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A Process Model for Route Choice in Risky Traffic Networks

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Abstract

The accurate modeling of travelers' route choice decision making when faced with unreliable (risky) travel times is necessary for the assessment of policies aimed at improving travel time reliability. Compared with econometric models, process models have not been investigated in travel decision making under risk. A process model aims to describe the actual decision making procedure and could potentially provide a better explanation to route choice behavior. A process model, the priority heuristic (Brandstatter *et al.* , 2006), is introduced to the travel choice context and its probabilistic version, the probabilistic priority heuristic (PPH) model, is developed in this study. With data collected from a stated preference survey, a rank-dependent expected utility (RDEU) model and two other alternative models are compared with the PPH model through cross validation. Results showed that the PPH model outperforms the RDEU model in both data-fitting and predictive performances. This suggests that the process modeling paradigm could be a promising new area in travel behavior research. Major drawbacks of the PPH model include the discontinuity of the choice probability with respect to outcomes and associated probabilities, the limited applicability in situations where one alternative dominates or almost dominates the other, and the non-trivial extension to multiple-alternative situations.

Keywords: decision under risk, route choice, process model, rank-dependent expected utility model

1. Introduction and the Priority Heuristic

Understanding travel decision making in an uncertain environment and predicting travel choices in such an environment are important components in the overall goal of building a more reliable and efficient transportation system. Econometric (random utility) models are the generally accepted paradigm for choice modeling in transportation. They are adjusted to tackle the decision under risk problem, ranging from simply adding a risk measure (e.g., travel time standard deviation, deviation of actual from estimated travel time) to the utility function (Lam & Small, 2001; Bogers & van Zuylen, 2005; Ben-Elia *et al.* , 2013), to probabilistic versions of non-expected utility models from behavioral economics that captures non-linear subjective perceptions of both probabilities and outcomes, such as cumulative prospect theory (CPT) and rank-dependent expected utility (RDEU) theory (Schwanen & Ettema, 2009; Razo & Gao, 2013). Econometric models, such as CPT and RDEU applied in a travel decision making context, assume that decision makers integrate the outcomes and the associated probabilities of an alternative into one single measure of its worth (utility) and the alternative with higher utility will be chosen. See de Palma *et al.* (2008) for a review of the cross fertilization of the theories of decision under risk and discrete choice models.

However, one area in decision theory that is missing in travel behavior modeling is the process modeling paradigm, which aims to capture a decision maker's actual decision process, usually with efficient and frugal heuristics rather than correlating the choices with explanatory variables through complicated mathematical formula as in econometric models.

One of the popular process models is the parameter-free priority heuristic (PH) proposed in Brandstatter *et al.* (2006). PH supposes that a decision maker does not make trade-offs between outcomes and probabilities, but uses information in a non-compensatory manner. The final decision is obtained through a series of comparisons of outcomes and/or probabilities (termed "reasons"). Specifically, in the situation of two alternatives with two outcomes (minimum and maximum in terms of the absolute values in the domain of gain or loss), the order of comparison is minimum outcome, probability of minimum outcome and maximum outcome.

- Step 1: compare two minimum outcomes. If the difference is larger than 1/10 (defined as the aspiration level) of the higher maximum outcome, the more attractive alternative (larger minimum outcome in the domain of gain, and smaller minimum outcome in the domain of loss) is chosen and the process stops. Otherwise, go to Step 2.
- Step 2: compare probabilities of two minimum outcomes. If the difference is larger than 0.1, the more attractive alternative (smaller minimum-outcome probability in the domain of gain, and larger minimum-outcome probability in the domain of loss) is chosen and the process stops. Otherwise, go to Step 3.
- Step 3: compare two maximum outcomes. The more attractive alternative (larger maximum outcome in the domain of gain, and smaller maximum outcome in the domain of loss) is chosen and the process stops.

We give an example to show how the PH works. Consider two alternatives in the domain of gain:

$$(4000, 0.2; 0, 0.8) \quad vs \quad (3000, 0.25; 0, 0.75).$$

At the first reason, both alternatives have the same minimum outcome (0), which is less than 1/10*4000, and thus we move to the second reason. At the second reason, the difference between the probabilities of minimum outcomes, 0.8 - 0.75, is less than 0.1, and thus we move to the third and last reason. At the third reason, the first alternative has a larger maximum outcome and thus it is chosen.

The priority heuristic is simple in several respects. It typically consults only one or a few reasons; even if all are screened, it bases its choice on only one reason. Probabilities are treated as linear (in contrast to the non-linear transformation of probabilities in CPT), and a 1/10 aspiration level is used for all reasons except the last, in which the amount of difference is ignored. No parameters for overweighting small probabilities and underweighting large probabilities or for the value function are built in.

Brandstatter *et al.* (2006) has shown that the PH can account for evidence at variance with expected utility theory, namely a) the Allais paradox, b) risk aversion for gains if probabilities are high, c) risk seeking for gains if probabilities are low (e.g., buying lottery tickets), d) risk aversion for losses if probabilities are low (e.g., buying insurance), e) risk seeking for losses if probabilities are high, f) the certainty effect, g) the possibility effect, and h) intransitivities. A wide range of choice problems were used in (Brandstatter *et al.* , 2006) to compare the predictive performance of PH and other well-known theories of decision under risk, including CPT model (Tversky & Kahneman, 1992) and TAX (Transfer of Attention Exchange) model (Birnbaum, 1997). PH model gave comparable or superior performance in most situations.

Some researchers are skeptical of the PH. Johnson *et al.* (2008) conducted an experiment in web browsers running MouselabWEB (Willemsen & Johnson, 2012) to collect subjects' actual behavior during a decision making. Attention and transitions across elements of each alternative were recorded. It was found that transitions between outcomes across alternatives were rare and outcomes-probabilities transitions were common. This finding contradicted with PH. In addition, it was hypothesized that when PH stopped at step 1, attentions between two minimums should be observed dominantly while attentions of two minimum-probabilities and maximums should not be observed or very few. The actual observations from experiment suggested that attentions were evenly distributed across outcomes and probabilities.

