Adaptive Route Choices in Risky Traffic Networks: A Prospect Theory Approach

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Abstract This paper deals with route choice models capturing travelers' strategic behavior when adapting to revealed traffic conditions *en route* in a stochastic network. The strategic adaptive behavior is conceptualized as a routing policy, defined as a decision rule that maps from all possible revealed traffic conditions to the choices of next link out of decision nodes, given information access assumptions. In this paper, we use a specialized example where a variable message sign provides information about congestion status on outgoing links. We view the problem as choice under risk and present a routing policy choice model based on the cumulative prospect theory (CPT), where utility functions are nonlinear in probabilities and thus flexible attitudes toward risk can be captured.

In order to illustrate the differences between routing policy and non-adaptive path choice models as well as differences between models based on expected utility (EU) theory and CPT, we estimate models based on synthetic data and compare them in terms of prediction results. There are large differences in path share predictions and the results demonstrate the flexibility of the CPT model to represent varying degrees of risk aversion and risk seeking depending on the outcome probabilities.

Keywords Adaptive Route Choice; Routing Policy; Traveler Information; Choice Under Risk; Prospect Theory.

1 Introduction

Traffic networks are inherently risky with random disruptions which create significant congestion, as described in the 2003 Urban Mobility Report by the Texas Institute of Transportation (Schrank and Lomax, 2003): "Crashes, vehicle breakdown, weather, special events, construction and maintenance activities greatly affect the reliability of transportation systems; these delays account for about 50 percent of all delay on the roads." In such a context, travelers' route choice behavior have two distinctive features. First, travelers can benefit from real-time information by making their actions adapt to actual traffic conditions revealed *en route*, rather than being committed to a fixed set of links *a priori*. Secondly, as the outcomes of travelers' decisions generally involve risks, attitudes toward risks play an important role in the decision process.

This study focuses on within-day adaptation to revealed network conditions *en route* as opposed to the day-to-day adjustment process of route choices. A direct consequence of this within-day context is that we deal with a general network topology, while most studies of route choice under risk (e.g. Katsikopoulos, Duse-Anthony, Fisher and Duffy, 2000; Katsikopoulos, Duse-Anthony, Fisher and Duffy, 2002; Avineri and Prashker, 2004; Avineri and Prashker, 2005; Ben-Elia, Erev and Shiftan, 2008) are carried out in simplified two-link networks. Moreover, travelers are assumed to know the travel time distributions on relevant links before making a trip. This knowledge can come from experience or predictions such as weather forecast. Information is used exclusively to denote the revealed link travel times (observed and/or predicted) based on which travelers can infer travel time distributions for the remainder of the trip. It is generally provided successively during a trip, e.g., a continuous radio broadcast of traffic conditions on major roads. This definition of information is different, for example, from Ben-Elia et al. (2008) who define information as the knowledge of travel time distributions before the trip.

In a previous paper (Gao, Frejinger and Ben-Akiva, 2008), an econometric model that captures travelers' within-day adaptive route choices has been developed, albeit with an assumption of fixed attitude toward risks. The following summarizes the common features of this paper and Gao et al. (2008) which distinguish them from other studies in the literature. First, the strategic behavior in travelers' adaptive route choices is conceptualized as a routing policy (also called hyperpath, strategy or shortest path with recourse in the literature), which can be viewed as a plan that takes into account all possible outcomes and defines an action for each possible situation. The choice made by a traveler is which plan (routing policy) to take before the trip, while the adaptation *en route* is an execution of the plan. Note that other studies in the literature on econometric adaptive route choice models or the impact of information on route choices (see literature review in Section 2.1) are focused on successive switchings of existing path choices, and inevitably ignore the strategic behavior. See Gao et al. (2008) for a detailed discussion on the difference between a routing policy and successive applications of path choices. Secondly, we develop econometric models of the routing policy choice which can be estimated from observed path choices, while other studies of strategic routing decisions do not deal with the estimation problem.

In this paper, we extend the model in Gao et al. (2008) with more realistic descriptions of risk attitudes by applying the cumulative prospect theory (CPT) proposed by Tversky and Kahneman (1992). Choice under risk has been a long researched topic in economics, with the theory evolving from expected utility (EU) to non-expected utility (non-EU). The latter is a generalization of EU theory, and arguably describes individuals' actual decision behavior better in risky situations. CPT is the most popular non-EU model (Starmer, 2000; de Palma, Ben-Akiva, Brownstone, Holt, Magnac, McFadden, Moffatt, Picard, Train, Wakker and J., 2008). We refer to Starmer (2000) for a review of non-EU models.

It is sometimes argued in the literature that CPT is applicable only to one-shot decisions where the probabilities of decision outcomes are conveyed by description. Barron and Erev (2003) and Erev and Barron (2005) show that in a feedback-based decision context, there is more risk seeking in the gain than in the loss domain and an underweighting of small probabilities, which are contradictory to experimental results in Kahneman and Tversky (1979). A possible explanation is the recency effect: rare events are underweighted because they are not likely to have occurred recently. Ben-Elia et al. (2008) show that there is prospect theory consistent behavior (risk seeking in the loss domain) at the beginning of an experiment with both experience- and description-based travel time distributions, while this tendency tends to diminish with longer experience.

We note that in route choices, the perception of travel time distributions could come from both personal experience (recurrent congestion) and descriptions from traveler information systems (non-recurrent congestion due to, e.g., bad weather, incident, and special event), mixed on an ongoing basis. In this complicated situations, it would be rash to reject the prospect theory without extensive empirical evidences. Furthermore, Myagkov and Plott (1997) support prospect theory (risk seeking in the loss domain) in an exchange market equilibrium experiment with repeated choices. The analogy of traffic equilibrium to economic market equilibrium leads one to explore the application of prospect theory to route choices which are known to result in traffic equilibrium in a congested network. The application and estimation of CPT models for within-day adaptive route choice under risk is, to the best of our knowledge, a new research area and this study serves as the first step in this direction.

The paper is organized as follows. First we present a literature review of adaptive route choice models, choice under risk and CPT. We then give an illustrative example to explain the concepts and show the difference between a routing policy and a path as well as EU and CPT. We then proceed to specify the routing policy choice model that is also used by Gao et al. (2008). Numerical results are discussed in Section 5 where we present estimations of four models (path and routing policy with EU and CPT utility functions respectively) based on synthetic data and compare them in terms of prediction results. Conclusions and a discussion of future research directions are provided in the end.

2 Literature Review

2.1 Adaptive Route Choice Models

Most discrete choice models for route choice analysis are based on static and deterministic networks. Examples of such models are Path Size Logit (Ben-Akiva and Ramming, 1998; Ben-Akiva and Bierlaire, 1999), C-Logit (Cascetta, Nuzzolo, Russo and Vitetta, 1996), Cross-Nested Logit (Vovsha and Bekhor, 1998), and Logit Mixture (Ramming, 2001; Bekhor, Ben-Akiva and Ramming, 2002; Frejinger and Bierlaire, 2007). In this paper we refer to these models as non-adaptive path choice models, since the fact that travelers can adjust their route choices en-route in response to revealed traffic conditions is ignored.

