Modeling Strategic Route Choice and Real-Time Information Impacts in Stochastic and Time-Dependent Networks

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***Abstract*—This paper establishes a general framework to study the impacts of real-time information on users' routing decisions and the system cost in a stochastic time-dependent traffic network under a generalized equilibrium condition. Users are assumed to make strategic routing decisions, and the rule that maps a user's current state, including node, time and information, to a decision on the next node to take is defined as a routing policy. This definition allows for a wide variety of information accessibility situations, thus excluding the usually simplified assumptions such as either no information or full information. A user's choice set contains routing policies, rather than simple paths. A fixed-point problem formulation of the user equilibrium is given and an MSA (method of successive averages) heuristic is designed. Computational tests are carried out in a hypothetical network, where random incidents are the source of stochasticity. System costs derived from three models with different information accessibility situations are compared. The strategic route choices lead to shorter expected travel times at equilibrium. Smaller travel time variances are obtained as a byproduct. The value of real-time information is an increasing function of the incident probability. However it is not a monotonic function of the market penetration of information, which suggests that in designing a traveler information system or route guidance system, the information penetration needs to be chosen carefully to maximize benefits.**

***Index Terms*—Advanced Traveler Information System, Strategic route choice, Routing policy, Dynamic traffic assignment, Stochastic time-dependent network.**

I. Introduction

TRAFFIC networks are inherently uncertain with random disturbances such as incidents, bad weather, work zone and fluctuating demand, which creates significant congestion and unreliability. According to the 2009 Urban Mobility Report of the Texas Transportation Institute [1], such random disturbances account for more than fifty percent of all delay on the roads and the associated wasted fuel and emission. Meanwhile, with the fast development of sensor and telecommunication technologies, real-time information is increasingly available for travelers to make potentially better decisions in such an uncertain system. There are various mechanisms for providing real-time information differing in the spatial and temporal availability, the quality, and the format of information provided, e.g. variables message signs (VMS), websites, radio, traveler information call centers, in-vehicle communication systems connected to traffic management centers and/or other vehicles. It follows that in formulating traveler information related research problems, information should be an explicit part so that traditional simplified assumptions such as full or no information can be avoided.

Travelers' route choices in a stochastic network with real-time information are conceivably different from those in a deterministic network. It is generally believed that flexible route choices that adapt to network conditions will save travel time and enhance travel time reliability. For example, in a network with random incidents, if one simply sticks to his/her habitual route, s/he could be stuck in the incident link for a very long time. However, if adequate information is available about the incident and the traveler makes use of it and takes a detour, s/he can save travel time. The detour also helps reduce the prohibitively high travel time in an incident situation, and thus provides more reliable travel time.

We distinguish between two types of responses to real-time information: reactive and strategic. The literature saw a large body of studies on diversion or compliance under information (e.g., at a VMS), yet the modeled adaptation behavior is basically reactive meaning that the travelers decisions before arriving at a VMS do not consider the fact that the VMS will provide updated traffic conditions in the future. In reality, travelers might decide to acquire information as long as there is a reasonable prospect of reward from it [2]. Therefore the fact that a branch of the network has a VMS installed could make it more attractive even before the traveler arrives at the VMS location. A strategic traveler, in this case, is one that considers the availability of information in all later decision stages, not just the current one. Recent empirical studies with computer-based or driving simulator-based experiments show that travelers can make strategic route choices [3], [4].

The problem of optimal strategic routing decision making for an individual traveler has been studied by various researchers [5]–[13], and a recent literature review can be found in [12]. A general conclusion from the above studies is that in a flow-independent stochastic time-dependent (STD) network, an individual user's travel objective function value (e.g., expected time, expected schedule delay, variance) from following optimal strategic routing decisions is no higher than that from following an optimal fixed path.

After understanding how an individual traveler makes strategic routing decisions, another research question arises: what will be the network-level impact if many travelers make strategic routing decisions? The demand-supply interaction in

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a stochastic dynamic network needs to be captured to answer the question. This interaction in a deterministic network (with possible perception errors from the demand side) is captured by a conventional dynamic traffic assignment (DTA) model. A comprehensive literature review on dynamic traffic assignment can be found in [14].

We give a brief overview of the development of user equilibrium traffic assignment models regarding the modeling of stochasticity. Early developments addressed stochasticity in static traffic assignment methods. [15] established the concept of Stochastic User Equilibrium (SUE): in a SUE network no user believes he can improve his travel time by unilaterally changing routes. In other words, users have random perception errors of the true travel times. A "large sample" approximation is used such that the proportion of travelers that take a given path equals its probability to be chosen by an individual traveler. In a later paper by [16], the SUE problem with flow-dependent link costs is studied, with a proof of convergence. These early works are extended later in two directions of modeling stochasticity. The first direction is to abandon the "large sample" assumption and treat the flows (path or link) as random variables, either in an equilibrium or day-to-day stochastic process context [17]–[23]. The other direction is to add more sources of stochasticity by treating the underlying travel times, capacities and/or origin-desitnation (OD) trips as random [24]–[29].

This paper establishes a dynamic traffic assignment model where users make strategic route choices in response to real-time information in a stochastic time-dependent network with correlated link travel times. A routing policy is defined as a decision rule which specifies what node to take next out of the current node based the current time and real-time information, essentially a mapping from network states to decisions on next nodes. A routing policy can manifest itself as different paths depending on the underlying stochastic process that drives a traffic network. A path is purely topological and is a special case of a routing policy where any decision on the next node is not dependent on the current time or real-time information. A detailed description of routing policy is provided in Section III-A.

A similar concept is called "strategy" in the literature of transit assignment. A strategy consists in a rule that assigns to each decision node a set of outgoing arcs, probably sorted in preference order (the so-called "attractive lines"). A traveler follows the first available arc from the preference set. Early studies assume flow-independent probabilities in accessing attractive lines [30]–[32].

[33] proposed a strategic model for dynamic traffic assignment, as an extension to the static model studied by [34]. The models assume hard arc capacity constraints and holding of traffic, which is suitable for transit networks, but might not be for a traffic network. The randomness in travel times comes from the fact that if the demand for a given arc exceeds its hard capacity, then a given traveler might not be able to access the arc and has to use another arc. No external random events such as incidents or demand fluctuations are modeled. This could limit the model's ability to assess an advanced traveler information system (ATIS) which usually plays an important role when random events happen.

Strategic assignment studies in a traffic network, to the best of our knowledge, have been restricted in static networks, e.g., [35] and [36]. Both studies assume limited spatial stochastic dependency, and that users learn the realized conditions of outgoing links upon reaching a node. [35] assume the revealed information is on link travel times, and use a sequential Logit model similar to [37] to do the loading and a method of successive average (MSA) is used to solve the equilibrium problem. [36] assume that the revealed information is on link capacities, formulate a mathematical programming problem and use an adapted Frank-Wolfe solution algorithm.

The routing policy based DTA model in this paper is the first research endeavor on assigning strategic travelers onto congested, stochastic, and time-dependent traffic networks, by integrating three major components: optimal routing policy generation [12], routing policy choice model [38] and routing policy loading model. It contributes to the state of the art through the following novel features of the integrated DTA model:

- Users' choice sets comprise of routing policies rather than paths to model travelers' strategic route choices. The strategic choices are dependent on the time-of-day and information on realized flow-dependent link travel times, with possible spatial and/or temporal limitations. Thus the usual simplified assumptions of no information or full information is avoided. The encapsulation of real-time information in the definition of a routing policy makes the general framework naturally suitable to handle heterogeneous information access situations over users.
- Link-wise and time-wise stochastic dependencies of link travel times (which is prominent in reality) are modeled by treating link travel times as jointly distributed time-dependent random variables.
- The equilibrium is in terms of probabilistic distributions of time-dependent link travel times and path flows. In other words, traveler's route choices are stable under the equilibrium in the sense of a stable distribution, rather than stable, fixed values.

The paper is organized as follows. In Section II, we give an illustrative example to motivate the development of a routing policy-based DTA model. In Section III, the strategic user equilibrium problem is defined and a conceptual framework is introduced with a fixed-point formulation of the equilibrium problem. Section IV presents a method of successive average as a solution heuristic to the equilibrium problem. In Section V, computational tests are set up to study the behavior of the proposed model and compare it with models that do not consider strategic route choices.

Throughout the paper, a symbol with a $\sim$ over it is a random variable, while the same symbol without the $\sim$ is one specific realization of the random variable. A "support point" is defined as a distinct value (vector of values) that a discrete random variable (vector) can take. A probability mass function (PMF) of a random variable (vector) is thus a combination of support points and the associated probabilities.

II. An Illustrative Example

We use an illustrative example to explain some of the key concepts, give the motivation for the research and provide some insights into the routing policy based assignment problem. The example is simplified in the following aspects: 1) the network is static; 2) a link performance function is used to describe the congestion effect rather than a more realistic network loading model, such as the one used later in the computational tests; and 3) the information avaiable to travelers is not expressed as link travel time realizations as later formally defined, but the occurance of an incident for the sake of convenience. Note that an equivalent representation of the informatoin in terms of realized link travel times is available. We use the following notation:

a	:	index for link
i	:	index for routing policy (including path)
x_a	:	flow on link a
f_i	:	flow on routing policy (path) i
$C_a(x_a)$	:	link travel time as a function of flow on link a
e_i	:	expected travel time of routing policy i

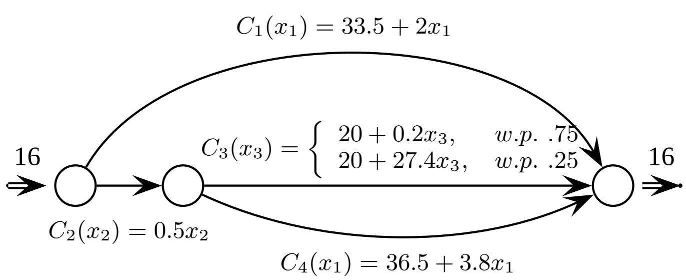

Fig. 1. An Illustrative Example

The network is shown in Figure 1 with four links and one origin-destination pair with a demand of 16. We can view link 1 as an arterial road, link 2 a connector to the highway link 3, and link 4 a detour to the highway. Three paths are available: path 1 composed of link 1; path 2 composed of links 2 and 3; and path 3 composed of links 2 and 4.