Glockner & Betsch (2008) pointed out that two strong restrictions have been imposed on the PH as described in Brandstatter *et al.* (2006): 1) the PH does not work in the situation where one alternative dominates the other one such as: (0, 1%, 1, 99%) vs (2, 50%; 30, 50%) (in the domain of gain) where PH model will make a wrong prediction. 2) The accuracy of the PH will decrease dramatically when the ratio of two alternatives' expected outcome values exceed 2, such as 20 vs (0, 1%; 100, 99%) (in the domain of gain). These two restrictions help the PH exclude more than 50% cases where it is not good at when all scenarios are randomly generated, and thus it is doubtful whether the PH can be used as a general theory of decision under risk.

While the debate about the PH is going on, we think it worthwhile to investigate its applicability in a travel decision making context. The original PH is suitable for predicting majority choices, but appears to be less suited to provide proportional predictions. In order to predict the percentage of demand for each route in the traffic network, we construct a probabilistic PH model (Rieskamp, 2008). It is estimated using an existing SP (stated preference) dataset collected by the authors for investigating the RDEU model (Razo & Gao, 2013).

RDEU model and two other alternative models are introduced for comparison with the process model. The two other models have no underlying behavioral theories, and are designed for data fitting. They potentially can provide upper bounds on the goodness-of-fit and enable a more thorough assessment of the PPH model. Cross validation is finally conducted to investigate the data fitting and predicting performance of these four models. Note that the model assessment is based on the prediction of the final choice, rather than observations of the actual decision process, which will require much more detailed process data (Johnson *et al.* , 2008).

2. Probabilistic Priority Heuristic (PPH) Model

2.1. Model Development

We develop a probabilistic version of the PH similar to Rieskamp (2008) to predict the proportion of demand for each route in a traffic network, while the deterministic PH is only able to predict the majority choice. In the initial application of the PH to our dataset, when the threshold used in the comparison of minimum outcomes changed from the default 10% to 20%, the predictive accuracy of the PH improved considerably. This finding suggested that the PH model could be improved by estimating threshold values rather than using the default 10%. Conceivably comparing minimum outcomes (min), probabilities of minimum outcomes (pr) and then maximum outcomes (max) is not necessarily the only comparing order. The other five potential orders should also be considered: 2) min, max and pr; 3) max, min and pr; 4) max, pr and min; 5) pr, max and min; 6) pr, min and max. The existence of different comparing orders has been discussed in Hilbig (2008).

We treat all travel times as losses and in the remainder of the paper we work in the domain of loss only. Consider two alternatives A and B, each with two probabilistic outcomes (in absolute values) and the associated probabilities in the domain of loss,

$$(A_{min}, A_{pr}; A_{max}, 1 - A_{pr}) \quad \text{and} \quad (B_{min}, B_{pr}; B_{max}, 1 - B_{pr}),$$

where the minimum (maximum) outcomes are defined by absolute values (e.g., a travel time of 10 minutes is a smaller loss than 15 minutes). Note that a lower loss (maximum or minimum) is more attractive, and a higher probability of the minimum loss is more attractive. Let R denote a reason, and $R = min, max, pr$.

Error terms ϵ_{AR} and ϵ_{BR} are added to the objective values of reason R for the two alternatives respectively. Error terms for different reasons are independent, but do not necessarily have the same variance (scale).

If R is not the last reason, the probability of choosing A at reason R is the probability that the difference (the direction of taking the difference depends on the reason) between the noise-added reason values is greater than a threshold δ_R between 0 and 1, multiplied by the maximum outcome in the choice situation $M = max(A_{max}, B_{max})$.

$$P_R(A) = Prob(-[(A_R + \epsilon_{AR}) - (B_R + \epsilon_{BR})] > \delta_R M), R = max, min. \tag{1}$$

$$P_R(A) = Prob((A_R + \epsilon_{AR}) - (B_R + \epsilon_{BR}) > \delta_R M), R = pr. \tag{2}$$

Similarly, the probability of choosing B at reason R if R is not the last reason is

$$P_R(B) = Prob(-[(B_R + \epsilon_{BR}) - (A_R + \epsilon_{AR})] > \delta_R M), R = max, min. \tag{3}$$

$$P_R(B) = Prob((B_R + \epsilon_{BR}) - (A_R + \epsilon_{AR}) > \delta_R M), R = pr. \tag{4}$$

When δ_R is positive, $P_R(A)+P_R(B) < 1$, and the probability of not making a decision at reason R and moving to the next reason is $1 - P_R(A) - P_R(B)$. If δ_R is zero, the model collapses to a utility maximization one, and $P_R(A) + P_R(B) = 1$.

If R is the last reason, a decision must be made, and thus δ_R is set to 0. The probability of choosing A at the last reason R is thus

$$P_R(A) = Prob(-(A_R + \epsilon_{AR}) > -(B_R + \epsilon_{BR})), R = max, min. \tag{5}$$

$$P_R(A) = Prob(A_R + \epsilon_{AR} > B_R + \epsilon_{BR}), R = pr. \tag{6}$$

The probability of choosing B at the last reason R is $1 - P_{AR}$.

For a given reason R, the difference of the error terms $\epsilon_{AR} - \epsilon_{BR}$ effectively adds noises to the threshold of the reason $\delta_R M$, and captures the fact that different decision makers could have different thresholds. Other potential contributors to the noise include perception errors of outcomes and probabilities, and missing attributes. Theoretically if certain independent continuous distributions are assumed for the perception errors of the two outcomes of an alternative, the designations of the maximum and minimum outcome might be reversed for some realizations of the error terms, compared to the objective designation. We believe that such situations rarely happen in reality as decision makers generally can differentiate a good outcome from a bad outcome. Therefore we maintain the maximum and minimum outcome designation based on their objective values. The perception error, as only one part of the error term, is assumed to be not large enough to reverse the ordering.