A seemingly natural way to build adaptive route choice models is to have a sequence of non-adaptive path choice models at each decision node, where the attributes of alternative paths reflect updated information. In this way, any of the above mentioned route choice models, with adequate incorporation of real-time information, could in principle be applied successively in a stochastic network to model adaptive route choice behavior. DynaMIT (Ben-Akiva, Bierlaire, Koutsopoulos and Mishalani, 2002) and DYNASMART (Mahmassani, 2001) are examples of dynamic traffic assignment models that apply adaptive path choice models. Calibration of DynaMIT's route choice model based on field data is reported in Balakrishna (2006) and Balakrishna, Ben-Akiva and Koutsopoulos (2007). Furthermore, Srinivasan and Mahmassani (2003) and Abdel-Aty and Abdalla (2006) are examples of studies that estimate route switching models applied successively on intermediate nodes.

There have been a large number of studies of path choice models with real-time information, both pre-trip and en-route, and a recent literature review can be found in Abdel-Aty and Abdalla (2006). Some models predict the decision to switch from a previous chosen or experienced route (e.g. Polydoropoulou, Ben-Akiva, Khattack and Lauprete, 1996; Abdel-Aty and Abdalla, 2004; Mahmassani and Liu, 1999; Srinivasan and Mahmassani, 2003); others are route choice models with explicit choice sets of paths (e.g. Bogers, Viti and Hoogendoorn, 2005; Peeta and Yu, 2005; Abdel-Aty and Abdalla, 2006).

In this paper we estimate routing policy choice models based on path observations. The definition of a routing policy depends on the underlying network and the information access (Gao and Chabini, 2006). An example of how it is defined here is given in Section 3. The literature includes a number of algorithmic studies of optimal routing policy problems (e.g. Hall, 1986; Polychronopoulos and Tsitsiklis, 1996; Marcotte and Nguyen, 1998; Pretolani, 2000; Miller-Hooks and Mahmassani, 2000; Miller-Hooks, 2001; Waller and Ziliaskopoulos, 2002; Gao, 2005; Gao and Chabini, 2006). Moreover, a sequential Logit loading of hyperpath flows in an equilibrium traffic assignment is proposed in Ukkusuri and Patil (2007), however the estimation problem is not addressed.

2.2 Choice Under Risk

Non-EU theory has been studied in the literature of economics for decades (see e.g. Kahneman and Tversky, 1979; Machina, 1989; Tversky and Kahneman, 1992), yet the application in risk attitudes toward travel times and transportation route choice is scarce. Katsikopoulos et al. (2000) conduct stated preferences experiments on drivers' route switching behavior, and find that participants are risk-averse when the average travel time along the alternative route is shorter than the certain travel time of the main route but risk-seeking when the opposite is true. This is consistent with the conclusions from Kahneman and Tversky (1979). de Lapparent (2004) studies business travelers' risk attitudes toward travel time losses using a rank dependent expected utility model which allows probabilities to enter non-linearly into an individual's objective function. Avineri and Prashker (2004) find violations of EU theory in a route choice stated preferences context through a series of experiments similar to those in Kahneman and Tversky (1979). Later on the same authors apply the CPT (Tversky and Kahneman, 1992), to the study of route choice with feedback and find that it might not be suitable to address repeated decision tasks (Avineri and Prashker, 2005). de Palma et al. (2008) discuss how the non-EU framework can be modeled and estimated within the framework of discrete choices in static and dynamic contexts. Ben-Elia et al. (2008) study the risk taking behavior in a route choice context where drivers' perceptions of travel time distributions come from both experience and description.

2.3 Cumulative Prospect Theory

The CPT developed by Tversky and Kahneman (1992) is a combination of the original prospect theory (Kahneman and Tversky, 1979) and the rank-dependent expected utility model (Quiggin, 1982). A brief review of CPT is provided below as further development of the routing policy choice model is based on CPT.

A prospect f is represented as a sequence of pairs (x_j, p_j), where x_j is the j-th outcome and p_j the associated objective probability. Individuals are assumed to evaluate alternatives in terms of gains and losses (positive and negative numbers respectively) with respect to some reference point and their preferences are modeled jointly with a value function and a weighting function. The value function is in a two-part power form:

$$v(x_j) = \begin{cases} x_j^\alpha, & \text{if } x_j > 0 \\ -\lambda(-x_j)^\beta, & \text{if } x_j \leq 0 \end{cases} . \tag{1}$$

In accordance with the principle of diminishing sensitivity, it is concave for gains ($\alpha \leq 1$) and convex for losses ($\beta \leq 1$). Moreover, it is steeper for losses than for gains ($\lambda > 1$) according to the principle of loss aversion.

Different specifications of the weighting function are presented in the literature (for an overview, see de Palma et al., 2008). Here we present the one discussed in Tversky and Kahneman (1992) with an inverted S-shape which overweights small probabilities and underweights moderate and high probabilities. The functional forms are presented below for gains and losses respectively:

$$w^+(p_j) = \frac{p_j^\rho}{\left(p_j^\rho + (1-p_j)^\rho\right)^{1/\rho}}, \quad w^-(p_j) = \frac{p_j^\delta}{\left(p_j^\delta + (1-p_j)^\delta\right)^{1/\delta}}. \tag{2}$$

Figure 1 plots the weighting function $w^-(p)$ using $\delta = 0.69$ and the EU case, $w^-(p) = p$ ($\delta = 1$), is shown with a dotted line.

The reference point is important for a CPT model and in a real choice context it can often be set in different ways. In a route choice situation, examples are free-flow travel time, habitual travel time and expected travel time. How to fix the reference point is an open research question, in particular, how it varies across different individuals and also how it changes for the same individual as he/she gains experience with repeated risky choices under similar conditions (de Palma et al., 2008). Given that we use synthetic data in this paper, there is no *a priori* belief which measure is more suitable as a reference point. We then choose to use the free-flow travel time, which is the shortest path travel time over all risky situations. Therefore we deal with losses only and our model actually corresponds to the rank-dependent expected utility model proposed by Quiggin (1982).

