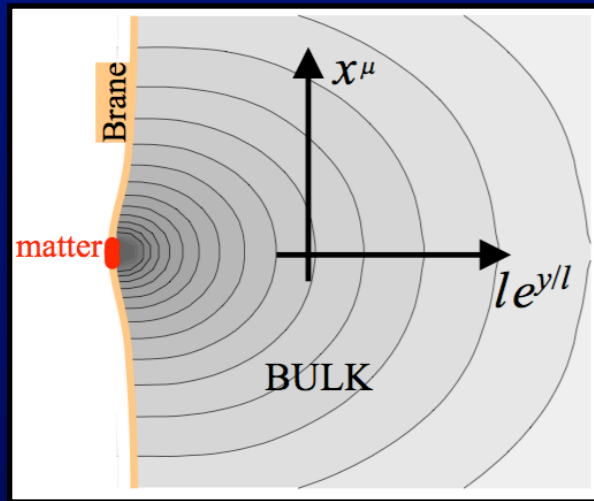
in the form of some (small) stress energy tensor $T_{\mu\nu}$

Allowed deformations of the system (scalar, vector, tensor) :

$$\begin{aligned}
 ds^2 &= (1 + 2\Phi) d\vartheta^2 + 2A d\vartheta d\varphi + (1 + 2C) \cos^2 \theta d\varphi^2 \\
 &+ 2(T_\mu + \partial_\mu T) d\vartheta dx^\mu + 2(V_\mu + \partial_\mu V) d\varphi dx^\mu \\
 &+ \left\{ \eta_{\mu\nu} (1 + 2\Psi) + 2E_{,\mu\nu} + E_{(\mu,\nu)} + h_{\mu\nu} \right\} dx^\mu dx^\nu \\
 \delta A_M &= \{a_\vartheta, a_\varphi, \partial_\mu a + a_\mu\}, \quad \delta\sigma, \quad \theta_{\text{brane}} = \bar{\theta} + \zeta(x)
 \end{aligned}$$

- Fix gauge
- Solve Einstein/Maxwell equations in the bulk
- Require regularity at the pole
- Require Z_2 symmetry at the equator
- Continuity+jumps at the brane (Israel junction conditions)

A crucial mode: the brane bending



In RS, crucial to get 4D gravity: couples to the trace of the matter stress-energy

Garriga and Tanaka 99,
Giddings, Katz and Randall 00

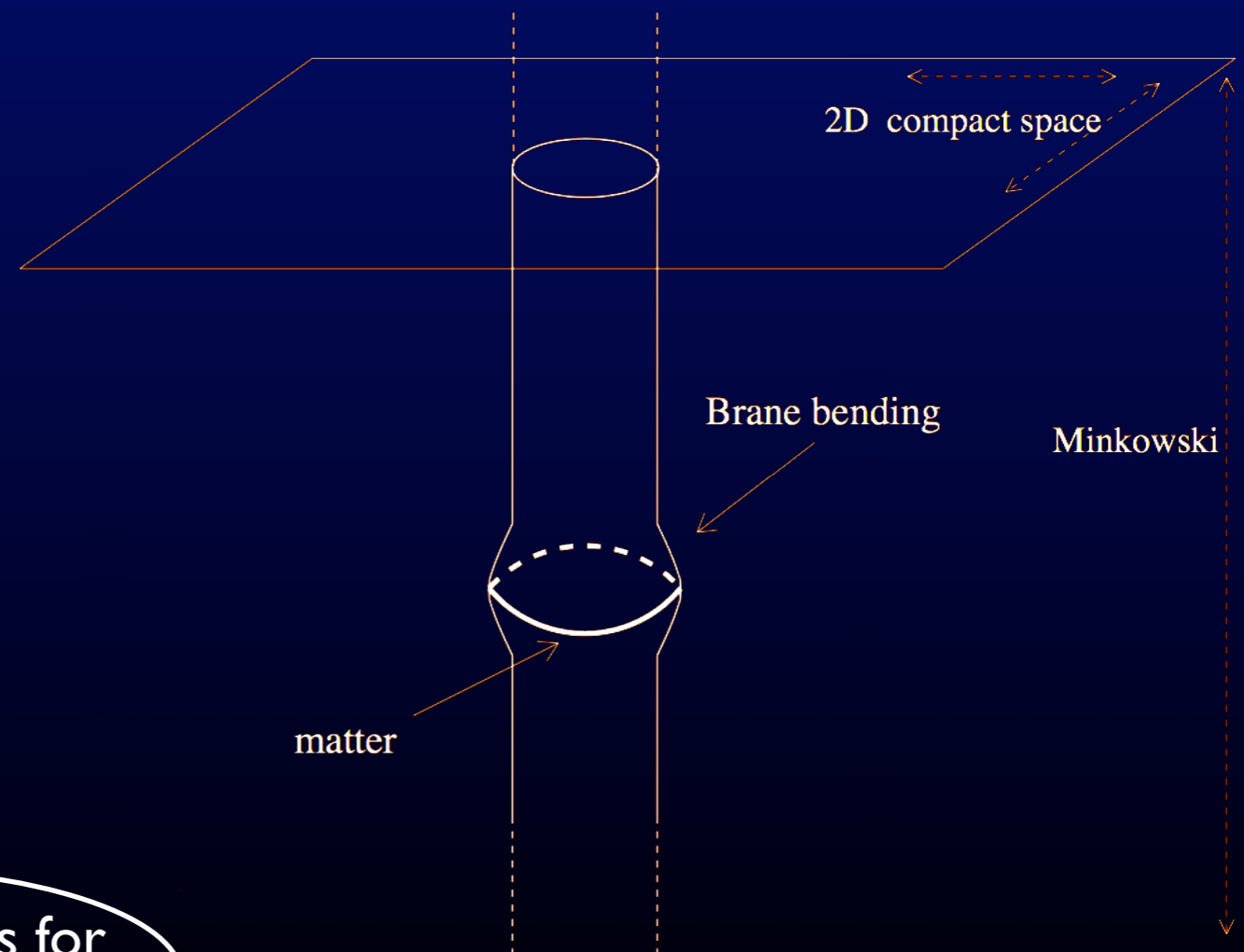
In our case, this is the mode that allows to put matter with arbitrary equation of state on the brane:

$$\left[\partial^2 \zeta \right]_J = \frac{T}{3M^4} \quad , \quad T \equiv T^\mu_\mu$$