The unconditional probability of choosing A is thus the sum of three components, each corresponding to a reason,

$$\begin{aligned} P(A) \;=\; & P_{R_1}(A) + & \textbf{(Reason 1)} \\ & P_{R_2}(A)(1 - P_{R_1}(A) - P_{R_1}(B)) & \textbf{(Reason 2)} \\ & P_{R_3}(A)(1 - P_{R_1}(A) - P_{R_1}(B))(1 - P_{R_2}(A) - P_{R_2}(B)). & \textbf{(Reason 3)} \end{aligned} \tag{7}$$

2.2. Discontinuity of the PPH Model

The choice probability of an alternative calculated from a PPH model can be discontinuous with respect to the outcomes and/or probabilities of the alternative outcome distributions, due to the discrete nature of defining the minimum and/or maximum outcomes. Two typical situations are discussed below, one with the probability and the other with the outcome.

Consider the comparing order of min, max, and pr. When the probability of the minimum outcome of alternative A, A_{pr} approaches 0 but remains a positive number, A_{min} remains the minimum outcome of alternative A. However when A_{pr} is exactly 0, the outcome distribution of alternative A collapse to a deterministic one $(A_{max}, 1)$ and the minimum outcome is the same as the maximum outcome A_{max}. The discontinuity in the change of the minimum outcome from A_{min} to A_{max} at $A_{pr} = 0$ will result in the discontinuity of the final probability of choosing A at the same location.

Consider again the comparing order of min, max, and pr. When A_{min} approaches A_{max} but is not equal to A_{max}, the probability of the minimum outcome remains A_{pr}. However when A_{min} is equal to A_{max}, the outcome distribution of alternative A collapse to a deterministic one, $(A_{min}, 1)$, and the probability of the minimum outcome becomes 1. The discontinuity in the change of the probability of the minimum outcome from A_{pr} to 1 at $A_{min} = A_{max}$ will result in the discontinuity of the final probability of choosing A at the same location.

The discontinuity could make it difficult to interpret model predictions at and close to the location of discontinuity. An example is shown later in Figure 4 with discussions provided in Section 6.1.

3. Test Design

We briefly present the test design detailed in Razo & Gao (2013). An abstract network is shown in Figure 1 and a screenshot in Figure 2. A subject had a choice between a path with a random travel time (Path A with a high travel time of t_H with probability p and low travel time of t_L with probability $(1 - p)$, the risky route), and a path with a deterministic travel time (Path B with a travel time of t_B, the safe route).

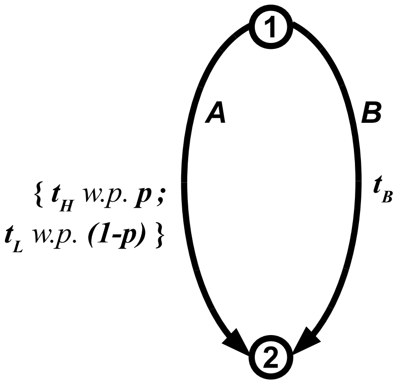

Fig. 1. The Abstract Network

With the advantage of simplicity and clarity as compared to describing the scenarios in written or verbal form, this SP survey was conducted using interactive graphical maps with a point-and-click interface (shown in Figure 2) presented to a subject on a computer screen. Routes in green color are assigned as buttons for subjects to click. The white labels, 30 and 40, indicate the usual travel time of the adjacent route with the unit of minute. The yellow label beside the risky link indicates the probability of a delay and the full travel time of this path in the event of a delay. With a factorial design, the probability of delay (p) could be 20%, 50% and 80%. t_L was fixed at 30 minutes throughout all scenarios. t_H took values 40, 50, and 60 while t_B took values from 35 to 55 with a step size of 5 such that the safe route was not dominated by the risky route. Including introduction, paperwork and the survey, each session lasted 40 to 60 minutes for each subject.

74 individual subjects were recruited from the University of Massachusetts Amherst students and staff community and surrounding areas. The mean age was 24.2 years and mean driving experience was 6.9 years. 54% of the subjects were male, 46% were female.

The survey included this map as a simple risk map to test subjects' risk attitude and another strategy map to investigate people's strategic route choice behavior. The strategic map data are not used in this study. Each subject made choices in 24 different scenarios in the simple risk map with a total of 1,767 observations (9 observations are missing due to problems in transmitting data).

4. Model Specification

The PPH model developed in the previous section is a general model without specifications of the distributions of random error terms. It is adapted to the actual choice problem in the survey. An alternative-specific constant (ASC) is added to the risky route for each reason, ASC_{min}, ASC_{max}, ASC_{pr}. These variables are used to capture potential biases towards either one of the two routes, e.g., the risky route has two segments and could be viewed as less desirable due to the extra effort involved in clicking on the map.