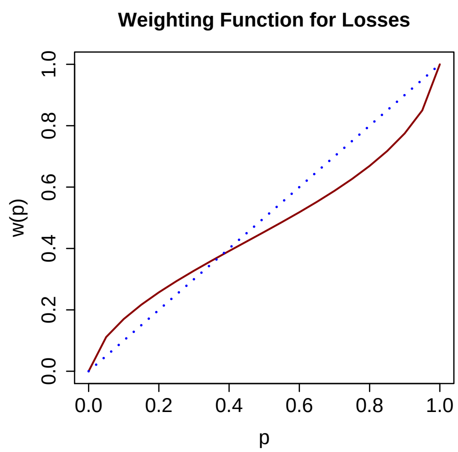

Figure 1: Weighting Function $w^{-}(p)$ with $\delta = 0.69$

The utility of a prospect f^{-} with $m+1$ loss outcomes is

$$Y(f^{-}) = \sum_{j=-m}^{0} \pi_j^{-} v(x_j), \tag{3}$$

where π_j^{-} is the decision weight for loss outcome j. The outcomes are arranged in increasing order and the negative subscripts indicate negative outcomes. Furthermore, π_j^{-} is calculated from the weighting functions of cumulative probabilities:

$$\pi_j^{-} = w^{-}(p_{-m} + \cdots + p_j) - w^{-}(p_{-m} + \cdots + p_{j-1}), \quad -m+1 \leq j \leq 0, \tag{4}$$

and $\pi_{-m}^{-} = w^{-}(p_{-m})$.

3 Illustrative Example

We use an example to illustrate the concept of routing policy and compare results from EU and CPT approaches. The network is shown in Figure 2 and has one origin node A and one destination node C with an intermediate node B. In the following we simplify the notation of a two-outcome prospect $(x, p; 0, 1-p)$ as (x, p) and a deterministic prospect $(x, 1.0)$ as (x). Both symbolic and numerical values (in brackets) are presented for travel times and probabilities. Numerical values are used in this section, while symbolic values are used in Section 5.

As discussed in the previous section, the reference point of the travel time between an origin-destination (OD) node pair is the shortest travel time across all paths and all risky situations. For any two-outcome prospect, the outcome without time loss is denoted as a good day, and the other a bad day. As an example, consider going from node B to C. If the travel time on link 3 is 30 minutes on a good day (with probability 0.75), it is 80 minutes on a bad day (with probability 0.25) and the travel time on link 2 is always 70 minutes. If we further assume the travel time on link 0 is 30 minutes such that the least possible travel time between origin node A and destination node C is 60 minutes, the travel time on link 1 is 110 minutes with probability 0.2 and 60 minutes with probability 0.8.

Link 3 is a highway that usually provides smooth travel, but there is a certain probability p_2 that it can be congested, due to for example an incident. Link 2 is a local road whose travel time is generally longer than that on the highway, but more reliable. It can serve as a diversion for the highway on a bad day. Link 1 is another highway with possible incident, but no diversion route. Incidents on the two highways are independent from each other.

A traveler knows the probabilistic distributions of the link travel times, and this is therefore a problem of choice under risk, as opposed to choice under uncertainty where the probabilities of outcomes are unknown. A variable message sign (VMS) is set up at node B such that the traveler learns about the realized travel time on link 3 only when he/she arrives at node B. We can make the behavioral assumption that a traveler decides on a fixed set of links (a non-adaptive path) to take, or he/she adapts the route choice at node B depending on the information displayed on the VMS. The latter behavior can be described as “choosing a routing policy”. There are 5 routing policies as shown in Figure 2. We use a 3-component vector to denote a routing policy. For example, (U,U,L) is the routing policy such that link 0 (upper link, U) is taken out of node A, followed by link 3 (upper link, U) on a good day when the time loss on link 3 is 0 and link 2 (lower link, L) on a bad day when the time loss on link 3 is 50 minutes. Out of the 5 routing policies, 3 are non-adaptive paths, namely (U,U,U), (U,L,L) and (L,-,-). We give aliases to four of the routing policies for the convenience of reference in Table 1. The name “Uncommitted” is used because a traveler taking routing policy (U,U,L) does not commit to a fixed path but adapts to information obtained at node B. Routing policy (U,L,U) is not named since it is clearly inferior to others and not relevant in our discussion.

Below we give an example of CPT utility and EU calculation for which we use the median estimates of the parameters given by Tversky and Kahneman (1992) for the value function (1) and weighting function (2); $\beta = 0.88$ and $\delta = 0.69$. λ does not affect the ordering of the utilities in a loss-only situation, so we arbitrarily set it to be 1 in this illustrative example. For the EU approach, we use the same value function as in the CPT approach so that the differences between the two approaches

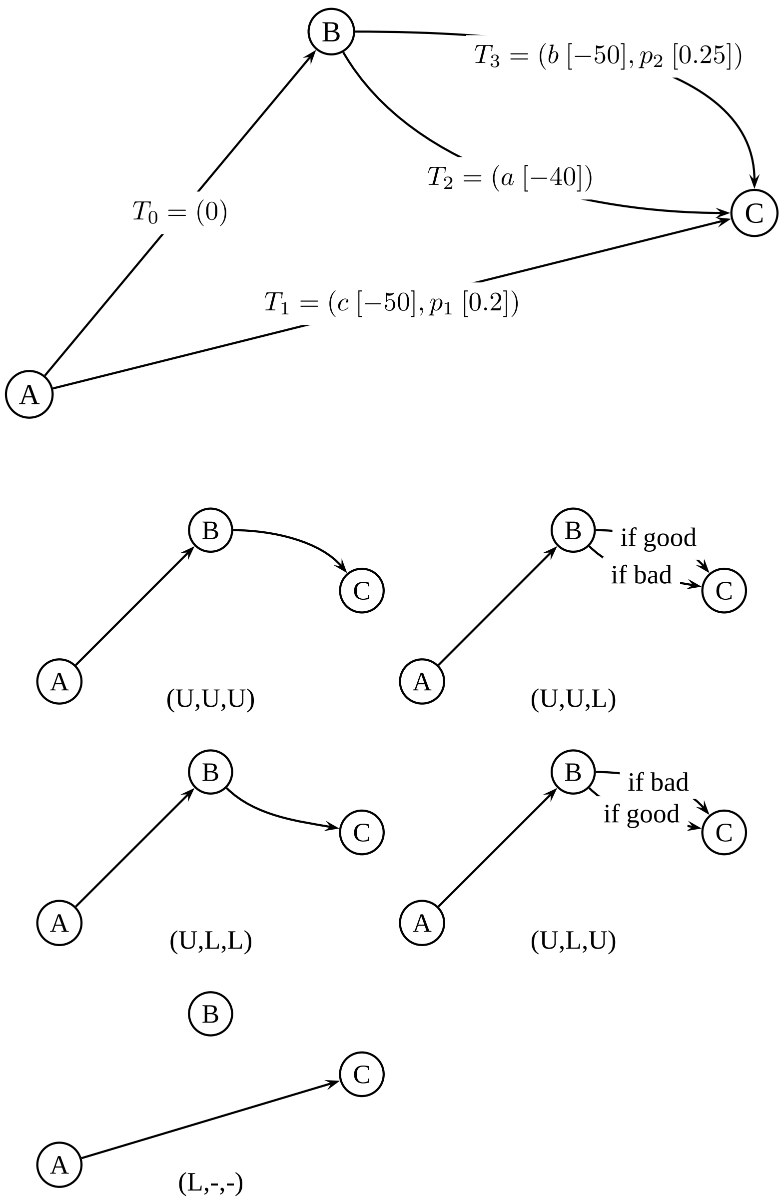

Figure 2: A Risky Traffic Network and All Routing Policies

are due to the weighting functions only. The EU of a prospect f^- is:

$$Y'(f^-) = \sum_{j=-m}^{0} p_j v(x_j). \tag{5}$$

Consider the routing policy Uncommitted. On a good day, it is manifested as the path with links 0 and 3 and thus the time loss is 0; on a bad day, it is manifested as the path with links 0 and 2 and thus the time loss is -40. Its prospect is therefore (-40,.25; 0,.75) simplified as (-40,.25). Note that the outcomes ($x_{-1} = -40$, $x_0 = 0$) are in increasing order with the corresponding probabilities $p_{-1} = 0.25$ and $p_0 = 0.75$. We carry out the step-by-step calculation based on (1)–(5) as follows:

$$\begin{aligned}
\pi_{-1} &= w^-(p_{-1}) = \frac{p_{-1}^{\delta}}{\left(p_{-1}^{\delta} + (1-p_{-1})^{\delta}\right)^{1/\delta}} = 0.29,\\
\pi_0 &= w^-(p_{-1}+p_0) - w^-(p_{-1}) = w^-(1) - w^-(p_{-1}) = 0.71,\\
v(x_{-1}) &= -\lambda(-x_{-1})^{\beta} = -25.69,\\
v(x_0) &= -\lambda(-x_0)^{\beta} = 0,\\
Y &= \pi_{-1}v(x_{-1}) + \pi_0 v(x_0) = -7.5,\\
Y' &= p_{-1}v(x_{-1}) + p_0 v(x_0) = -6.4.
\end{aligned}$$

Table 1 shows the prospects of all the 5 routing policies and the EU and CPT utility respectively. First let us take a look at the non-adaptive choice and adaptive choice under the CPT framework. If the traveler does not make use of the information collected *en route*, three non-adaptive alternatives, Left Highway, Local Street and Right Highway, are available at the origin. Right Highway will be chosen since, in this case, it has the highest utility. If the traveler adapts to traffic conditions at node B, Uncommitted will be chosen which has a higher utility than Right Highway. Therefore following a routing policy rather than a non-adaptive path can make the traveler better off under the CPT framework. Combined with previous studies under the EU framework (Gao et al., 2008), we can see that a routing policy is better than (or at least as good as) a non-adaptive path, whether the traveler is an EU or CPT utility maximizer. Note that under the EU framework, in this example, the optimal non-adaptive path is also the best among all routing policies.

The next observation is that the optimal routing policy is different under the EU and CPT framework: Uncommitted for CPT and Right Highway for EU. This can be explained by the fact that in the CPT framework the probability 0.2 is overweighted more proportionally than 0.25 is. This property is also termed subadditivity for small p, i.e. $\pi(rp) > r\pi(p)$, for $0 < r < 1$ (Kahneman and Tversky, 1979). Specifically

$$\frac{\pi(.2)}{\pi(.25)} > \frac{.2}{.25}.$$

Table 1: Routing Policy Prospects ($p_1 = 0.2, p_2 = 0.25$)

Routing Policy	Non-Adaptive Path	Prospect	Alias	CPT (Y)	EU (Y')
(U,U,U)	$\surd$	(-50,.25)	Left Highway	-9.2	-7.8
(U,U,L)		(-40,.25)	Uncommitted	-7.5	-6.4
(U,L,L)	$\surd$	(-40)	Local Street	-25.7	-25.7
(U,L,U)		(-50,.25; -40,.75)		-48.4	-49.8
(L,-,-)	$\surd$	(-50,.2)	Right Highway	-8.0	-6.3

Therefore the loss of 50 minutes from Right Highway has a bigger impact on a CPT utility maximizer than an EU maximizer, which results in the choice of Uncommitted for a CPT utility maximizer.

4 Model Specification

In this section we present a discrete choice model formulation for routing policy choice in a stochastic network (first introduced in Gao et al., 2008). The choice of routing policy is latent - it can be viewed as a plan in the traveler's mind, and only the result of the plan execution is observed, which is the manifested path. We therefore estimate the model based on path observations. The formulation applies to a general definition of routing policy (Gao and Chabini, 2006; Gao, 2005), including the one used in this paper.

The joint probability distribution of all the link travel time discrete random variables is described by a set of support points $R = \{r_1, r_2, ...\}$, where each support point is one distinctive vector of values that all the link travel time random variables can take. The probability of a given support point r is denoted as $p(r)$, and $\sum_{r \in R} p(r) = 1$. For example, the link travel time joint distribution of the network in Figure 2 has four support points, due to the two independent two-outcome travel time random variables on links 1 and 3. Table 2 shows the support points in detail. Note that travel time losses are shown rather than actual travel times.

A routing policy is manifested as a path for a given support point. For example, routing policy Uncommitted is manefested as Left Highway for support point 1 or 2, and as Local Street for support point 3 or 4. A support point can be fully defined by the realized travel times on all random links. Therefore the prospect of a routing policy can be obtained by executing the routing policy in all the support points. In each support point, the attribute of the manifested path (e.g., travel time, cost) and the support point probability constitute one element of the prospect. We assume that the realized support point for each path observation is known to the modeler

Table 2: Support Points of the Network in Figure 2 (Actual Travel Times)

Link Travel Time Random Variable	Support Point 1	2	3	4
T_0	0	0	0	0
T_1	0	-50	0	-50
T_2	-40	-40	-40	-40
T_3	0	0	-50	-50
Probability	.6	.15	.2	.05

through, for example, adequately dispersed GPS observations or probe vehicles that cover all random links.

We use a Policy Size Logit model (Gao, 2005) for the probability that individual n chooses routing policy γ in choice set $\mathcal{G}_n$. It is a routing policy version of the Path Size Logit model (Ben-Akiva and Ramming, 1998; Ben-Akiva and Bierlaire, 1999; Frejinger, Bierlaire and Ben-Akiva, 2009), where a term, Policy Size (PoS), is added to the deterministic utility correcting for correlations among routing policies. More precisely, the model is defined as

$$P(\gamma|\mathcal{G}_n;\Phi) = \frac{e^{V_{\gamma n}}}{\sum_{k\in\mathcal{G}_n} e^{V_{kn}}}, \tag{6}$$

where $V_{\gamma n} = \theta \ln \mathrm{PoS}_{\gamma n} + Y_{\gamma n}$, and $Y_{\gamma n}$ is the CPT utility value calculated according to Equations (1) through (4) for routing policy γ and individual n. Φ is the vector of parameters to be estimated: $\Phi = (\theta, \lambda, \beta, \delta)$, where λ, β and δ are parameters of the value function and weighting function for losses. The formulation of PoS

$$\mathrm{PoS}_{\gamma n} = \sum_{r\in R}\left(\sum_{\ell\in\mathcal{N}_\gamma^r}\left(\frac{T_\ell^r}{T_\gamma^r}\right)\frac{1}{M_{\ell n}^r}\right)P(r) \tag{7}$$

may be viewed as "expected Path Size (PS)" with notations

R	:	the set of support points of link travel time distribution;
$\mathcal{N}_\gamma^r$	:	the set of links on the realized path of routing policy γ for support point r;
T_ℓ^r	:	travel time on link ℓ for support point r;
T_γ^r	:	realized travel time on routing policy γ for support point r;
$M_{\ell n}^r$	:	the number of routing policies in choice set $\mathcal{G}_n$ using link ℓ for support point r;
$P(r)$	:	probability of support point r.