Link 3 has a random travel time function, whose coefficient is normally 0.2, but can be 27.4 one out of four times, due to for example an incident. Real-time Information is provided at the source node of link 3 to indicate whether an incident has occurred and allows travelers opportunities to divert to link 4.

The evaluation of the impacts of such an information system requires the modeling of travelers' route choices and the resulting network-wide performance under the incident with the real-time information. A common approach in the literature and practice is to do an equilibrium traffic assignment from the origin based on the three paths, and for travelers who reach the intermediate information node, a route switching model is then applied to redistribute the flows based on the incident and information. An implicit assumption of this approach is that travelers simply react to the information on the spot and cannot plan ahead. We will label this type of traffic assignment as reactive in the following discussion.

Depending on whether the potential incident is accounted for when the traffic assignment is performed at the origin, the predicted flow distributions from the reactive traffic assignment will be different. For the sake of illustration simplicity, we assume travelers are risk neutral and minimize their own mean travel times, thus at a user equilibrium, all used paths have the same and minimum mean travel time. If the incident is ignored at the origin, $f_1 = 0, f_2 = 16, f_3 = 0$ with $e_2 = 31.2$ that is no larger than the free-flow travel time on path 1 or 3. Note that the actual mean travel time should be higher than 31.2, since later on part of the 16 units of flow could experience an incident and part of them will divert to a longer route. If the incident is accounted for at the origin, $f_1 = 9, f_2 = 4, f_3 = 3$ with an equal mean travel time of 51.5 on all three paths.

Some travelers, however, could make strategic route choices by taking link 2 first and then link 3 if there is no incident and link 4 if there is one. They do not commit themselves to a particular path at the origin as the aforementioned reactive assignment model assumes. Such a strategic choice is termed as a routing policy in this paper. A path is a special routing policy, where the decision of taking link 3 or 4 is not affected by the information. We generalize the path-based user equilibrium condition to the routing policy-based one such that all used routing policies have the same and minimum mean travel time. There are five routing policies in the network. Routing policies 1 through 3 are paths 1 through 3, and routing policy 4 is the one discussed before. Routing policy 5 is the opposite of routing policy 4: first take link 2, and if there is an incident, take the incident link, otherwise take the detour. Routing policy 5 is conceivably not efficient. The equilibrium assignment result is $f_1 = 4, f_2 = 2, f_3 = 0, f_4 = 10, f_5 = 0$ with a mean travel time of 41.5 on all three used routing policies, among which two are fixed paths.

On the one hand, the reactive model overestimates flows on the highway and its detour if the incident is not accounted for at the origin (the network is treated as deterministic), as the potential degradation of the highway link is ignored. On the other hand, the reactive model underestimates flows on the highway and its detour if the incident is accounted for at the origin, as the potential travel time savings from being strategic are not considered. This example demonstrates the necessity to develop a traffic assignment model that explicitly considers travelers' strategic route choices in a stochastic network.

III. Modeling Framework and Fixed-Point Formulation

A. The Network and Routing Policy

Let $G = (N, A, T, \widetilde{C})$ be a stochastic time-dependent network. N is the set of nodes and A the set of directional links. A directional link is uniquely defined by a node pair (j, k) where j is the source node and k the sink node. T is the set of time periods $\{0, 1, ..., K-1\}$. Travel time on each link (j, k) during each time period t is a random variable $\widetilde{C}_{jk,t}$; the vector of all link travel time random variables at all time periods is denoted $\widetilde{C}$. Let $\{C^1, C^2, ..., C^R\}$ be the set of finite number of discrete support points of the link travel time joint distribution. The r-th support point has a probability p^r, and $\sum_{r=1}^{R} p^r = 1$. $C^r_{jk,t}$ is the travel time on link (j,k) at time t for the r-th support point. Beyond time period $K-1$, travel times are static, i.e. the travel time on link (j,k) at any time $t \geq K-1$ is equal to $C^r_{jk,K-1}, \forall r$.

We assume the traveler knows *a priori* the link travel time joint distribution. The decision is what node k to take next, based on the state $x = \{j,t,I\}$, where j is the node, t the time, and I the information. Information I is defined as a set of realized link travel times at a given time and node that are useful for making inferences about future link travel times. It represents the traveler's knowledge about the network conditions. This knowledge could be dependent on the time and location of the traveler, and therefore I should be regarded as $I(j,t)$. We omit the arguments of I because in this paper information is always associated with a state where j and t are well defined. An ideal case is when travelers have perfect information about the whole network, but generally the information is local, e.g. one learns the travel time realization of a link when s/he arrives at the node from which the link emanates.

The traveler continues the trip after making the choice. The next state is denoted as $\widetilde{y} = \{k, \widetilde{t'}, \widetilde{I'}\}$. The arrival time $\widetilde{t'}$ at node k is in general a random variable, because the travel time on link (j,k) at time t conditional on I is generally random. The next current information $\widetilde{I'}$ is also generally random, and includes realizations of link travel time random variables between time t and $\widetilde{t'}$, in addition to those already in I. For a given state and decision, probabilities of all possible next states can be evaluated from the link travel time joint distribution.

A trip can be characterized by a series of states that a traveler experiences, defined as a state chain. One can experience multiple possible state chains in the stochastic time-dependent network, and we have the following definition.

Definition III.1 (Routing Policy)**.** *A routing policy $\mu : x \mapsto k$ is a mapping from network states $x = \{j,t,I\}$ to decisions on next nodes k.*

An illustrative example is provided in the appendix. This definition indicates that route choices in a stochastic time-dependent network are not set *a priori*. Rather, they are closely related to network conditions, and this notion is critical in applications involving traveler information systems. For a given set of realizations of all link travel times (a deterministic network), a routing policy from a given origin at a given departure time manifests itself as a path. That is, a traveler always ends up taking a path every day, although this path is not known until the end of the trip, and s/he could end up taking different paths on different days.

Let $e_\mu(x)$ denote the expected travel time to the destination node d when the initial state is x and the routing policy μ is applied. Define $A(j)$ as the set of downstream nodes of node j, $\tilde{C}_{jk,t}|I$ as the travel time random variable for link (j,k) at time t conditional on information I, and $\tilde{I}'|I$ as the information random variable at the next node k and at time $t+\tilde{C}_{jk,t}|I$. We make the assumption that there exists at least one path from any node to the destination node d under any possible value of the link travel time vector.

Denote $Z(j,t)$ as the set of all possible information at node j and at time t. For $\forall j \in N-\{d\}, \forall t \in T, \forall I \in Z(j,t)$, let $e_{\mu^*}(x)$ and μ^* be the solutions of the following system of equations:

$$\begin{aligned} e_{\mu^*}(j,t,I) \quad &= \quad \min_{k\in A(j)}\{E_{\tilde{C}_{jk,t}}[\tilde{C}_{jk,t} \\ &\qquad +E_{\tilde{I}'}[e_{\mu^*}(k,t+\tilde{C}_{jk,t},\tilde{I}')|\tilde{C}_{jk,t}]|I]\} \end{aligned} \tag{1}$$

$$\begin{aligned} \mu^*(j,t,I) \quad &= \quad \arg\min_{k\in A(j)}\{E_{\tilde{C}_{jk,t}}[\tilde{C}_{jk,t} \\ &\qquad +E_{\tilde{I}'}[e_{\mu^*}(k,t+\tilde{C}_{jk,t},\tilde{I}')|\tilde{C}_{jk,t}]|I]\} \end{aligned} \tag{2}$$

with the boundary conditions: $e_{\mu^*}(d,t,I) = 0, \mu^*(d,t,I) = d, \forall t \in T, \forall I \in Z(d,t)$. Since $\tilde{I}'|I$ is dependent on $\tilde{C}_{jk,t}|I$, we first take the expectation over $\tilde{I}'|I$ with a given realization of $\tilde{C}_{jk,t}|I$ and then take the expectation over $\tilde{C}_{jk,t}|I$. As shown in [39], $e_{\mu^*}(x)$ and μ^* are optimal if I contains all links up to the current time, and are not necessarily optimal if I contains delayed, pre-trip, limited location or no link travel time realizations. Dynamic programming algorithms has been developed in [12] and [39] to solve the system of equations, where all the states $\{j,t,I\}$ need to be generated in pre-processing, and the backward induction starts from the static period where a conventional shortest path algorithm can be applied. The optimality conditions and solution algorithms can be extended to other link-additive optimization criteria, e.g., generalized cost as a weighted sum of travel time and monetary cost.

B. Modeling Framework

We present a conceptual framework for the routing policy based dynamic traffic assignment model as shown in Figure 2. The inputs to the DTA model are the stochastic dynamic demand and supply $(\widetilde{D}, \widetilde{S})$, represented by a joint distribution with R support points $\{(D^1,S^1),(D^2,S^2)...,(D^R,S^R)\}$, each of which has a probability $p^r, r = 1,...,R$. The demand is assumed to be independent of travel costs (inelastic). $D^r = \{D^r_{od,t}, t = 0,1,2,...,\forall \text{ OD pair}\{o,d\}\}$, where $D^r_{od,t}$ is the number of trips between origin o and destination d for departure time t in the r^{th} support point. $S^r = \{S^r_{jk,t}, t = 0,1,2,...,\forall\{j,k\} \in A\}$, where $S^r_{jk,t}$ is the capacity of link $\{j,k\}$ at time t in the r^{th} support point.