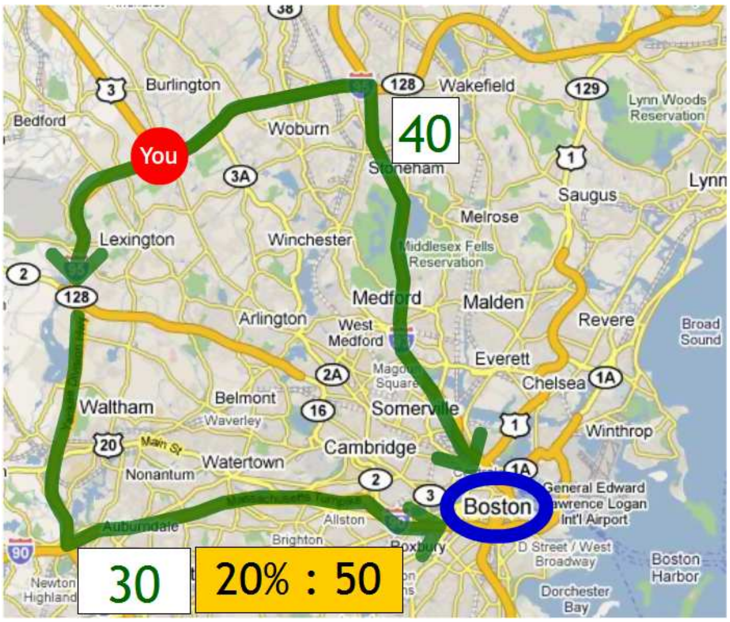

Fig. 2. Screenshot of the Survey Interface

The error terms are assumed to be i.i.d. Gumbel across observations and alternatives for the same reason. We simplify the variance structure across reasons, by assuming that error terms for min and max have the same standard deviation, and that the standard deviation of the error terms for pr is 1/60 of that for min and max (60 is an approximate magnitude of the travel times in the survey). These assumptions reduce the number of scale parameters to only one, λ for pr.

The probabilities of choosing A (risky route) and B (safe route) at reason R if R is not the last reason ($R = min, max$) are respectively

$$P_R(A) = \frac{1}{1 + \exp\{-(\lambda/60)[-(ASC_R + A_R - B_R) - \delta_R M]\}}, \tag{8}$$

$$P_R(B) = \frac{1}{1 + \exp\{-(\lambda/60)[-(B_R - ASC_R - A_R) - \delta_R M]\}}. \tag{9}$$

The probabilities of choosing A and B at reason pr if pr is not the last reason, are respectively

$$P_{pr}(A) = \frac{1}{1 + \exp\{-\lambda[(ASC_{pr} + A_{pr} - B_{pr}) - \delta_{pr} M]\}}, \tag{10}$$

$$P_{pr}(B) = \frac{1}{1 + \exp\{-\lambda[(B_{pr} - ASC_{pr} - A_{pr}) - \delta_{pr} M]\}}. \tag{11}$$

The probability of choosing A at the last reason $R = min, max$ is

$$P_R(A) = \frac{1}{1 + \exp\{-(\lambda/60)[-(ASC_R + A_R - B_R)]\}}. \tag{12}$$

The probability of choosing A at the last reason pr is

$$P_{pr}(A) = \frac{1}{1 + \exp\{-\lambda[ASC_{pr} + A_{pr} - B_{pr}]\}}. \tag{13}$$

The unconditional probability of choosing A, $P(A)$ can be obtained by substituting these probabilities into Eq. (7), and $P(B) = 1 - P(A)$.

To account for the panel effect that a subject made choices in multiple scenarios, the ASC for the first reason is treated as a normally distributed random variable across subjects and its mean and standard deviation are estimated.

Seven parameters are thus to be estimated: two threshold values for the first two reasons (δ_{min}, δ_{max}, δ_{pr} depending on which two are the first), one scale (λ), three ASCs for three reasons (ASC_{min}, ASC_{max}, ASC_{pr}), and the standard deviation of the first ASC.

The major differences of our model from that of Rieskamp (2008) include: 1) ASCs are included to capture innate biases in a travel choice context, while the choice scenarios used in Rieskamp (2008) are based on stated lotteries and do not entail ASCs in general; 2) The panel effect is accounted for while Rieskamp (2008) ignores it; 3) The error terms are Gumbel distributed instead of normal to enhance the tractability of the model.

5. Estimation Results

Table 1. Estimation Results of PPH models (Values in parentheses are robust standard errors. 74 subjects and 1,767 observations. 7 parameters for each model.)

	PPH_1 min,pr,max	PPH_2 min,max,pr	PPH_3 max,min,pr	PPH_4 max,pr,min	PPH_5 pr,max,min	PPH_6 pr,min,max
Scale λ	28.8 (2.30)	21.3 (1.23)	24.6 (1.41)	19.9 (0.989)	31.1 (1.99)	27.9 (1.82)
ASC_{min}	μ:-19.3 (1.29) σ:1.66 (0.261)	μ:9.51 (0.430) σ:2.09 (0.409)	-29.1 (1.93)	15.6 (0.408)	-72.0 (0)	8.91 (0.446)
ASC_{max}	-15.5 (0.711)	-11.0 (0.485)	μ:26.2 (0.737) σ:1.87 (0.397)	μ:-22.4 (0.582) σ:0 (0)	11.2 (1.09)	-7.79 (0.478)
ASC_{pr}	0.195 (0.00513)	0.334 (0.0293)	0.396 (0.0197)	0.514 (0.00920)	μ:0.517 (0.00537) σ:0.0239 (0.0071)	μ:0.517 (0.00594) σ:0.0284 (0.00915)
δ_{min}	0.627 (0.0243)	0.135 (0.00672)	0.802 (0.0336)	NA (NA)	NA (NA)	0.139 (0.0071)
δ_{max}	NA (NA)	0.111 (0.00879)	0.784 (0.0107)	0.391 (0.00677)	0.402 (0.0205)	NA (NA)
δ_{pr}	0.0323 (0.00589)	NA (NA)	NA (NA)	0.349 (0.00862)	0.338 (0.00595)	0.343 (0.00717)
FLL	-816.539	-826.779	-800.085	-880.671	-831.959	-828.892
$\overline{\rho}^2$	0.328	0.319	0.341	0.275	0.315	0.318

PPH models with all six potential comparing orders are estimated in BIOGEME Python 2.0 (Bierlaire, 2003, 2008) with 1,000 simulation draws for the normally distributed ASCs. Results are shown in Table 1. FLL stands for the final log likelihood. $\overline{\rho}^2 = 1 - (\text{FLL} - K)/L_0$ is the measure of fitness (Ben-Akiva & Lerman, 1985), where L_0 is the log likelihood of the naive (equal-probability) model, and K is the number of parameters. All parameters are significantly different from zero, except the standard deviation of ASC_{max} in PPH_4. PPH_3 (max, min and pr) has the best model fit ($\overline{\rho}^2 = 0.341$) and the order is different from the original order (min, pr and max) posited in Brandstatter *et al.* (2006). This can be explained by the following two observations. 1) Travelers are generally very concerned about delays, and likely to consider the maximum outcome (delay on the risky route) first. 2) The survey scenarios were grouped by delay probability (for reasons not related to this study), and thus subjects were likely not paying attention to the probability while it remained constant.