Note that all variables could be time-dependent, in which case the time subscript will be added to each variable. The value of Policy Size indicates the level of overlapping. If a routing policy does not overlap with any other routing policy in the choice set, it has a Policy Size of 1. Conversely, if J routing policies completely overlap, each of them has a Policy Size of $1/J$.

We then model the probability of a path observation i conditional on support point r and choice set of routing policies $\mathcal{G}_n$ as

$$P_n(i|r) = \sum_{\gamma \in \mathcal{G}_n} P(i|\gamma, r) P(\gamma|\mathcal{G}_n; \Phi). \tag{8}$$

Since several different routing policies can be manifested as the same path for a given support point, we sum over all routing policies in $\mathcal{G}_n$ and multiply the routing policy choice probability $P(\gamma|\mathcal{G}_n; \Phi)$ with a binary variable $P(i|\gamma, r)$ which equals 1 if γ is manifested as i for support point r and 0 otherwise.

5 Numerical Results

The setup for the numerical experiment is designed to

- validate that the CPT routing policy choice model can be consistently estimated based on (non-adaptive) path observations; and
- illustrate differences in terms of prediction results between CPT and EU routing policy choice and non-adaptive path choice models.

In this context it is appropriate to use synthetic data generated with a postulated model, since the true parameter values and decision rule are known. We postulate a CPT routing policy choice model to generate (non-adaptive) path observations and estimate four models based on the synthetic data:

- CPT routing policy and non-adaptive path choice models;
- EU routing policy and non-adaptive path choice models.

The CPT routing policy model is chosen as the postulated model, since it is the most complex among the four: routing policy is a generalization of non-adaptive path and CPT utility a generalization of EU.

The CPT routing policy choice model is used for validation and should have unbiased parameter estimates when compared to the postulated model. For the same reason, we expect the CPT routing policy model to have the best goodness-of-fit.

Postulating a CPT routing policy choice model is not only important for validation but also for the prediction results. Since empirical evidences suggest that individuals are not necessarily EU maximizers when making choices under risk (discussion in Sections 1 and 2), it is interesting to illustrate how EU models perform when the individuals' choice behavior follow a CPT model. This is especially so, considering that EU models are the most commonly used for analyzing route choice behavior.

5.1 Observation Generation

We use the network in Figure 2 and create a synthetic dataset that is used for estimation. 6000 path observations are generated with the postulated CPT Policy Size Logit model. The probability of a routing policy γ is given by Equations (6) and (7) with parameters $\theta = 1$, $\lambda = 2$, $\beta = 0.88$ and $\delta = 0.69$. The choice set $\mathcal{G}_n$ contains the same five routing policy alternatives for all observations but the link travel times vary.

Each path observation is generated in three main steps. First we sample link travel time losses a, b and c from a uniform distribution $[-60, 0]$. The actual travel times on links 0 and 3 are sampled from a uniform distribution $]0, 60]$. They are not used in the value function, but to compute PoS (PS) attributes. Link travel time probabilities (p_0 and p_1) are sampled from a uniform distribution $[0, 1]$. Second, we compute the probability $P(\gamma|\mathcal{G}_n),\ \forall\, \gamma \in \mathcal{G}_n$, and randomly draw one routing policy that is labeled as chosen. Third, we sample a support point from the set of four support points based on their probabilities and associate a path with the chosen routing policy.

5.2 Estimation

Four models are estimated as described in the introduction of this section. The deterministic utilities are the same in the four models, except for the weighting functions which equal the probabilities of the prospects (δ is fixed to one) in the EU models.

BIOGEME (Bierlaire, 2003; Bierlaire, 2005) is used for all model estimations and the results are reported in Table 3. For each model we give the parameter estimates as well as the t-test values with respect to (w.r.t.) the postulated parameter values.

First we note that the estimation is validated by the CPT routing policy choice model. It has the same formulation as the postulated model but is estimated based on path observations instead of routing policy observations. As expected, the parameter estimates are unbiased w.r.t. their true values.

Except for $\widehat{\theta}$, the parameter estimates in the CPT path choice model are significantly different from the postulated values. $\widehat{\beta}$ equals 0.999 and is not significantly different from one (t-test statistic: -0.04) which means that this model does not capture diminishing sensitivity as travel time increases. Moreover, the parameter estimate associated with the weighting function, $\widehat{\delta}$, is closer to one than the postulated value. The weighting function is therefore closer to the EU formulation. That is, small probabilities are less overweighted and high probabilities less underweighted compared to $\delta = 0.69$. Note however that the estimate is significantly different from one (t-test statistic: -12.17).

We now turn our attention to $\widehat{\beta}$ in the routing policy choice and path choice EU models. These estimates are greater than one and significantly different from one (t-test statistics of 6.95 and 3.80, respectively). As opposed to expectation, the value functions are hence strictly concave meaning that an increasing sensitivity to travel time (as travel time increases) is modeled. Recall that the postulated model has a CPT formulation. The EU models can therefore be viewed as misspecified which may explain these counter intuitive estimates. We go into further details when analyzing the prediction results in the following section.