The outputs include two major parts: 1) an equilibrium joint distribution of link travel times $\widetilde{C}$, represented by R support points $\{C^1, C^2, \ldots, C^R\}$ and the associated probabilities $\{p^1, p^2, \ldots, p^R\}$, where C^r is a vector of time-dependent link travel times: $\{C^r_{jk,t}, \forall\{j,k\} \in A, \forall t\}$; and 2) the corresponding routing policy splits $f = \{f^i_{od,t}\}$, the fraction of the flow between OD pair $\{o,d\}$ at time t assigned to routing policy i. Other measures of effectiveness of interest, such as the distribution and summary statistics of link volumes, OD travel times, and path flows can be obtained from the equilibrium distribution of link travel times and routing policy splits. Note that the distributions of all relevant traffic random variables are discrete, since the definition of a routing policy is based on a discrete distribution of link travel times. It is an interesting future research question to work with continuous distributions.

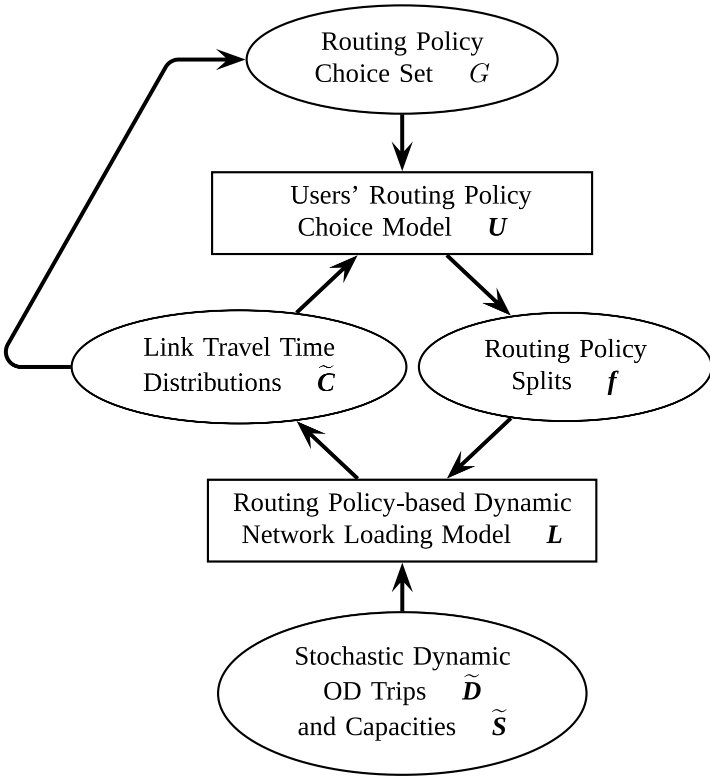

Fig. 2. A Conceptual Framework of Strategic Dynamic Traffic Assignment Model

C. *A Fixed-Point Formulation*

The routing policy-based DTA model comprises of two sub-models:

- the users' routing policy choice model, denoted as U, and
- the routing policy based dynamic network loading model, denoted as L.

The users' routing policy choice model takes as input a set of routing policies $G = \{\mu_1, ..., \mu_i, ...\}$, and a joint distribution of link travel times $\widetilde{C}$ generated by the routing policy-based dynamic network loading model. The link travel time distribution plays two roles. First, it provides the basis for the definition of a routing policy - note that a path is purely a topological concept by contrast. Secondly, the attributes of candidate routing policies, such as expected OD travel time and travel time standard deviation are evaluated based on the link travel time distribution. We further assume that the splits of OD flows among the candidate routing policies f are determined by a Policy-Size Logit model [38], [40], where the utility function includes the aforementioned attributes of routing policies that might affect travelers' decision making.

$$f = U(G(\widetilde{C}), \widetilde{C}) \tag{3}$$

We can classify the users according to various characteristics, as is typical in path-based DTA models. One important characteristic enabled by the use of routing policies is the classification by information access, which is embedded in the definition of a routing policy.

The demand is loaded onto the network according the routing policy splits, by the routing policy based dynamic network loading model. Note that we use "splits" rather than "flows" here: routing policy splits are deterministic, while routing policy flows could be stochastic when the demand is stochastic. For each support point of the random demand and supply, the network loading model outputs a single realization of the link travel time distribution. Therefore through the loading, we obtain the PMF of link travel times from the PMF of the demand and supply. Note that although the input demand/supply support points are distinct from each other, the output link travel time realizations are not necessarily distinct. This is why the word "realization" is used here, rather than support point. Nevertheless, the PMF of link travel times is still represented by the R realizations with the corresponding probabilities. The loading model is presented as follows:

$$\widetilde{C} = L(f, \widetilde{D}, \widetilde{S}) \tag{4}$$

Fixed-point formulations of user equilibrium are first developed by [41], and later extended by a number of studies [18], [32], [42]. Combining the above two mappings, a fixed-point formulation of the routing policy based equilibrium can be expressed in terms of the joint probabilistic distribution of all random link travel times $\widetilde{C}$:

$$\widetilde{C} = L(U(G(\widetilde{C}), \widetilde{C}), \widetilde{D}, \widetilde{S}). \tag{5}$$

The formulation states that a distribution of link travel times represents an equilibrium if, after travelers make decisions based on the link travel time distribution and the network then reacts to the decisions that have been made, the resulting joint distribution of link travel times is the same as the initial distribution, so that travelers have no reason to revise their decisions.

This paper does not examine the mathematical properties (existence and uniqueness) of this fixed-point problem, but rather uses it as a conceptual tool to understand the equilibrium and a guide in developing a heuristic algorithm to solve the routing policy-based DTA problem for the purpose of evaluating the impacts of real-time information. During the computational tests, solutions from the heuristic will be checked experimentally to determine whether fixed points are found. Mathematical properties of the fixed-point problem will be of interest for future research.

IV. Solution Algorithms

A. *The Dynamic Network Loading Model*

The dynamic network loading model takes as input the routing policies splits, and the PMF of random demand/supply, and outputs a PMF of link travel times. Two features distinguish the routing policy based stochastic dynamic network loader from a path based deterministic dynamic network loader.

First, the demand and supply can both be stochastic, and are described by a joint discrete set of support points $\{D^1, S^1, D^2, S^2, ..., D^R, S^R\}$ with associated probabilities $\{p^1, p^2, ..., p^R\}$. Stochastic factors in the supply side can generally be modeled as changes in capacities. For example, an incident may block several lanes, and thus reduce the capacity of the link. The starting time, the location, duration and severity of an incident can all be random. Therefore the stochastic time-dependent supply can be modeled using time-dependent random capacities. The modeling of stochastic demand is straightforward by treating the number of OD trips at each time period as a random variable. The stochastic demand/supply feature actually does not necessitate fundamental changes to a path-based deterministic loader. What is needed is the repeated runs of a loader for R times, for each support point of the random demand/supply, to generate the PMF of link travel times.

We consider the routing policy-based "single-loading" problem, where the demand and supply are set at a given support point, say D^r and S^r. The model we seek to build is an adaptive loader, denoted as AL and $C^r = AL(f, D^r, S^r)$, where f represents the routing policy splits. In a routing policy-based loading model, users follow routing policies, rather than non-adaptive paths. A routing policy is a decision rule based on real-time information. To determine the next link of routing policy μ for all links at time t, we must translate the currently available link travel times to a current information at time t used in the definition of routing policy μ. Then we move the users according to the computed next links, and repeat the above operation when they reach the end of the next links. Conceivably, the actual path taken by a user is not known until the end of the trip. This is different from a path-based (non-adaptive) network loader, where the sequence of links a traveler will take is known before the trip starts.

A conventional network loader can be revised to load routing policy flows by adding a module at each decision node to determine the next link for a routing policy. However in this paper we adopt an iterative approach that treats an existing path-based dynamic network loader as a black box. Recall that any routing policy will be manifested as a path with a given link travel time realization, represented by the function $h = V(f, C^r)$. At the end of the iterative process, we obtain path flows that if loaded onto the path-based loader will produce a realization of link travel times, based on which the routing policy flows will be manifested as the path flows just loaded. The statement for the iterative process is presented as follows, where NAL stands for a conventional non-adaptive loader.

Iterative Adaptive Dynamic Loading Algorithm with Support Point D^r, S^r

Step 0 (Initialization)
0.1: N = maximal number of iterations;
0.2: $n = 0$ (the iteration counter);
0.3: $C^r_{(n)}$ = free flow travel times;
0.4: $h_{(n)} = V(f, C^r_{(n)})$
0.5: $n = n + 1$
Step 1 (Main Loop)
1.1: Non-adaptive loader: $C^r_{(n)} = NAL(pf_{(n-1)}, D^r, S^r)$
1.2: Policy-to-path translation: $h = V(f, C^r_{(n)})$
1.3: Path flow update: $h_{(n)} = (1-\alpha)h_{(n-1)} + \alpha h$,
where $\alpha = 1/n$
Step 2 (Stopping Criterion)
If $n = N$, then $C^r = NAL(h_{(N)}, D^r, S^r)$ and STOP
Otherwise, $n = n + 1$, and go to Step 1

The iterative process will be stopped after a maximum number of iterations is reached. In Section V where computational tests are carried out, a convergence check is performed for this fixed point problem, ensuring the difference between path flows from two successive iterations is small enough.