Thresholds of min, pr and max differ very much in different models. Estimated values of δ_{min} and δ_{max} are 0.802 and 0.784 respectively for PPH_3, much higher than the original 0.1. Similar high values are found in previous studies (Rieskamp, 2008). It is not entirely clear why such high values of thresholds exist, and future research is needed to understand whether people truly use such high thresholds or they are the result of a wrong underlying theory.

It is not straightforward to interpret ASCs in a PPH model. In a typical utility maximization model such as the REDU model discussed in the next section, ASC is used to capture the bias towards a certain alternative and its sign indicate the direction of the bias. In the PPH model, however, the final probability of choosing a given alternative is a complex function of all three ASCs. Unless all three ASCs have the same sign, it is not straightforward what effect it will have on the choice probabilities. The only way to find out is to calculate the probability of choosing an alternative assuming the two alternatives have the same travel time distribution ("everything else equal"). However, "everything else equal" can be ambiguous. When the two alternatives have the same travel time distribution so that A_R and B_R cancel out for all reasons, the probability still depends on M, and thus varies across contexts.

6. Three Alternative Models

6.1. RDEU model

One of the most popular non-expected utility (non-EU) theories (Starmer, 2000) is the rank-dependent expected utility (RDEU) theory (Quiggin, 1982; Schmeidler, 1989). A decision maker is supposed to maximize

$$V(\mathbf{x}, \mathbf{p}) = \sum_{i=1}^{m} \pi_i u(x_i), \tag{14}$$

where $\mathbf{x}$ and $\mathbf{p}$ denote vectors of travel time outcomes (in absolute values) and associated probabilities respectively with a size of m.

$u(x)$ is a value function of outcomes and takes a power functional form.

$$u(x) = -x^{\beta}. \tag{15}$$

$\beta < 1$ indicates a decreasing sensitivity to outcome.

π_i is the decision weight for outcome i. It is related to the weighting function $w(p)$ that takes the form

$$w(p) = \frac{p^{\delta}}{(p^{\delta} + (1-p)^{\delta})^{1/\delta}}, \delta > 0.279, \tag{16}$$

and describes distorted perceptions of objective probabilities following Tversky & Kahneman (1992). A smaller δ suggests a more pronounced inverted S-shape. See Figure 3 for an illustration (red solid line). The blue dotted line shows a perfect perception with $\delta = 1$. $w(0) = 0$ and $w(1) = 1$ suggest that people have no problem perceiving impossibility and certainty. The sensitivity to probability diminishes when moving away from the two extreme points $p = 0$ and $p = 1$, represented by a flatter curve toward the middle point between 0 and 1.

With a sorted outcome sequence $x_1, x_2, \ldots, x_m$ by absolute values, the decision weight of outcome i is

$$\pi_i = w(p_i + p_{i+1} + \cdots + p_m) - w(p_{i+1} + \cdots + p_m), \tag{17}$$

and $\pi_m = w(p_m)$.

When the RDEU model is applied to a route choice scenario from the survey, the utility of risky route with the parameter vector $\boldsymbol{\phi} = \{ASC, \lambda, \beta, \delta\}$ is

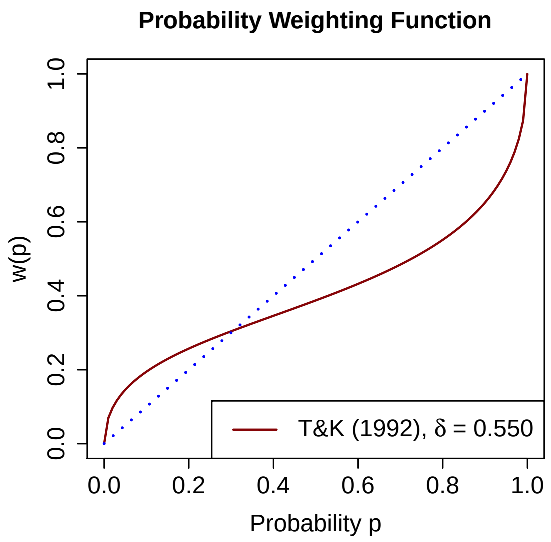

Fig. 3. Probability Weighting Function

$$V(A) = ASC + \lambda \left[(t_H)^{\beta} w(p) + (t_L)^{\beta} (1 - w(p)) \right]. \tag{18}$$

Note that the negative sign before the power function in Eq. (15) is absorbed by the scale parameter λ. ASC denotes a bias towards the risky route, and is a normally distributed random variable across subjects to account for the panel effect.

The utility of the safe route is

$$V(\text{Safe}) = \lambda (t_B)^{\beta}. \tag{19}$$

The utilities are applied to a Logit function to yield the probability of a given choice observation.

$$P(i) = \frac{\exp(V(i))}{\sum_{j=A,B} \exp(V(j))} \tag{20}$$

Table 2 presents estimation results of the RDEU model with 1,767 route choice observations from 74 subjects. The negative sign of the mean of the ASC (μ = -0.933) indicates subjects' average preference towards the safe route when everything else is equal. The diminishing sensitivity to outcome is confirmed by $\beta < 1$, as well as an inverted S-shaped weighting function ($0.279 < \delta < 1$). Comparing Tables 2 and 1 we find that 5 out of 6 PPH models obtain better goodness-of-fit than the RDEU model. Note that the REDU model has already been shown to outperform a number of other models, including the mean-variance, mean-standard deviation models and their variations and the expected utility model (Razo & Gao, 2013).