	CPT		**EU**	
Choice model	Routing Policy	Path	Routing Policy	Path
Number of obs.	6000	6000	6000	6000
Null log-likelihood	-8150.419	-6591.674	-8150.419	-6591.674
Final log-likelihood	-477.716	-1611.63	-720.33	-1643.572
Adjusted rho sq.	0.941	0.755	0.911	0.750
$\widehat{\lambda}$ **(value fn)**	**2.09**	**0.276**	**0.495**	**0.185**
Robust std error	0.141	0.0272	0.0349	0.0172
t-test w.r.t. 2	0.64	-63.38	-43.12	-105.52
$\widehat{\beta}$ **(value fn)**	**0.88**	**0.999**	**1.13**	**1.09**
Robust std error	0.0126	0.0226	0.0187	0.0237
t-test w.r.t. 0.88	0.00	5.27	13.37	8.86
$\widehat{\delta}$ **(weighting fn)**	**0.688**	**0.832**		
Robust std error	0.00935	0.0138		
t-test w.r.t. 0.69	-0.21	10.29		
$\widehat{\theta}$ **(PS/PoS coef.)**	**1.02**	**0.915**	**1.48**	**1.4**
Robust std error	0.105	0.115	0.0862	0.114
t-test w.r.t. 1	0.19	-0.74	5.57	3.51

Table 3: Estimation Results

5.3 Prediction

The four estimated models are applied to predict route choices in the same topological network, but with a fixed set of hypothetical link travel time variables: $a = -40, b = c = -50, p_1 = 0.8p, p_2 = p$. The reference travel times for (A, B) and (B, C) are both 30 minutes, and thus 60 minutes for (A,C). The value of p, viewed as incident probability, is a parameter of the prediction tests and varies from 0 to 1, with an increment of 0.05.

Table 4: Routing Policy Prospects as Functions of p

Routing Policy	Non-Adaptive Path	Prospect	Alias
(U,U,U)	$\surd$	(-50, p)	Left Highway
(U,U,L)		(-40, p)	Uncommitted
(U,L,L)	$\surd$	(-40)	Local Street
(U,L,U)		(-50, p; -40,1-p)	
(L,-,-)	$\surd$	(-50, $0.8p$)	Right Highway

The prospects of the five routing policies as functions of p are shown in Table 4 along with aliases for the relevant routing policies. Of the most interest is the comparison between Uncommitted and Right Highway. They have the same expected travel time for all p which is minimum among all routing policies, but Right Highway has a larger variance and is thus riskier than Uncommitted.

Since the network is stochastic with all the support points known, we obtain distributions of variables such as path shares and path travel times over the support points, whose probabilities are functions of p. We present the summary statistics (expected value and/or standard deviation) in Figures 3 through 6. In all the figures, the x-axis represents the incident probability p.

Figure 3 shows expected path shares predicated by routing policy models with EU and CPT utility functions. We first examine the EU results (dotted line). As p increases, expected shares of the two risky highways decrease, since more likely incidents make the highways less desirable compared to Local Street which is unaffected by incidents. The increased share of Local Street could come from both travelers committed to it and Uncommitted. The committed flows increase because the utility of Local Street increases relative to other alternatives, and the uncommitted flows increase because more diversions occur now that the incident on Left Highway is more likely.

As discussed previously, the estimate of the power function exponent for the EU routing policy model is 1.13. The value function is thus concave and we would expect risk-averse behavior regardless of p. However we see that for most values

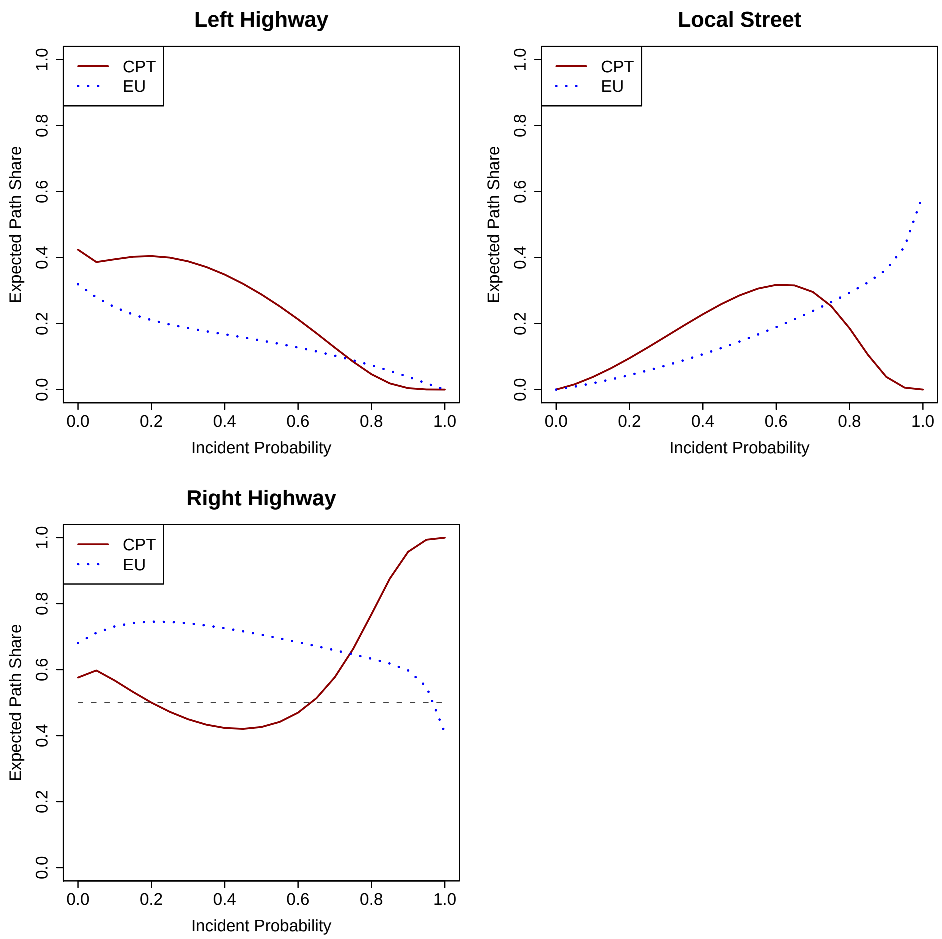

Figure 3: Expected Path Shares with Routing Policy Model

of p, more than half of the flows go to the riskier Right Highway (the dashed line indicates a path share of 0.5). Recall that there are two variables in the systematic utility of the choice model: Policy Size and EU value. On one hand, Right Highway is riskier and thus has a lower EU value compared to the left branch due to the concavity of the value function. On the other hand, Right Highway does not overlap with any routing policies, and therefore has the largest Policy Size. As we can see from Table 3, compared with the CPT Policy Size model, the coefficient for EU value ($\widehat{\lambda}$) is seriously underestimated and that for Policy Size ($\widehat{\theta}$) is moderately overestimated. Therefore Policy Size dominates EU value and more flows go to Right Highway.

Next we examine CPT results, which are different from EU results. In an EU model, the risk attitude is determined by the value function; while in a CPT model, it is determined jointly by the value function and the decision weights derived from the weighting function. Assume the value function for losses is convex. Under EU, this implies that a traveler is risk seeking everywhere: (a, p) is preferred to (ap) since $v(a)p > v(ap)$. Under CPT, the story is different. When p is intermediate or large, $p > \pi(p)$. Due to the convexity of the value function in the loss domain, we have

$$\frac{v(ap)}{v(a)} > p.$$

Combine the two inequalities we have (note $v(a) < 0$)

$$\frac{v(ap)}{v(a)} > \pi(p) \Longrightarrow v(ap) < v(a)\pi(p),$$

and thus the traveler prefers (a, p) to (ap) and is risk seeking. However, when p is small, $p < \pi(p)$, and there could exist some p such that

$$\frac{v(ap)}{v(a)} < \pi(p) \Longrightarrow v(ap) > v(a)\pi(p).$$

The traveler prefers (ap) to (a, p) and is risk averse.