(Jump across the brane)

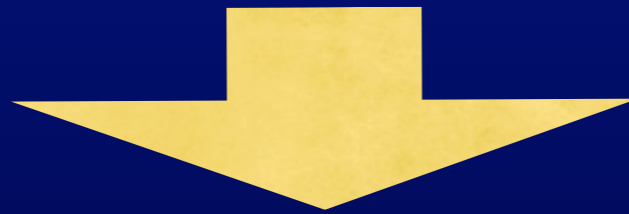
No scalar modes for the delta like brane

Graesser, Kile and Wang 04



Assumptions

We assume that the brane radius is hierarchically smaller than the bulk radius



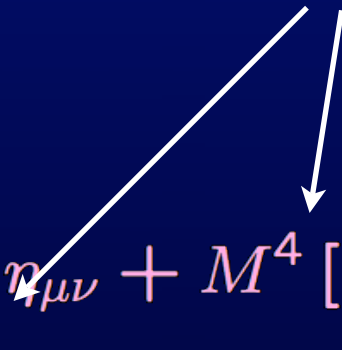
Modes are constant in the ϕ direction,
nontrivial dependence on θ

Analysis limited only to zero modes
(enough for long-range gravity)

Getting 4D gravity

Result is analogous to Garriga and Tanaka:
perturbative equations of motion get contributions
from tensor modes and from bending mode

Israel junction conditions give:

$$\partial_\vartheta (\cos \theta \partial_\vartheta h_{\mu\nu}) + \cos \theta \partial^2 h_{\mu\nu} = -2\delta(\theta - \bar{\theta}) \frac{\cos \bar{\theta}}{M^4} \left(T_{\mu\nu} - \frac{T}{3} \eta_{\mu\nu} + M^4 [\zeta_{,\mu\nu}]_J \right)$$


We are interested only in the induced 4D metric $g_{\mu\nu}^{(4)}$
and we define an effective 4D stress energy tensor as
the integral along ϕ of the brane stress energy tensor:

$$T_{\mu\nu}^{(4)} = \int_0^{2\pi R\beta} d\phi \sqrt{\gamma_{\phi\phi}^{(0)}} T_{\mu\nu} = 2\pi R\beta \cos \bar{\theta} T_{\mu\nu}$$

Einstein equations on the brane

$$\Upsilon = \left(-2\Psi + \frac{2\pi R\beta}{V_2} \cos \bar{\theta} [\zeta]_J \right) \Big|_{\text{brane}}$$

$$R_{\mu\nu}^{(4)} = \frac{1}{M^4 V_2} \left[T_{\mu\nu}^{(4)} - \frac{T^{(4)}}{2} \eta_{\mu\nu} \right] + \left(\frac{1}{2} \partial^2 \Upsilon \eta_{\mu\nu} + \Upsilon_{,\mu\nu} \right)$$

ADD M_p^2
Standard 4D gravity

Brans-Dicke scalar

Scalar-tensor theory

(Not surprisingly, $\Delta V_2 \propto \Upsilon$)

...but the Brans-Dicke scalar is heavy!

Indeed, the relation between Υ and T can be found explicitly
by replacing into Einstein's equations we get

$$R_{\mu\nu}^{(4)} = \frac{1}{M_p^2} \left[T_{\mu\nu}^{(4)} - \frac{T^{(4)}}{2} \eta_{\mu\nu} \right] + \mathcal{F}(\beta, \bar{\theta}) \frac{R^2}{M_p^2} \left(\frac{1}{2} \eta_{\mu\nu} \partial^2 + \partial_\mu \partial_\nu \right) \left(\frac{T^{(4)}}{3} - T_\phi^{(4)} \phi \right)$$

regular and $\mathcal{O}(1)$

extra term is negligible for $L \gg R$, where L is the
scale of variation of matter on the brane

**Standard 4D gravity
is recovered**

Stabilization fails at scales compatible
with compactification radius

Can be understood as an effect of
causality in the bulk:

The stabilization mechanism takes a time $\sim R$ to act

What happens in the limit of a delta-like defect?

This limit corresponds to $\bar{\theta} \rightarrow \pi/2$

The brane stress-energy tensor obeys

$$T_{\mu\nu}^{(4)} = 2\pi R\beta \cos \bar{\theta} T_{\mu\nu}$$

where we want to keep $T_{\mu\nu}^{(4)}$ constant



$T_{\mu\nu}$ has to diverge

From $[\partial^2 \zeta]_J = \frac{T}{3M^4}$, bending mode diverges

unless $T^{(4)}=0$. **Strong coupling problem**

Conclusions

- Built a regular version of the football (brane Universe in flux compactification)
- 4D gravity is recovered at distances $\gg R$
- Pathologies emerge in the delta-like codimension-2 limit
- First complete analysis (to our knowledge) of weak gravity in a stabilized 6D brane-worlds

Next...

- Cosmological solutions (de Sitter brane)
- Tachyon modes?
- How does the self tuning (not) work in this system?