Figure 4 shows the probabilities of choosing the risky route calculated from the PPH_3 (green line) and REDU (red line) models as functions of the delay probability (0 to 1) in five situations. All five situations have the same travel time probabilistic outcomes on the risky route, 30 and 60. The travel time on the safe route varies from 35 to 55 with a step size of 5 across the five situations. The dots at 0 and 1 for the PPH_3 model indicate its discontinuity at those locations.

The general decreasing trends (except at p = 1 for the PPH model) are consistent with the intuition that a higher chance of delay on the risky route reduces its attractiveness. The shapes of the curves however are considerably different. The REDU curves seem smoother, while the PPH_3 curves have five distinctive sections: two discontinuous locations at 0 and 1, two relatively flat sections close to 0 and 1, and one decreasing section in the middle. The cause of discontinuity is discussed in Section 2.2. As such it is

Table 2. Estimation Results of the RDEU Model (Values in parentheses are robust standard errors.)

Risky Branch Bias ***ASC***	μ: -0.933 (0.144) σ: 0.529 (0.115)
Scale $\boldsymbol{\lambda}$	-1.48 (0.570)
Value Func. $\boldsymbol{\beta}$	0.720 (0.0749)
Weight. Func. $\boldsymbol{\delta}$	0.616 (0.0232)
Final LL	-840.872
$\bar{\rho}^2$	0.309

difficult to interpret PPH_3 model results at and close to the two extreme locations, which are consistent with the original PH's restrictions discussed in Brandstatter *et al.* (2006) that it models difficult decisions, not all decisions. It does not apply to pairs of alternatives in which one alternative dominates the other one (the delay probability is 0 or 1), and it also does not apply to "easy" problems in which the expected values are strikingly different (the delay probability is close to 0 or 1). It is of interest for future research to extend the PPH model so that it is applicable to extreme cases.

6.2. Dummy Model

The other two models have no underlying decision theories and are simply fitting the data. Therefore they probably have better data-fitting performance and could produce an upper limit on the goodness-of-fit so that the performance of the PPH model can be better assessed.

The first of the two is a dummy model with a large number of dummy variables to fit the choice proportion for each possible scenario. It was proposed by graduate students from the Department of Statistics at the University of Massachusetts Amherst.

The utility of the safe route is assigned as 0. The utility of the risky route is:

$$\begin{aligned} V(A) = ASC &+ \beta_1 * 1_{p=0.5} + \beta_2 * 1_{p=0.8} + \beta_3 * 1_{t_H=50} \\ &+\beta_4 * 1_{t_H=60} + \beta_5 * 1_{t_H=120} + \beta_6 * 1_{t_B=40} \\ &+\beta_7 * 1_{t_B=45} + \beta_8 * 1_{t_B=50} + \beta_9 * 1_{t_B=55}. \end{aligned} \tag{21}$$

ASC is a normally distributed random variable across subjects, following the same assumption in the PPH and RDEU models. 1_{event} is a 0-1 variables that is equal to 1 if the event is true and 0 otherwise.

The probability to choose the risky route over safe route is:

$$P(A) = \frac{1}{1 + exp(-V(A))} \tag{22}$$

6.3. EER Model

Another similar logit regression model was proposed by Ernan Haruvy and a winner in a choice prediction competition (Erev *et al.* , 2010). The prediction competition produced two data sets, one for estimation that was provided to the competing groups and one for prediction that was not provided. Submitted models were estimated based on the estimation set only, and they competed in terms predictive accuracies based on the prediction set calculated by the organizers.

The utility of the safe route is assigned as 0. The utility of the risky route is

$$V(A) = ASC + \beta_1 * t_H + \beta_2 * t_L + \beta_3 * t_B + \gamma_1 * p + \gamma_2 * EXP. \tag{23}$$

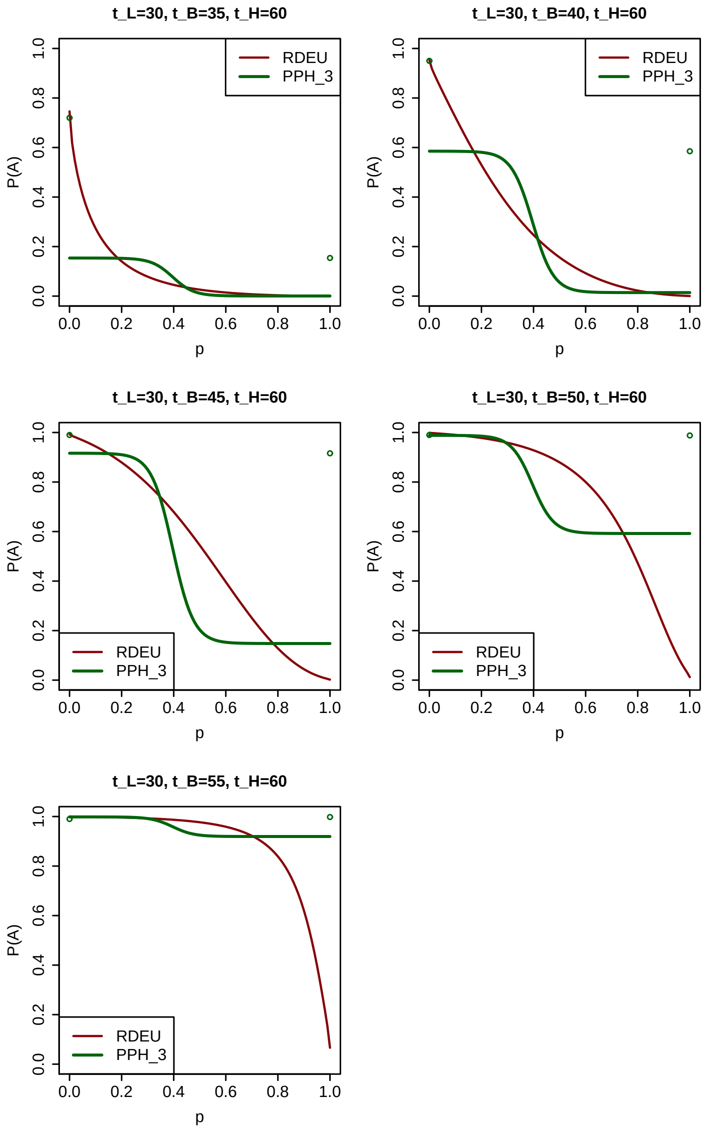

Fig. 4. Probabilities of Choosing the Risky Route as Functions of the Delay Probability

EXP is the expected travel time of the risky route. ASC is a normally distributed random variable across subjects. Eq. (22) can be applied here to calculate the probability to choose the risky route. The model is named EER after the first three authors of Erev *et al.* (2010).