As mentioned before, Right Highway is riskier than Uncommitted. The increase of the Right Highway share under high incident probability shows that when the probability of losses is high enough, higher probability induces more risky behavior. This is the so-called certainty effect (Kahneman and Tversky, 1979) where people treat almost certain losses as certain and thus shield away from it to take a riskier choice. Specifically in our example, when $p = 0.6$, Uncommitted (-40,.6) and Right Highway (-50,.48) might seem equally unattractive. However when $p = 0.8$, Uncommitted (-40,.8) could look like a sure loss and a traveler might think: why not take Right Highway (-50,.64) as there is a lower risk of loss and, in addition, a loss of 50 minutes is not too bad compared to 40 minutes?

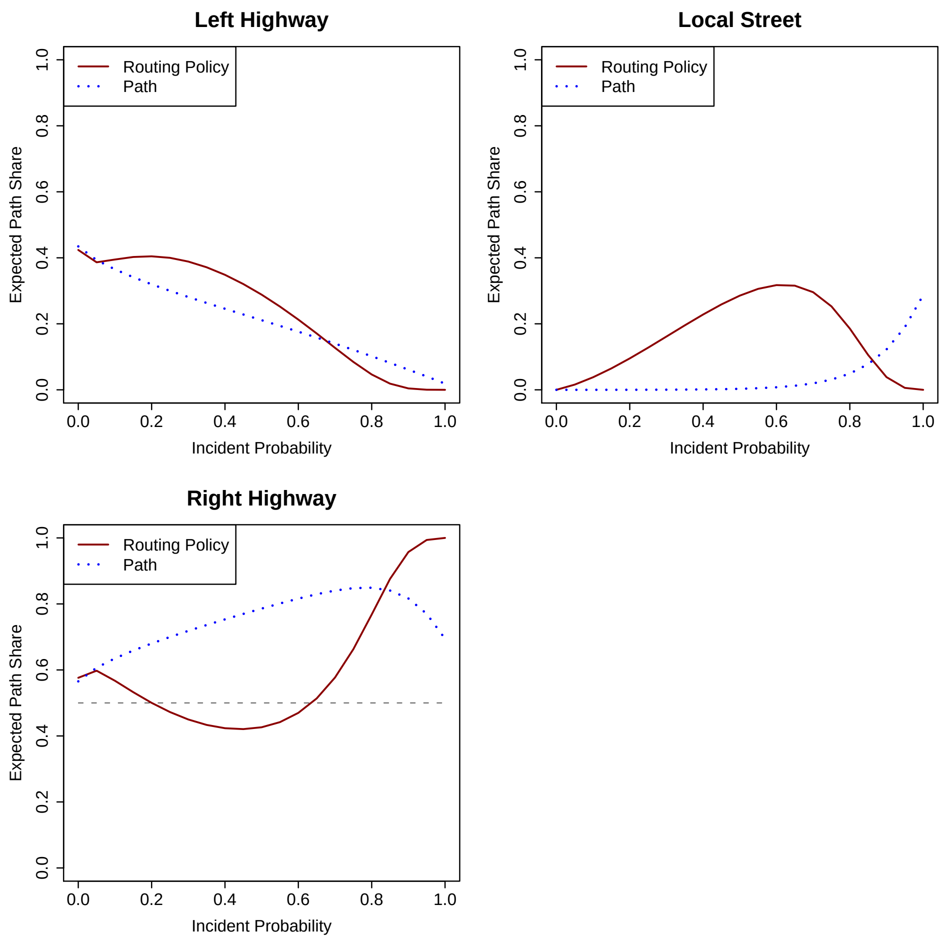

Figure 4: Expected Path Shares with CPT Function

Figure 4 shows the results of the routing policy model and the non-adaptive path model (dotted line), both with the CPT utility function. There are large differences between these two in terms of flows on Right Highway and Local Street. For most of the range of p, Right Highway flows predicated by the non-adaptive path model is much higher than those by the routing policy model. This is because Uncommitted is not considered by the non-adaptive path model, that is, the adaptation to real-time information on incident occurrence is not modeled. Uncommitted generally is the most attractive choice and the lack of it in the non-adaptive path model makes flows shift significantly to the second-best choice, Right Highway. When p is close to 1, Righ Highway flows predicted by the non-adaptive path model is less than those by the routing policy model. This is because Uncommitted is almost identical to Local Street with an almost sure incident on Left Highway, and the adaptive advantage diminishes. Meanwhile, the value function and weighting function parameter estimates of the path model (0.999 and 0.832 respectively) are closer to 1 than those of the routing policy model (0.88 and 0.688 respectively), suggesting a less convex value function and less underweighting of high probabilities. Therefore the path model predicts less risk-seeking behavior at a high probability and less flows on the riskier Right Highway.

Figure 5 shows the expected average travel time and the standard deviation of average travel time of the routing policy model and path model: the average is taken over all paths and the expectation over all support points. It is not surprising that the two models are similar in terms of expected average time, since the best choice from the routing policy model (Uncommitted) and the best choice from the non-adaptive path model (Right Highway) have the same expected travel time for all p. Although the difference is small, we can still tell that it is larger when p is in the medium range when the network is more unpredictable. A slightly different setting where Local Street has a loss of 30 instead of 40 minutes leads to a clearer picture as shown in Figure 6; the option value provided by Local Street to Left Highway is larger when the network is more unpredictable. Note that a similar conclusion is drawn in Gao et al. (2008) where a constant attitude of risk aversion is assumed.

The right parts of Figures 5 and 6 show the variance of the average travel time across support points for each model. A general observation is that the variances are first increasing and then decreasing with p, although the turning points are different depending on the model type and the value of Local Street loss. Note that the network is deterministic when $p = 0$ and almost deterministic when $p = 1$ (the incident probability on Right Highway $p_2 = 0.8p = 0.8$). It is most unpredictable at some p value between 0 and 1. Therefore both the routing policy and non-adaptive path models predict higher variances in the middle and lower variances when p goes to 0 or 1.

We further inspect the difference between routing policy and path choice models in terms of travel time variance in Figure 5. As both models have CPT utility functions, the twofold risk attitude applies to both. When incident probability is low, travelers are risk averse and are able to achieve less travel time variance by being adaptive, indicated by the fact that routing policy model predicts less variance than the path model does with small p. When p is high, the variance from the routing policy model increases and eventually becomes larger than that from the non-adaptive path model. This is consistent with the analysis of Figure 4 that the path model predicts less risk-seeking behavior than the routing policy model does at a high p.