Based on the single loading $C^r = AL(f, S^r, D^r)$, a generic algorithm for the routing policy based dynamic network loading model, $\widetilde{C} = L(f, \widetilde{D}, \widetilde{S})$, is proposed.

Routing Policy Based Dynamic Network Loading (DNL) Algorithm

Step 0 (Initialization)
0.1 $r = 1$ (the counter for the number of demand/supply PMF support points)
Step 1 (Loading)
1.1 Perform a single loading: $C^r = AL(f, S^r, D^r)$
1.2 $p^r =$ the probability associated with (S^r, D^r)
Step 2 (Stopping Criterion)
If $r = R$, STOP
Otherwise $r = r + 1$, and go to Step 1

For each support point of the demand/supply, we do a single loading to obtain one support point of link travel times. The corresponding probability is the same as that for the demand/supply support point. We repeat the process for all the support points and thus obtain the PMF of link travel times.

B. Solution Algorithm to the DTA Problem

We adopt a method of successive averages (MSA) to solve the fixed-point problem (Equation 5). At each iteration, the routing policy splits are updated by combining the splits from the current iteration and previous iterations. Since no proof of convergence is available at this moment, the method is heuristic for the DTA problem. The algorithm is presented as follows:

Routing Policy-Based DTA Heuristic

Step 0 (Initialization)
0.1: N = maximal number of iterations;
0.2: MSA counter $i = 1$
0.3: $C^r_{(0)}$ = free flow link travel times, $r = 1, ..., R$
0.4: Policy choice set $G_{(0)}$= { paths }
0.5: Policy splits $f_{(0)} = 0$
Step 1 (Main Loop)
1.1: Generate an optimal routing policy based on $\widetilde{C}_{(i-1)}$ for each OD pair
1.2: Choice set update $G_{(i)} = G_{(i-1)} \cup \{\mu_i\}$
1.3: Users' choice model $f' = U\left(G_{(i)}, \widetilde{C}_{(i-1)}\right)$
1.4: MSA update $f_{(i)} = (1-\alpha)f_{(i-1)} + \alpha f'$, where $\alpha = 1/i$

1.5: Loader $\widetilde{C}_{(i)} = L\left(f_{(i)}, \widetilde{D}, \widetilde{S}\right)$

Step 2 (Stopping Criterion)

If $i = N$, STOP

Otherwise, $i = i + 1$, and go to Step 1

A reasonable value for the maximum number of iterations will be obtained by running the heuristic for a sufficiently large number of iterations and observing the convergence property. Experimental results on this topic will be presented in the next section. In Step 0.4, we initialize the routing policy choice set to include all paths that would have been included in a choice set for a path-based DTA model. Note that for each OD pair and each departure time, there is a choice set, and the initialization is done for all choice sets. The subscripts for OD pair and departure time are omitted to avoid heavy notation. In Step 0.5, we initialize routing policy splits to be zeros for all OD pairs and departure times. These are infeasible splits which is just for the convenience of writing a formula in Step 1.4. $f_{(0)}$ is not taken into account in the MSA update, as when $i = 1$, its coefficient is zero.

In each MSA iteration, we generate a new PMF of link travel time distribution, based on the PMF of link travel time distribution from the last MSA iteration. The process is a sequential application of the optimal routing policy algorithm, the users' policy choice model, and the dynamic network loading model. The PMF of link travel time distribution from the last iteration is used in two places: first an optimal routing policy is generated based on this PMF and added to the choice set, then attributes of routing policies in the choice set are evaluated based on this PMF and used in the users' routing policy choice model. A Policy-Size Logit is used for the routing policy choice model [38], where the policy-size is analogous to path-size as used in a Logit path choice model [43] to correct for overlapping of alternatives. The major difference between a path and routing policy choice model is that the choice set in a routing policy choice model is composed of routing policies (paths are special cases of routing policies). Routing policy splits obtained from the choice model are then combined with those from the last iteration, multiplied by the demand and loaded into the network to generate the PMF of link travel times.

Note that in the users' choice model, attributes of an alternative routing policy are calculated based on the link travel time distribution from the last iteration $\widetilde{C}_{(i-1)}$, not based on the link travel time distribution that defines the specific routing policy. As a matter of fact, each routing policy in the choice set is defined over different link time distribution, as each of them is generated in different MSA iteration. In the calculation, we will encounter the problem that we might not be able to find an exact match between link travel times in a routing policy's definition and those at the current iteration. In this case, some measures that describe the similarity (or difference) between two vectors are used to make an approximate match. Detailed discussion of the problem can be found in [44].

V. Computational Tests

We present a specific computer implementation of the DTA model, and carry out computational tests. The objectives of the computational test are as follows:

- Study the convergence properties of the MSA heuristic;
- Study the behavior of the routing policy based DTA model through sensitivity analyses;
- Explore the effect of real-time information by comparing results from path-based model and routing policy-based model.

This section is organized as follows. First we present the three models that will be implemented: the base model, the path model and the routing policy model. The first two models are implemented as specializations of a general routing policy model. The test design is then described, including the network, probabilistic descriptions of the random supply, specifications of users' choice model and information access, and the simulation parameters. Results from the tests then follow. We give the emperical convergence performance of the MSA heuristic proposed in Section IV. Origin-destination travel times at equilibrium are compared across the three models both under typical support points and in expected values. Two sets of sensitivity analyses are provided with respect to incident probability and market penetration of real-time information.

A. The Three Models

The motivation for the routing policy based DTA model is to be able to model users' strategic choices in a network with random travel times. Some interesting questions then arise. What if the random disturbances to the network are ignored in calculating equilibrium? What if we do not model the strategic choices? The comparison of results from different models will give us a better understanding of the routing policy based model. We develop three models for comparison purposes as shown in Table I.

	Base	Path-Based	Policy-Based
Random Capacities	No	Yes	Yes
Adaptive choices	No	No	Yes

TABLE I
Three Equilibrium Models

In each column, we have one equilibrium model: base model, path-based model and policy-based model, respectively. We have two features listed: random capacities and real-time information, which specify respectively whether random capacities are considered and whether travelers are assumed to make strategic routing choices. We elaborate on the models one by one.

The first model is the base model with conventional assumptions of a DTA model in the literature. It is an equilibrium assignment model in a deterministic network, where stochastic disturbances are ignored, e.g. incidents are not considered at all. It corresponds to the case where users have no idea about the incident and just follow their habitual paths. After the equilibrium path flows are obtained, they are loaded onto the network with random capacities, and the resulting measures of effectiveness are calculated.

The second model is the path-based model with equilibrium in distribution. In this model, the distributions of random supply are known to the travelers, but no strategic choice behavior is assumed. The equilibrium is expressed as distributions, not average values of link travel times. Behaviorally both the base model and the path model assume travelers follow paths, yet the underlying network in the path model is random. By comparing results of the base model and the path model, we can study the value of *a priori* information on stochasticity of supply.

The third model is the routing policy-based model with equilibrium in distribution. Again the underlying network is random as in the path model. In addition, travelers make use of real-time information as compared to those in previous models, and therefore travelers' choice sets consist of routing policies rather than paths. By comparing results of the path model and the routing policy model, we can study the value of real-time information.

Computational tests on an additional model have also been conducted but not presented due to the space limit. This model assumes that a traveler takes into account real-time travel information at a given node and time, but the choice is myopic in the sense that a shortest path is sought, implicitly assuming no future diversions will take place. In all the tests, results from this myopic model are very close to those from the routing policy-based model, suggesting that the myopic nature does not impose significant disadvantage compared to the look-ahead strategies. More computational tests in larger and more complex network settings are desired to study the difference between the two types of adaptive models.

We implemented the three models using the C++ programming language in a Red Hat Linux environment. The MSA algorithm for a routing policy-based DTA presented in Section IV is a general description of all the three models. The base and path model algorithms are implemented by specializing the policy model algorithm.

The network loading model is based on the mesoscopic traffic simulator of DynaMIT [45]. We make the DynaMIT loader deterministic by suppressing all random factors in the loader. This is to make sure that the stochasticity of the model comes solely from random incidents, and thus test results across models are comparable.

B. The Test Network

We conduct computational tests on the simple hypothetical network shown in Figure 3. The network has six nodes and eight directed links.

We deal with one OD pair between nodes 0 and 5, and assume zero flows between any other OD pair. Four paths exist for OD pair (0,5) as shown in Figure 3, with real-time diversion possibilities at nodes 0, 1 and 2. The study period is from 6:30am to 8:00am. The time resolution is 1 minute for the optimal routing policy algorithm and users' behavior model. The DynaMIT supply simulator works at a finer resolution (5 sec) for the simulation, but the post-processed link (path) travel times are also by minute. Therefore we have 90 time periods in the tests.

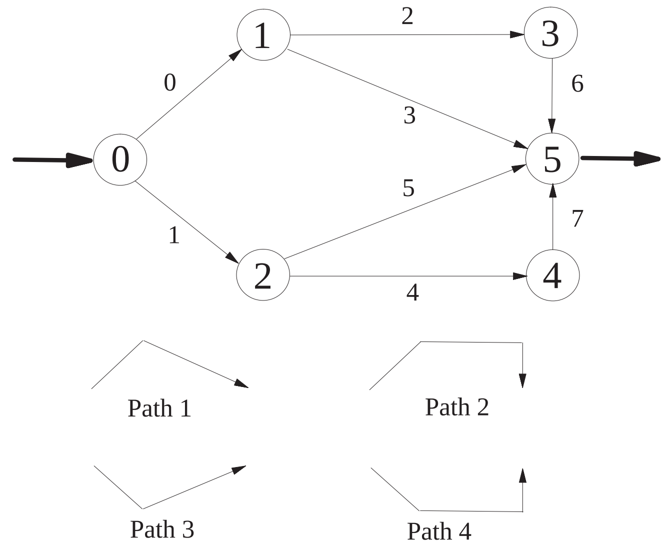

Fig. 3. Test Network

C. Random Incidents

There are random incidents in the network. An incident is defined by the link ID, start time, duration and capacity reduction factor. If an incident starts from 8:00am and lasts for 20 minutes with a capacity reduction factor 0.5 on link 0, the output capacity of link 0 will be $0.5 \times 1.1 = 0.55$ veh/link/sec from 8:00am to 8:20am, and will revert to the original value of 1.1 veh/link/sec from 8:20am on.