7. Cross Validation

Cross validation is a method to assess different models' forecasting ability within the same data population. First, we generate 10 independent data sets from the original data set (1,767 observations of 74 subjects). Each time, 2/3 of the subjects' data are randomly chosen as the training set for model estimation, while the remaining 1/3 subjects' data are used as the validation set to test models' predictive performance. For example, in the 1st data set, 44 subjects with 1,054 observations are randomly chosen for model estimation. The remaining 30 subjects' 713 observations are used for validation.

7.1. Criteria

The mean squared distance (MSD) is used to compare the performance in addition to the adjusted rho squared. The squared distance (SD) is defined as the squared difference between the calculated probability to choose the risky route and the observed proportion of subjects that choose the risky route in one scenario. MSD is then an average of SDs over all scenarios.

7.2. Results

Table 3. Average Performance Measures of the Four Models

		Dummy	PPH_3	EER	RDEU
Average over 10 estimation data sets	FLL	-515.432	-535.013	-543.619	-561.170
	$\bar{\rho}^2$	0.355	0.336	0.328	0.306
	MSD	0.0075	0.0157	0.0189	0.0240
Average over 10 prediction data sets	FLL	-259.595	-268.095	-273.97	-281.778
	$\bar{\rho}^2$	0.338	0.327	0.318	0.299
	MSD	0.0136	0.0221	0.0256	0.0299
No. of Param.		11	7	5	5

Four competing models' estimation and prediction results in 10 data sets can be found in Tables 4 and 5. Average performance measures are presented in Table 3, and the ranking in terms of both estimation and prediction performance is (from best to worst): Dummy, PPH_3, EER and RDEU. In general we see a drop of performance level in the prediction set compared with the estimation set. The overall performance of PPH_3 model is better than that of the RDEU model. Surprisingly it is also better than the EER model, which is a winner of a choice prediction competition. It is not surprising that the dummy model gives an overall best performance, due to its data-fitting nature. These results suggest that the process modeling paradigm is a valid candidate for studying travel choice behavior under risk.

However, for a specific data set, the ranking does not necessarily hold. For example, in the 7th data set, the prediction FLL of the RDEU model (-258.51) is a little better than that of the PPH_3 model (-259.16) and EER model (-260.848). In the 9th data set, the prediction $\bar{\rho}^2$ of the dummy model (0.281) is a slightly worse than that of the PPH_3 model (0.290). In the 7th data set, the prediction MSD of the RDEU model (0.0208) is smaller than that of the EER model (0.0243). These confirm the notion that a model that best fits the data does not necessarily have the best prediction accuracy.

This cross validation is not a generalizability test - it simply tests the model robustness across subjects with the same set of scenarios. The general applicability of these four models can be ranked conceivably as: $RDEU > PPH > Dummy = EER$. RDEU is able to handle decisions of multiple alternatives with multiple outcomes and the estimated model can be applied to any other scenarios. PPH model is good at comparing two alternatives and the extension to multiple alternatives is feasible but not trivial. The dummy and EER models cannot be applied to scenarios other than those in the estimation set and are the most limited.

8. Conclusions and Future Directions

This study introduces a process model for studying route choice with risky travel times. A probabilistic version of the priority heuristic (Brandstatter *et al.* , 2006) is developed and estimated with an SP survey data set. The PPH model has superior estimation and prediction performance than a previously developed REDU model, which itself has been shown to be better than a number of other models, including the mean-standard deviation and expected utility models (Razo & Gao, 2013). We conclude that the process modeling paradigm is a valid candidate for studying travel behavior under risk. Note that the comparison is based on a particular dataset, specifically the subjects are mostly from the university student body, and therefore generalization to other situations should be made cautiously.

Table 4. Estimation Results of Four Models

		Dummy	PPH_3	EER	RDEU
1st data set estimation	FLL	-457.431	-482.961	-485.338	-499.542
	$\overline{\rho}^2$	0.359	0.329	0.329	0.309
	MSD	0.0072	0.0158	0.0181	0.0225
1st data set prediction	FLL	-320.256	-321.507	-333.477	-345.292
	$\overline{\rho}^2$	0.330	0.335	0.315	0.291
	MSD	0.0163	0.0195	0.0285	0.0339
2nd data set estimation	FLL	-522.492	-541.728	-556.421	-568.765
	$\overline{\rho}^2$	0.356	0.338	0.322	0.307
	MSD	0.0062	0.0137	0.0191	0.0225
2nd data set prediction	FLL	-250.761	-259.833	-260.195	-272.711
	$\overline{\rho}^2$	0.340	0.327	0.331	0.300
	MSD	0.0139	0.0219	0.0222	0.0298
3rd data set estimation	FLL	-559.192	-575.55	-578.36	-598.314
	$\overline{\rho}^2$	0.339	0.325	0.324	0.301
	MSD	0.0083	0.0151	0.0162	0.0219
3rd data set prediction	FLL	-214.849	-227.031	-238.878	-244.607
	$\overline{\rho}^2$	0.376	0.353	0.326	0.310
	MSD	0.0128	0.0297	0.0331	0.0363
4th data set estimation	FLL	-517.45	-541.252	-553.939	-569.253
	$\overline{\rho}^2$	0.361	0.338	0.325	0.306
	MSD	0.0066	0.0156	0.0203	0.0245
4th data set prediction	FLL	-256.699	-260.223	-263.172	-273.48
	$\overline{\rho}^2$	0.326	0.327	0.325	0.299
	MSD	0.0151	0.0229	0.0221	0.0286
5th data set estimation	FLL	-518.298	-538.256	-544.427	-559.936
	$\overline{\rho}^2$	0.347	0.328	0.323	0.303
	MSD	0.0074	0.0142	0.0178	0.0221
5th data set prediction	FLL	-255.333	-263.395	-272.097	-281.897
	$\overline{\rho}^2$	0.356	0.347	0.330	0.307
	MSD	0.0121	0.0249	0.0257	0.0314
6th data set estimation	FLL	-581.01	-597.095	-611.531	-626.835
	$\overline{\rho}^2$	0.349	0.336	0.322	0.305
	MSD	0.0075	0.0145	0.0190	0.0227
6th data set prediction	FLL	-192.864	-204.978	-204.939	-215.538
	$\overline{\rho}^2$	0.354	0.328	0.334	0.301
	MSD	0.0191	0.0302	0.0298	0.0379
No. of Param.		11	7	5	5