Note that if Local Street has a time loss of 30 minutes, it is clearly more attractive than Right Highway. At a high incident probability on Left Highway (p), the best choice from the routing policy model (Uncommitted) is manifested as Local Street most of the time. It follows that both the routing policy and path models predict most path flows to the risk-free Local Street. Therefore the variances from the two models are getting close for high values of p as shown in the right part of Figure 6.

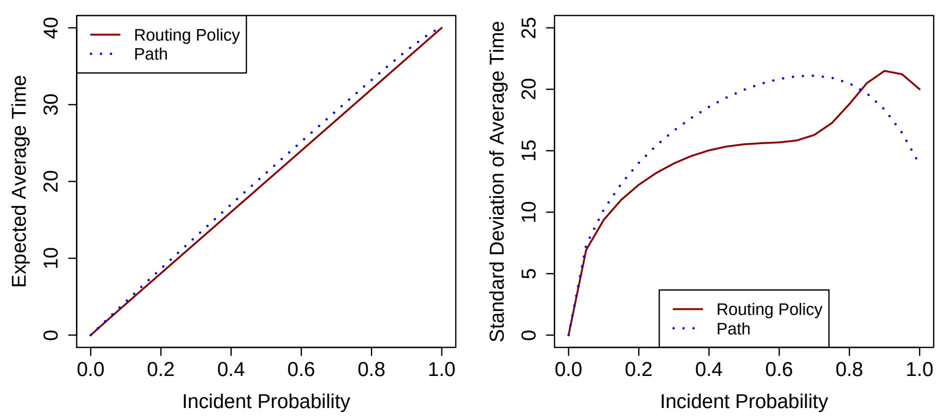

Figure 5: Mean and Standard Deviation of Average Travel Time with CPT Function (Loss on Local Street = 40 min)

6 Conclusions

This paper develops the first econometric specification for a routing policy choice model with a CPT utility function to capture travelers' route adaptation to real-time

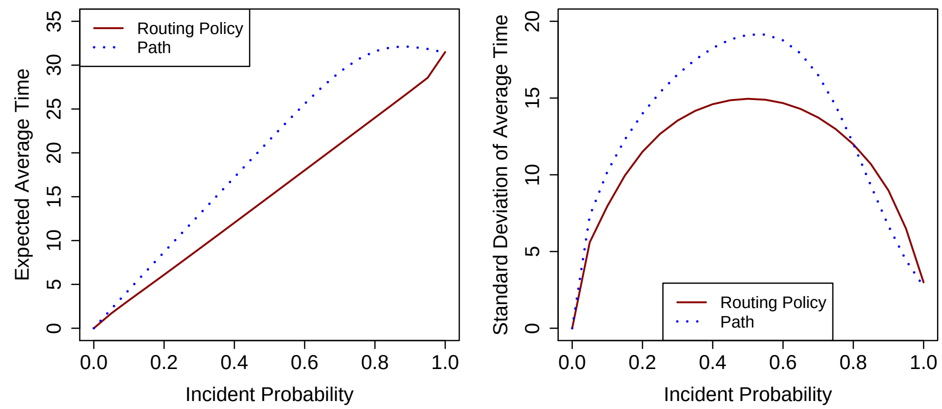

Figure 6: Mean and Standard Deviation of Average Travel Time with CPT Function (Loss on Local Street = 30 min)

information and risk attitudes more realistically. A routing policy is a decision rule that maps from all possible network states to next links out of decision nodes. The concept of routing policy explicitly captures travelers' strategic route choice adjustments according to information on realized network conditions in stochastic networks.

The CPT is more general than the EU theory in explaining decisions under risk. Specifically it allows the probabilities to enter a utility function nonlinearly, and thus the risk attitudes are determined jointly by the value function and weighting function. This provides explanations to some famous "paradoxes" under the EU framework (see e.g. Kahneman and Tversky, 1979).

The CPT routing policy choice model is compared to a CPT non-adaptive path model and two EU models for routing policy and non-adaptive path choices respectively. Prediction results show that the CPT model predicts the twofold attitudes toward travel time losses: risk seeking under high probability and risk averse under low probability, which cannot be captured by the EU model. Under the CPT framework, the routing policy model captures the option value of diversion while the path model does not. The difference between the routing policy model and the path model is larger in terms of expected travel time, if the network is more unpredictable.

7 Future Directions

A number of issues have been raised in Gao et al. (2008) regarding the estimation of a routing policy choice model on data collected in a real network, including deriving link travel time distributions, collecting data on travelers' information access, choice set generation and inertia in route choice. In the following, we focus on research directions regarding CPT in the route choice context.

In this paper we assume travelers know the probabilistic distributions of link travel times and thus our problem is "decision under risk". There is another class of problems where the distributions are not completely known to the decision makers, and the problem is called "decision under uncertainty". In the context of route choice, one could argue that it is more realistic to assume travelers do not have a full picture of the travel time distributions. Their knowledge might be incomplete or biased. For example, Henn and Ottomanelli (2006) use possibility theory as an alternative to probability theory to represent uncertainty in route choice. It is of interest to explore theories in choice under uncertainty and apply them to the routing policy choice model.

Weighting functions play an important role in CPT. There are a number of functional forms in the literature besides the one proposed by Tversky and Kahneman (1992). It is interesting to apply other functional forms and compare their performances. It is also hypothesized that the weighting function parameters vary across individuals, therefore it would be interesting to build models such that these variations can be identified. Note that although an inverted S-shape function is used in this paper for illustrative purpose, estimates from empirical data may or may not agree with such a shape. For example, the underweighting of small probabilities in feedback-based experiments (Barron and Erev, 2003) might suggest an S-shape weighting function.

As the prospect theory deals with gains and losses instead of absolute quantities, the choice of reference travel times is critical. However, the determination of reference point is still an open question (de Palma et al., 2008). Reference points are likely to be context-dependent in addition to being individual specific. For example, on a rainy day the reference point might be larger, since the travelers already expect higher travel times. One possible way to estimate reference point is to treat it as a discrete latent class variable. For example, we could assume there are three possible reference points in route choice: the free flow (minimum possible) travel time, the worst travel time (stuck in a snowstorm for five hours) and the mean travel time. The empirical data could then reveal the probabilities that a traveler will fall in one of the three classes, together with estimates of other parameters.

Finally, it may be interesting to postulate alternative types of models to CPT routing policy choice, using other decision rules such as lexicographic rules, regression trees and other non-EU maximization schemes than CPT. It could also be interesting to evaluate how the CPT routing policy choice model performs when travelers' behavior (postulated model) follows EU maximization.

Acknowledgments

We would like to thank André de Palma, Matthieu de Lapparent and the two reviewers of this journal for their comments. We are also grateful to the five reviewers of the 18th International Symposium on Transportation and Traffic Theory (ISTTT18) who have helped to improve this paper.

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