The random incident is set up according to the following rules:

- There is at most one incident during the study period for any given day;
- The probability of one incident in the network on any given day is p;
- The incident has a positive probability on links 0, 2, 3 and 6, but zero on links 1, 4, 5 and 7;
- The incident probability on a link is proportional to the link's length (for links 0, 2, 3 and 6);
- If an incident occurs on a link, the start time can be 6:30am, 6:40am, 6:50am, ..., 7:50am with equal probability;
- The duration of any incident is fixed as 10min, and the capacity reduction factor is fixed as 0.3.

Based on the above assumptions, an random incident can be defined by the joint distribution of the incident location and the start time. As there are 4 possible link locations and 9 possible start times, the joint distribution has 37 support points in total: 36 support points for the case of one incident, and an additional 1 support point for the case of no incident.

Note that all links travel times at all time periods are jointly distributed random variables due to traffic flow propagations, even though the incident could only occur at one link in a given support point.

D. Demand

We assume deterministic demand between OD pair (0, 5) with 2880 veh/hr between 6:30am and 7:00am and 4680 veh/hr between 7:00am and 8:00am.

Users are assumed to minimize expected travel time with perception errors. The coefficient of the expected travel time in the Policy-Size Logit model is negative with a large enough absolute value (-6.0) to approximate a fastest routing policy (path) choice situation. All users have perfect real-time information in the routing policy model, i.e. knowledge of travel time realizations on all links up to the current time (see Algorithm DOT-SPI in [12] for details of the perfect real-time information variant of the optimal routing policy problem). Obviously, users have no real-time information in the base model or the path model.

E. Results

We set 7:00-7:30 as the statistics collection period based on the following two considerations. First, an implicit assumption of the incident distribution is that there is zero probability of incidents outside the study period (6:30-8:00). This assumption is not realistic, as naturally random incidents can happen during any time of the day. In order to eliminate the effect of this assumption, we choose a statistics collection period such that incidents before 6:30 or after 8:00 have no effect on travelers departing during the period. Secondly, since OD travel times are needed in the equilibrium assignment, we require that the computation of OD travel times are correct, that is, travelers departing during the statistics collection period must leave the network before 8:00.

1) Convergence Study: Recall that a routing policy is defined over a link travel time distribution. At each MSA iteration, one routing policy is generated based on the link travel time distribution obtained in that iteration. However, as we start from free flow travel times, link travel time distributions at early iterations could be quite different from the equilibirum distribution, and routing policies defined over them can be misleading and make the MSA stuck in some non-equilibrium point. This problem does not exist in a path-based equilibrium assignment model, as a path is defined topologically and has nothing to do with travel times. An intuitive solution idea is that routing policies should be defined over distributions that are close to (if not equal to) the equilibrium values. We then adopt the reset method proposed by [46], which resets the MSA counter to 1 after x, $2x$, $3x$,... iterations for a chosen value of x. By resetting the MSA counter, we eliminate the effects of all early routing policies and effectively use the latest link travel time distribution as an initial solution to a new round of MSA iterations.

An inconsistency norm of link travel time for each support point is used to check for the convergence of the fixed-point problem, as an equilibrium in the distribution of link travel times is sought. Let C^r be the vector of all link travel times at all time intervals for support point r and $F(\widetilde{C})^r$ the r-th support point of the combined mapping of $\widetilde{C}$ as defined in Equation 5. The inconsistency norm is defined as

$$\parallel C^r - F(\widetilde{C})^r \parallel = \sqrt{\frac{\sum_{(j,k)\in A}\sum_{t\in T}(C^r_{jk,t} - F(\widetilde{C})^r_{jk,t})^2}{|A| \times |T|}}. \tag{6}$$

Figure 4 shows the convergence process of the routing policy model when $p = 0.9$. The inconsistency norm is plotted against the number of iterations. Each plot in the figure is one of the 37 discrete supporting points for the distribution of OD travel times. The x-axis represents departure time, while the y-axis represents OD travel time. The incident link ID and incident start time corresponding to a given support point are listed on the top of each graph. We see that after around 30 iterations, the inconsistency norms become stable and fairly small, which is an indication of achieving an approximate fixed point. The spikes at iterations 7 and 21 are due to the reset of the MSA counter.

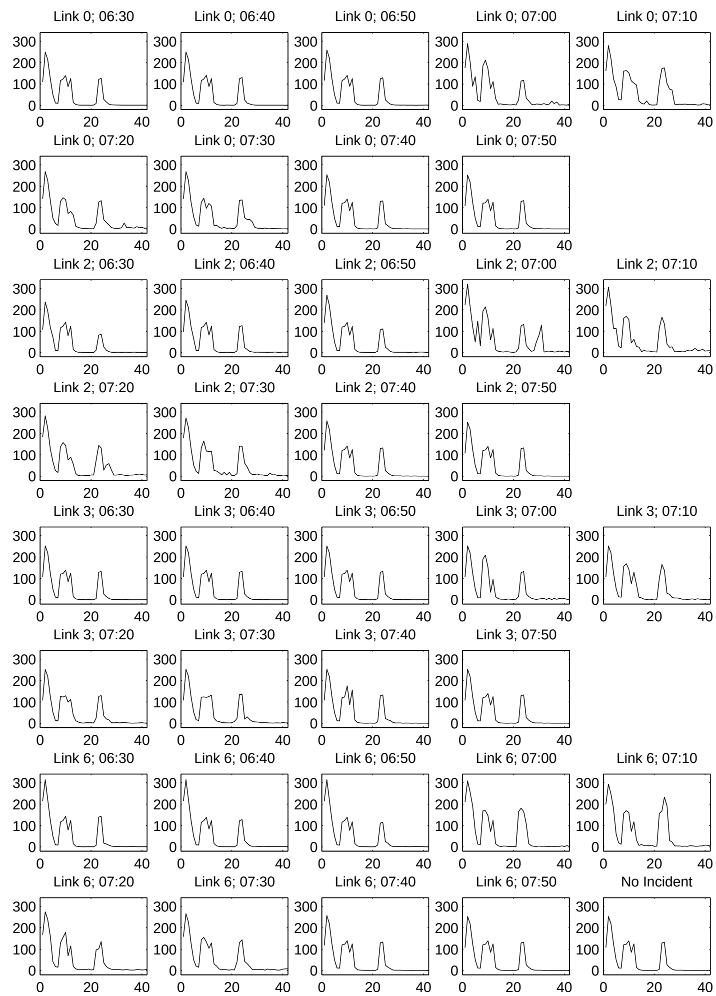

Fig. 4. Convergence of Routing Policy Model (X-Axis: Number of Iterations; Y-Axis: $\parallel C^r - F(\widetilde{C})^r \parallel$; $p = 0.9$; Each subgraph represents a support point and the title indicates the incident location and starting time with the exception of the last one.)

A commonly accepted measure of the distance from a user equilibrium is the relative gap, which is the relative difference between the total system costs under the current flow assignment and when all travelers follow the optimal routing policy based on the current travel costs. In general the relative gap is not good for our model, since perception errors are considered by utilizing a random utility choice model and the optimal routing policy perceived by a traveler is not necessarily the same as the "true" one based on actual link costs.

2) Solution Discussion: We discuss the solutions of the three models and compare them when appropriate. Note that we focus on the statistics collection period 7:00am through 7:30am, although statistics for all time intervals are presented. Special caution should be taken when reading statistics close to 8:00am, when unfinished trips make the calculation of travel times unreliable.

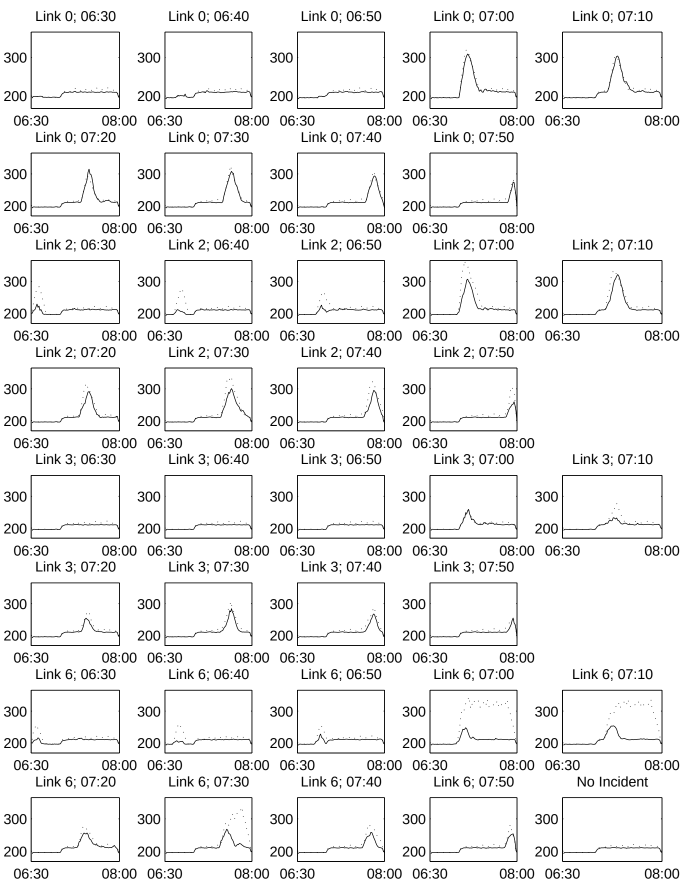

Fig. 5. OD Travel Time Distributions of Path Model (dotted) and Routing Policy Model (solid) (X-Axis: Departure Time; Y-Axis: OD Travel Time (sec); $p = 0.9$; Each subgraph represents a support point and the title indicates the incident location and starting time with the exception of the last one.)