The PH is extended to multiple-outcome situations in (Brandstatter *et al.* , 2006) where decisions are based on maximum and minimum outcomes and their associated probabilities. Outcomes in the middle are not used in the decision making. In a travel choice context, it is more plausible to assume travelers recognize certain travel time categories (e.g., free flow, normal, congested, jam) rather than a continuous distribution of travel times. Observed travel time data are inherently discrete and thus support the categorization of travel time outcomes. Therefore the maximum and minimum travel times and their associated probabilities can be readily obtained, and the PPH model can be applied. Note that the assumption of only maximum and minimum outcomes are utilized need to be validated, and intermediate outcomes/probabilities might be added to the decision process.

The PH could also be extended to multiple-attribute situations. The PH effectively treats minimum out-

Table 5. Estimation Results of 4 Models. (Continued)

		Dummy	PPH_3	EER	RDEU
7th data set estimation	FLL	-526.821	-547.532	-556.79	-583.677
	$\bar{\rho}^2$	0.349	0.329	0.320	0.288
	MSD	0.0076	0.0182	0.0203	0.0286
7th data set prediction	FLL	-248.378	-259.16	-260.848	-258.51
	$\bar{\rho}^2$	0.349	0.332	0.333	0.339
	MSD	0.0149	0.0158	0.0243	0.0208
8th data set estimation	FLL	-464.62	-474.046	-483.669	-503.599
	$\bar{\rho}^2$	0.374	0.366	0.357	0.331
	MSD	0.0096	0.0149	0.0186	0.0245
8th data set prediction	FLL	-310.729	-329.422	-334.664	-339.654
	$\bar{\rho}^2$	0.308	0.277	0.270	0.259
	MSD	0.0081	0.0216	0.0257	0.0284
9th data set estimation	FLL	-513.691	-540.455	-550.812	-568.794
	$\bar{\rho}^2$	0.379	0.352	0.342	0.320
	MSD	0.0071	0.0178	0.0213	0.0263
9th data set prediction	FLL	-262.729	-263.146	-268.815	-275.883
	$\bar{\rho}^2$	0.281	0.290	0.280	0.262
	MSD	0.0131	0.0173	0.0212	0.0257
10th data set estimation	FLL	-493.319	-511.257	-514.907	-532.984
	$\bar{\rho}^2$	0.337	0.319	0.317	0.293
	MSD	0.0079	0.0174	0.0184	0.0242
10th data set prediction	FLL	-283.351	-292.253	-302.613	-310.206
	$\bar{\rho}^2$	0.365	0.355	0.337	0.320
	MSD	0.0110	0.0170	0.0232	0.0265
No. of Param.		11	7	5	5

come, maximum outcome and the associated probabilities as different attributes. The underlying assumption is that no systematic trade-off is made; rather, a series of comparisons over the different attributes are made and a choice is made if the difference of a certain attribute exceeds an aspiration level. In this sense, the PH follows the perspective of method of "elimination by aspect" proposed by (Tversky, 1972). The PH thus is well suited to handle multiple-attribute situations in travel choice, e.g, travel time and cost, and the order of comparison likely depends on the saliency or importance of an attribute.

The PPH model's deficiencies include the discontinuity and limited application in "easy" problems. Furthermore, for simplicity a decision with only two alternatives is investigated in this study. A decision with more than two alternatives is common in a travel decision context, and an extension of the PPH model is needed for its application in real life transportation problems.

We made a strong assumption in the PPH model that all subjects adopted the same comparing order for all scenarios during the survey. It is reasonable to suppose that subjects can make use of more than one comparing order, and the comparing orders vary across subjects and contexts. Moreover, we hypothesize that different decision strategies, such as RDEU and PPH might be used in different contexts and/or by different people. Therefore, it will be worthwhile to combine these two paradigms (and possibly others) and investigate which strategies are more likely to be used in different decision contexts. This combination could also potentially resolve the aforementioned problem of the PPH model not working well with "easy" problems and the discontinuity at extreme points.

All the probabilities in the survey were directly presented to subjects. In real life, however, travelers experience outcomes and delays and perceive event frequencies without explicit descriptions of probabilities. Research has shown a difference between decision from description and decision from experience (Rakow & Newell, 2010), for example, small probabilities are underweighted in decision from experience, in contrast to the overestimation in decision from description. Future research should focus on decision from experience as travelers learn about the uncertain environment through experience in most situations. The fact that travelers' choices collectively affect the network performance through congestion effects should also be adequately captured (Lu *et al.* , 2011; Ben-Elia *et al.* , 2013). There are indeed situations where a combination of both theories is desired, such as for modeling choice behavior when real-time traffic information describes event probabilities, and a traveler has the decision environment both experienced and described.

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