We present the equilibrium OD travel time distribution of the path model (dashed lines) and routing policy model (solid lines) together in Figure 5. Generally, the routing policy model gives lower OD travel time, and the savings are quite outstanding in some cases (e.g. when incidents are on link 6 and start from 7:00 and 7:10). This can be attributed to the flexibility gained through strategic routing where the realized link travel times tells the travelers about the development of the incident and travelers plan ahead for such type of information.

Figure 6 gives time-dependent path flow distributions for path 2. We omit the presentation of other path flow distributions for the sake of brievity. A very distinctive feature is the decrease of the path flow around the duration of the incident in most support points. As we arrange the graphs by the start time of incident, we can see a moving "pit" in the path flow. This is more intelligent than fixed path flows in the path model, as path 2 could be severely affected by an incident. Routing policy flows are deterministic, however a routing policy will manifest itself as different paths in different support points, and thus path flows are random and we can talk about path flow distributions.

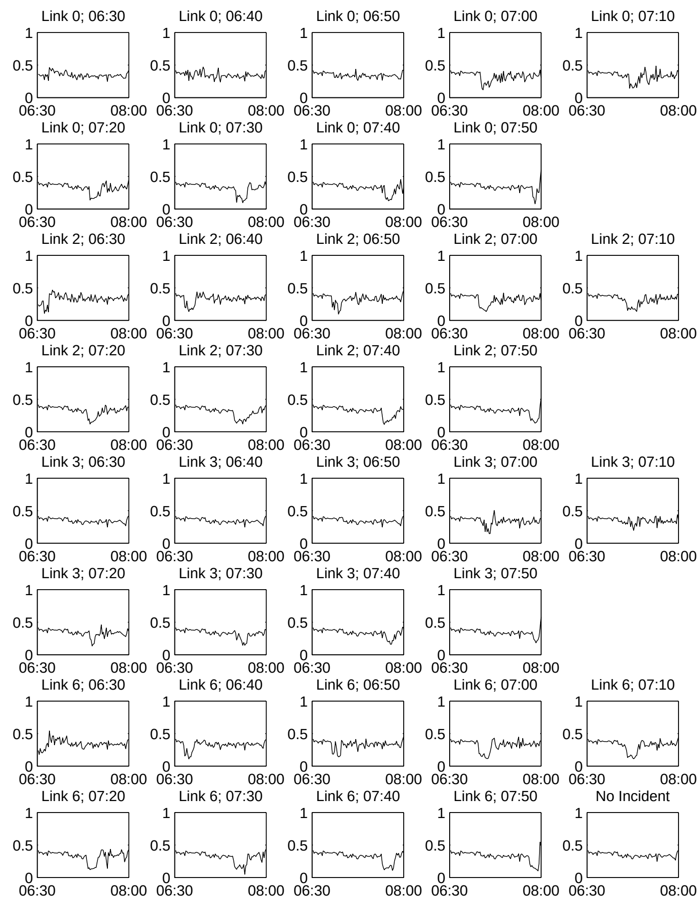

Fig. 6. Path 2 Flow Distribution of Routing Policy Model(X-Axis: Departure Time; Y-Axis: Path Share; $p = 0.9$; Each subgraph represents a support point and the title indicates the incident location and starting time with the exception of the last one.)

Next we compare expected OD travel times from the three models in Figure 7. Expected OD travel time is the major measure of effectiveness in our tests. The path model gives lower expected OD travel times than the base model, and the routing policy model provides further travel time savings. Figure 8 gives the time-dependent OD travel time standard deviations. Although travelers are minimizing expected travel time only, their travel time variances are also reduced by taking strategic routing choices. This is due to the fact that their travel times are reduced in incident scenarios, and thus smoother across support points.

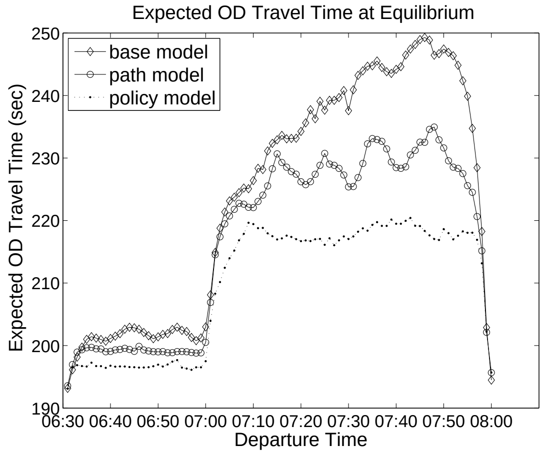

Fig. 7. Expected OD Travel Time at Equilibrium ($p = 0.9$)

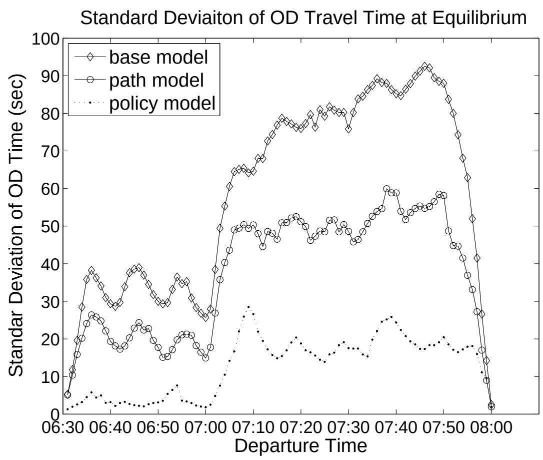

Fig. 8. OD Travel Time Standard Deviations at Equilibrium ($p = 0.9$)

3) Sensitivity Analysis: We are interested in the behavior of the models when we vary the incident probability. We define a single measure of effectiveness (MOE) to be compared in the sensitivity analysis, which is the expected OD travel time averaged over the 30 departure times in the statistics collection period.

We vary p from 0 to 1.0 by a step size of 0.1. The result is plotted in Figure 9. For each of the models, the average expected OD travel time increases as incident probability increases, but different model has different increasing rate. This increasing function seems intuitively correct, as a more likely incident increases the probability that a network is congested, and thus a higher expected travel time. We note that the routing policy model gives a higher value for $p = 0.9$ (216.06) than for $p = 1.0$ (216.00). We believe that this difference is too small to be significant, and is likely due to numerical reasons.

The relationship for the base model is linear. The explanation is as follows. The path flows are the same for all incident probabilities, since the base model does not consider

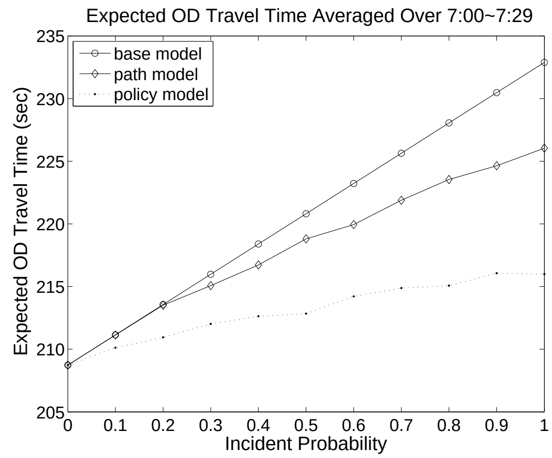

Fig. 9. Average Expected OD Travel Times as Functions of Incident Probability

incidents at all. The OD travel time for each support point is calculated, and a weighted average is taken to obtain the expected OD travel time. The weight is the probability of a support point which is a linear function of p. Therefore the expected OD travel time is also a linear function of p. While in the other two models, random incidents are considered in the equilibrium process and equilibrium path (routing policy) flows differ when p differs. Therefore the relationship is in general non-linear.

The path model gives less expected travel time than the base model, and the routing policy model gives less expected travel time than the path model. The savings (path over base, and routing policy over path) increase as the incident probability increases, both in absolute value and in percentage. The relative saving of the path model over the base model is in the range of $0 \sim 2.9\%$, and the relative saving of strategic models over the path model is in the range of $0 \sim 4.4\%$. This increasing function suggests that values of both *a priori* and real-time information are more evident when traffic conditions are worse. This could be reasonable in reality when traffic conditions without incident are not too congested, as then there is enough room for diversion. This is actually the setting of our tests, as traffic is almost in free flow state with no incident. We expect that when a network is already quite congested without incident, this function might become flat after a certain value of p.

Next we carry out sensitivity analysis with respect to market penetration of real-time information. For a given penetration k, which is a value between 0 and 100%, we assign k of the demand to take minimum expected travel time routing policies, while the remaining $1 - k$ of the demand to take minimum expected travel time paths. Equilibrium is sought by an MSA heuristic that updates the path splits and policy splits simultaneously.

We have the results for $p = 0.1$ in Figure 10. The average system expected OD travel time is at its largest value when market penetration of real-time information is zero. At that time, if one traveler is intelligent enough and take a routing policy rather than a path, he/she can save travel time. More and more of them find the benefits of real-time information, and they gain travel time savings and thus bring down the average expected travel time. However, in a congested traffic network, the changing of users' behavior changes the network-wide traffic conditions through interaction between supply and demand. As seen from the figure, the reduction in expected travel time becomes less evident when penetration goes from 20% to 40% and from 40% to 60%. Later on, higher penetrations of real-time information actually result in increases in the expected travel time. We then conclude that the total system travel time reduction resulted from travelers adapting to real-time information is not a monotonic function of the market penetration of real-time information. Furthermore, despite the varying effect of real-time information, travel time reductions are always positive with real-time information, compared to the no-real-time-information case. Results for $p = 0.5$ and $p = 0.9$ also show the non-monotonicity of benefits of real-time information. Note that these results are valid for the specific settings in this paper, and caution should be taken in generalizing them.

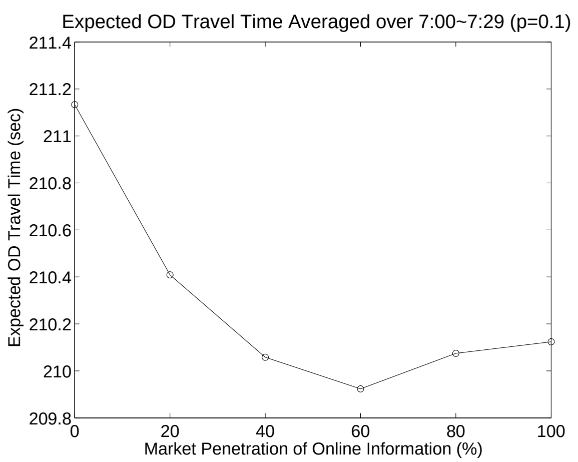

Fig. 10. Average Expected OD Travel Time as a Function of Market Penetration ($p = 0.1$)

VI. Conclusions and Future Directions

The development of traffic models in the presence of traveler information is pertinent and timely given the advent of ATIS. This paper establishes a user equilibrium dynamic traffic assignment model where users make strategic routing choices in a stochastic time-dependent network. Travel times on all links at all discrete times are jointly distributed random variables whose distribution is endogenous to the DTA model. A traveler is assumed to know *a priori* the joint distribution, and during a trip s/he has access to real-time information on realized link travel times based on which a decision on what node to take next is made. Such a traveler is said to follow a routing policy, defined as a mapping from all possible states to next nodes to take, where a state is a triple of node, time and real-time information. A simple path is a special case of a routing policy, where the decision on next node is not dependent on time or real-time information.

A general framework of the routing policy-based DTA model is established and a fixed-point problem formulation of the user equilibrium is given. The DTA model takes as input a discrete distribution of random dynamic origin-destination trips and/or supply variables, and generates equilibrium routing policy splits and an equilibrium discrete distribution of link travel time distribution. The random supply includes incidents, work zones, breakdowns and bad weather, and can be expressed through random changes of network parameters, e.g. link capacity reduction. The DTA model has two components: the routing policy choice model and the routing policy based dynamic network loading model.

An MSA heuristic is designed to solve the fixed-point problem of routing policy based DTA. Computational tests are carried out in a hypothetical network where random incidents are the source of stochasticity. The heuristic converges satisfactorily in the test network under the proposed test settings. The strategic feature in the routing policy based model leads to shorter expected travel times at equilibrium. Smaller travel time variances are obtained as a byproduct. The value of real-time information is an increasing function of the incident probability when the traffic with no incident is relatively light. However it is not a monotonic function of the market penetration of real-time information, which suggests that in designing a traveler information system or route guidance system, the information penetration needs to be chosen carefully to maximize benefits.

For future research, tests on a real-world network with actual data are of great interest. Computational efficiency is a potential concern for real-world applications, with the large number of feasible routing policies and support points and a doubly iterative solution process (DTA and DNL). However the problem is not as serious as it seems. First, no explicit routing policy enumeration is needed, as the choice set is expanded by adding the optimal routing policy from each MSA iteration. Secondly, the number of support points (and thus the number of simulation runs in the main DNL loop) can be controlled by the users to make a good trade-off between modeling accuracy and tractability, and needs not increase exponentially with the network size. Thirdly, the iterative process to load routing policy flows is not necessary, and only a short-cut to make use of currently available non-adaptive network loaders. Conceptually a conventional chronological network loader can be revised to load routing policy flows by adding a module that determines the next link for a routing policy at each node, based on realized travel times from loaded flows up to the current time.

The model and solution algorithm can be applied directly to multiple OD scenarios, even though a single OD is assumed in the current case study. It is hypothesized that the optimal penetration rate for a given OD pair will decrease with more OD pairs, as the advantage of a detour in case of disruptions will decrease if the detour is already saturated by flows from other OD pairs.

Tests with users classified by their real-time information accessibilities are also of great interest, as in reality users are heterogeneous. Note that a path is a special type of routing policy, where the next node is fixed regardless of the time and real-time information. In this sense, the encapsulation of real-time information in the definition of a routing policy enables the general framework to handle as many information access situations as possible: each additional user class only requires one more routing policy definition.

The capability of the routing policy-based DTA to deal with a wide range of information access situations also allows for the systematic study of information system design problems, e.g., the location [47], content, coverage and dissemination frequency of information.

The routing policy as defined in this paper uses every node as a potential decision node, while in reality a traveler has limited cognitive capacity and cannot store and process information in such a fine resolution. It is found in a recent driving simulator study [4] that travelers can plan ahead for information downstream and furthermore, are less strategic in more complex networks. Conceivably in a real network, only a few decision nodes exist for a traveler, e.g., a diversion point between two major highways. The current DTA framework can accommodate this more realistic definition of routing policies, however more work is needed to design algorithms to calculate optimal routing policies with limited decision points. Furthermore, travelers might not be able to accurately perceive travel times, and thus routing policies defined on coarser states, e.g., levels of service, could be used under the same DTA framework.

Appendix
An Illustrative Example for Routing Policy Definition

Figure 11 shows a network with three nodes, three links and three time periods. All possible values of travel times are in Table II. Each of the eight support points has a probability of 1/8.

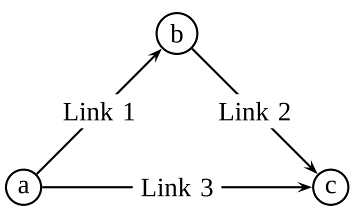

Fig. 11. A Small Network

We describe the routing decision process as follows. Assume the traveler has perfect real-time information that includes all link travel time realizations up to the current time, regardless of the current node. Therefore the information I at time t would be one of the joint realizations of $\widetilde{C}_{1,0}$, $\widetilde{C}_{2,0}$, $\widetilde{C}_{3,0}$, $\ldots$, $\widetilde{C}_{1,t}$, $\widetilde{C}_{2,t}$, and $\widetilde{C}_{3,t}$. Note that we use a single subscript to denote a link rather than a pair of nodes for the sake of simplicity. There are three possible I at time 0: (1, 1, 1), (1, 1, 4) and (1, 1, 3); six possible I at time 1: (1, 1, 1, 1, 2, 3), (1, 1, 1, 1, 1, 2), (1, 1, 4, 1, 2, 2), (1, 1, 4, 1, 1, 1), (1, 1, 3, 1, 2, 3) and (1, 1, 3, 1, 1, 2); and eight possible I at time 2 and beyond: $C^1, C^2, \ldots, C^8$. With three nodes this results in $(3+6+8)\times 3 = 51$ possible states. A routing policy is then a mapping from the fifty-one possible states to feasible nodes to take next.

t	Link	C^1	C^2	C^3	C^4	C^5	C^6	C^7	C^8
0	1	1	1	1	1	1	1	1	1
	2	1	1	1	1	1	1	1	1
	3	1	1	1	4	4	4	3	3
1	1	1	1	1	1	1	1	1	1
	2	2	2	1	2	2	1	2	1
	3	3	3	2	2	2	1	3	2
≥ 2	1	1	1	2	1	1	1	2	2
	2	1	2	1	1	1	1	2	1
	3	3	2	2	3	4	3	5	2
p^r		1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8

TABLE II
Joint Distribution of Link Travel Times

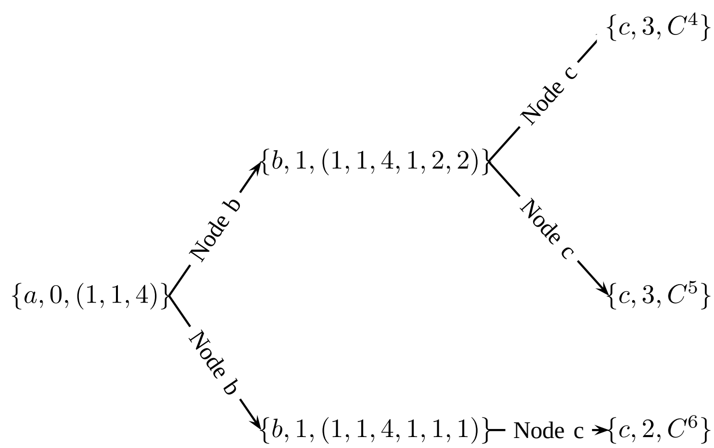

Fig. 12. State Chains for a Routing Policy

Consider a traveler traveling from nodes a to c following a routing policy. Suppose the initial state is $\{a,0,(1,1,4)\}$ as shown in Figure 12. We can see from the joint distribution table that the network could be C^4, C^5, or C^6. Suppose the routing policy maps the initial state to node b, upon the arrival at which the traveler could be in two possible states: $y = \{b,t',I'\} = \{b,1,(1,1,4,1,2,2)\}$ or $\{b,1,(1,1,4,1,1,1)\}$. The only choice at node B is node c, and after that the traveler arrives at the destination. The ending states are also random. From the state $\{b,1,(1,1,4,1,2,2)\}$, the traveler could end up at state $\{c,3,C^4\}$ or $\{c,3,C^5\}$. From the state $\{b,1,(1,1,4,1,1,1)\}$, the traveler could end up at state $\{c,2,C^6\}$. The OD travel time is a random variable: 3 with probability (w.p.) 2/3, and 1 w.p. 1/3.

References

[1] D. Schrank and T. Lomax, "The 2009 annual urban mobility report," Texas Transportation Institute ($http://mobility.tamu.edu/$), Tech. Rep., 2009.

[2] P. Bonsall, "Traveler behavior: decision-making in an unpredictable world," *Journal of Intelligent Transportation Systems*, vol. 8, pp. 45–60, 2004.

[3] M. Razo and S. Gao, "Strategic thinking and risk attitudes in route choice: Stated preference approach," *Transportation Research Record: Journal of the Transportation Research Board*, vol. 2156, pp. 28–35, 2010.

[4] H. Tian, S. Gao, D. L. Fisher, and B. Post, "Route choice behavior in a driving simulator with real-time information," in *90th TRB Annual Meeting DVD*, 2011.
[5] R. W. Hall, "The fastest path through a network with random time-dependent travel times," *Transportation Science*, vol. 20, no. 3, pp. 91–101, 1986.
[6] G. Polychronopoulos and J. N. Tsitsiklis, "Stochastic shortest path problems with recourse," *Networks*, vol. 27, pp. 133–143, 1996.
[7] P. Marcotte and S. Nguyen, Eds., *Equilibrium and Advanced Transportation Modeling*. Kluwer Academic Publishers, 1998, ch. 9.
[8] D. Pretolani, "A directed hyperpath model for random time dependent shortest paths," *European Journal of Operational Research*, vol. 123, pp. 315–324, 2000.
[9] E. D. Miller-Hooks and H. S. Mahmassani, "Least expected time paths in stochastic, time-varying transportation networks," *Transportation Science*, vol. 34, no. 2, pp. 198–215, 2000.
[10] E. Miller-Hooks, "Adaptive least-expected time paths in stochastic, time-varying transportation and data networks," *Networks*, vol. 37, no. 1, pp. 35–52, 2001.
[11] S. T. Waller and A. K. Ziliaskopoulos, "On the online shortest path problem with limited arc cost dependencies," *Networks*, vol. 40, no. 4, pp. 216–227, 2002.
[12] S. Gao and I. Chabini, "Optimal routing policy problems in stochastic time-dependent networks," *Transportation Research Part B*, vol. 40, no. 2, pp. 93–122, 2006.
[13] R. Claes, T. Holvoet, and D. Weyns, "A decentralized approach for anticipatory vehicle routing using delegate multiagent systems," *Intelligent Transportation Systems, IEEE Transactions on*, vol. 12, no. 2, pp. 364 –373, 2011.
[14] S. Peeta and A. K. Ziliaskopoulos, "Foundations of dynamic traffic assignment: the past, the present and the future," *Networks and Spatial Economics*, vol. 1, no. 3-4, pp. 233–265, 2001.
[15] C. F. Daganzo and Y. Sheffi, "On stochastic models of traffic assignment," *Transportation Science*, vol. 11, pp. 253–274, 1977.
[16] Y. Sheffi and W. B. Powell, "An algorithm for the equilibrium assignment problem with random link times," *Networks*, vol. 12, pp. 191–207, 1982.
[17] E. Cascetta, "A stochastic process approach to the analysis of temporal dynamics in transportation networks," *Transportation Research Part B*, vol. 23, no. 1, pp. 1–17, 1989.
[18] G. E. Cantarella and E. Cascetta, "Dynamic processes and equilibrium in transportation networks: Toward a unifying theory," *Transportation Science*, vol. 29, no. 4, pp. 305–329, 1995.
[19] G. A. Davis and N. L. Nihan, "Large population approximations of a general stochastic traffic assignment model," *Operations Research*, vol. 41, no. 1, pp. 169–178, 1993.
[20] M. L. Hazelton, S. Lee, and J. W. Polak, "Stationary states in stochastic process models of traffic assignment: A Markov Chain Monte Carlo approach," in *Proceedings of the 13th symposium on transportation and traffic theory*, J.-B. Lesort, Ed. Oxford: Pergamon, 1996, pp. 341–359.
[21] M. Hazelton, "Some remarks on stochastic user equilibrium," *Transportation Research Part B*, vol. 32, no. 2, pp. 101–108, 1998.
[22] D. Watling, "A second order stochastic network equilibrium model, I: Theoretical foundation," *Transportation Science*, vol. 36, no. 2, pp. 149–166, May 2002.
[23] ——, "A second order stochastic network equilibrium model, II: Solution method and numerical experiments," *Transportation Science*, vol. 36, no. 2, pp. 167–183, May 2002.
[24] P. Mirchandani and H. Soroush, "Generalized traffic equilibrium with probabilistic travel times and perceptions," *Transportation Science*, vol. 21, no. 3, pp. 133–152, 1987.
[25] H. X. Liu, X. Ban, B. Ran, and P. Mirchandani, "Analytical dynamic traffic assignment model with probabilistic travel times and perceptions," *Transportation Research Record*, vol. 1708, pp. 125–133, 2002.
[26] H. Lo and Y. K. Tung, "Network with degradable links: Capacity analysis and design," *Transportation Research Part B*, vol. 37, no. 4, pp. 345–363, 2003.
[27] A. Chen and Z. Zhou, "The alpha-reliable mean-excess traffic equilibriuim model with stochastic travel times," *Transportation Research Part B*, vol. 44, pp. 493–513, 2009.
[28] R. D. Connors and A. Sumalee, "A network equilibrium model with travellers' perception of stochastic travel times," *Transportation Research Part B*, vol. 43, pp. 614–624, 2009.
[29] A. Unnikrishnan, S. D. Boyles, and S. T. Waller, "User equilibrium under uncertain demand: Formulation, properties, and solution methods," *Transportation Research Part B*, Under review.
[30] S. Nguyen and S. Pallottino, "Equilibrium traffic assignment for large scale transit network," *European Journal of Operational Research*, vol. 37, no. 2, pp. 176–186, 1988.
[31] H. Spiess and M. Florian, "Optimal strategies: a new assignment model for transit networks," *Transportation Research Part B*, vol. 23, no. 2, pp. 83–102, 1989.
[32] G. E. Cantarella, "A general fixed-point approach to multimode multi-user equilibrium assignment with elastic demand," *Transportation Science*, vol. 31, no. 2, pp. 107–128, May 1997.
[33] Y. Hamdouch, P. Marcotte, and S. Nguyen, "A strategic model for dynamic traffic assignment," *Networks and Spatial Economics*, vol. 4, pp. 291–315, 2004.
[34] P. Marcotte, S. Nguyen, and A. Schoeb, "A strategic flow model of traffic assignment in static capacitated networks," *Operations Research*, vol. 52, no. 2, pp. 191–212, 2004.
[35] S. Ukkusuri and G. Patil, "Exploring user behavior in online network equilibrium problems," in *Proceedings of the 11th International Conference on Travel Behavior Research*, Kyoto, Japan, August 2006.
[36] A. Unnikrishnan and T. Waller, "User equilibrium with recourse," *Networks and Spatial Economics*, vol. 9, pp. 575–593, 2009.
[37] R. B. Dial, "A probabilistic multipath traffic assignment model which obviates path enumeration," *Transportation Research*, vol. 5, pp. 83–111, 1971.
[38] S. Gao, E. Frejinger, and M. E. Ben-Akiva, "Adaptive route choice models in stochastic time-dependent networks," *Transportation Research Record*, vol. 2156, pp. 28–35, 2008.
[39] S. Gao and H. Huang, "Real-time traveler information for optimal adaptive routing in stochastic time-dependent networks," *Transportation Research Part C*, 2011, doi: 10.1016/j.trc.2011.09.007.
[40] S. Gao, E. Frejinger, and M. Ben-Akiva, "Adaptive route choices in risky traffic networks: A prospect theory approach," *Transportation Research Part C*, vol. 18, no. 5, pp. 724–740, 2010.
[41] C. F. Daganzo, "Stochastic network equilibrium with multiple vehicle types and asymmetric, indefinite link cost jacobians," *Transportation Science*, vol. 17, pp. 282–300, 1983.
[42] D. E. Kaufman, R. L. Smith, and K. E. Wunderlich, "User-equilibrium properties of fixed points in dynamic traffic assignment," *Transportation Research Part C*, vol. 6, pp. 1–16, 1998.
[43] M. E. Ben-Akiva and M. Bierlaire, *Transportation Science Handbook*. Kluwer Academic Publishers, 1999, ch. Discrete choice methods and their applications to short term travel decisions.
[44] S. Gao, "Adaptive routing and traffic assignment in stochastic time-dependent networks," Ph.D. dissertation, Massachusetts Institute of Technology, Feb. 2005.
[45] M. E. Ben-Akiva, M. Bierlaire, J. Bottom, H. Koutsopoulos, and R. Mishalani, "Development of a route guidance generation system for real-time application," in *Proceedings of the 8th IFAC Symposium on Transportation Systems*, M. Papageorgiou and A. Pouliezos, Eds., Chania, Greece, June 1997, pp. 433–438.
[46] E. Cascetta and M. N. Postorino, "Fixed point approaches to the estimation of o/d matrices using traffic counts on congested networks," *Transportation Science*, vol. 35, no. 2, pp. 134–147, 2001.
[47] B. Bartin and K. Ozbay, "Determining the optimal configuration of highway routes for real-time traffic information: A case study," *Intelligent Transportation Systems, IEEE Transactions on*, vol. 11, no. 1, pp. 225 –231, 2010.

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