Advanced Designed Experiments

**COMPLETELY RANDOMIZED DESIGN**

- the simplest and least restrictive

  with \( p \) treatments and \( n > p \) experimental units >>> we can assign each treatment \((r_i)\) to units selected randomly from among \( n \) and do this until the \( p \)th treatment is assigned

  the only restriction is that before treatments are assigned each unit must have an equal chance of being assigned to any treatment

  this would be random selection without replacement (versus selection with replacement)

Advantages

1. Flexibility – any number of treatments and replications may be used, and the number of replications need not be the same among treatments (although comparisons are most precise when the treatments are equally replicated).

2. Statistical analysis is simple even with unequal replication, and it is not complicated by loss of data or missing observations.

3. The design provides maximum degrees of freedom for error.

Disadvantages

1. Precision is low if the experimental units are not uniform. Blocking can increase precision.

Uses

1. It is most precise when experimental material is uniform.

2. It is useful when a large fraction of the units may not respond or may be lost during the experiment.

3. It may be useful for experiments in which the total number of units is limited.

**Randomization**

- treatments are assigned to the experimental units completely at random in a number of ways; if we have 4 treatments each replicated 3 times, we will have 12 units; 2 methods of randomization are:

  1. By Lot – label 3 pieces of paper A, B, C, D >> pick at random for each unit
  2. By Random Number Table – select a starting point and a direction >> assign random number each experimental unit (1-2) >> lowest 3 numbers get A, next 3 get B, etc.
Fig. 1. Completely randomized design with 4 treatments and 12 experimental units.

– data analysis involves:

1. calculate the means for 4 treatments
2. test for differences among treatment means

   \[ H_0: A = B = C = D \quad \text{versus} \quad H_a: A \neq B \neq C \neq D \]

3. use Analysis of Variance (ANOVA) – remember that ANOVA will tell you if there is a difference among treatments, it will not tell you where the differences lie

4. analysis can take place with equal or unequal replication

**Fixed and Random Effects Models**

– fixed effects models = assume that the treatments used in an experiment are the only ones available
– random effects models = treatments may be a random sample from a large pop. of similar treatments

   e.g., a fixed effects model would focus on 4 specific soil types while a random effects model would involve 4 soil types to represent all existing soil types

   fixed effects models are the rule in designed experiments while random effects models are more common in sample surveys

**RANDOMIZED BLOCK DESIGN**

**Blocking**

– completely randomized designs are valid, but if we know something about the material we are testing or the areas where we are doing research, grouping experimental units into homogeneous units or blocks is better

– then we can make comparisons of treatments on units within blocks, and thus differences among blocks can be eliminated from experimental error; this results in an increase in precision

– blocking can be done by area (e.g., adjacent plots tend to be more alike than plots at some distance); by breed, genetic background, sex, or age; by time of treatment; by batch of raw material
– in general, any grouping of units into blocks is valid so long as it is done before applying treatments; any property
of the units that can be determined before the experiment begins can be used as a basis for blocking

– blocking will be effective, however, only if the variance among units within blocks is smaller than the variance over the whole set of units

**Randomized Block Design**

– the most basic experimental design which includes blocking, and probably the most used overall

– first group the units into \( r \) blocks of \( p \) units each so that blocks are as nearly alike as possible, then assign the treatments to units within blocks so that each treatment occurs only once per block

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<th>BLOCKS</th>
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<tbody>
<tr>
<td>I</td>
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<td>1</td>
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<td>3</td>
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<td>4</td>
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<td>5</td>
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Fig. 2. Assignment of numbers to units blocked to remove effects of a gradient.
– then treatments are assigned randomly to the units within the blocks; a separate randomization process is used in each block

<table>
<thead>
<tr>
<th>BLOCKS</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
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<tbody>
<tr>
<td>1</td>
<td>B</td>
<td>1</td>
<td>D</td>
<td>A</td>
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<tr>
<td>2</td>
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<tr>
<td>5</td>
<td>C</td>
<td>5</td>
<td>B</td>
<td>B</td>
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</tbody>
</table>

Fig. 3. Final experimental plan with treatments assigned randomly to units within blocks in a randomized block design.

Advantages

1. Increases precision by removing one source of variation from experimental error.
2. Can use any number of blocks and treatments so long as each treatment is replicated equally in each block.
3. Statistical analysis is relatively simple.

Disadvantages

1. Missing data can cause difficulty in analysis.

**LATIN SQUARE**

– allows the control of ≥2 sources of variation; units are assigned based on row and column classification
– treatments are assigned in such a way that each occurs once and only once in each row and column
– to construct a Latin square design for \( p \) treatments we need \( p^2 \) experimental units (4 treatments thus 16 units)
Fig. 4. Basic design for a 4x4 Latin square in a field plan to remove effects of slope & fertility gradients.

Advantages

1. There is one principal advantage of the Latin square, and this constitutes the primary reason for its use: it allows the experimenter to control two sources of variation.

Disadvantages

1. Requires many units, thus limiting number of treatments to \( \leq 10 \) (!).
2. Analysis becomes complicated if there are missing data or misassigned treatments.

– more complicated designs are possible, such as multiple Latin squares and Graeco-Latin squares; the latter allow the researcher to control 3 sources of variation.

**FACTORIAL EXPERIMENTS**

– \( \geq 1 \) treatments are often involved in experiments; for example, testing a new variety of corn might involve variables such as timing of planting, seeding rate, use of fertilizer, use of irrigation, etc. These variables could be tested one at a time – besides being inefficient, this one-factor-at-a-time approach does not account for interactions among variables

– sometimes factors act independently of each other >> i.e., changing the level of one factor produces the same results at all levels of another factor; often, however, the effects of two or more factors are not independent >> when two factors do not behave independently, they are said to interact

– interaction occurs when the response in changes in one factor is conditioned by the level of another factor

– if interactions exist, we must use multilevel, multifactor experiments >> the most common is the factorial experiment >> these experiments consist of combinations of two or more factors each at two or more levels such that each level of every factor occurs together with each level of every other factor

– factorial refers to the treatment combinations, not the type of experimental design >> factorial set of treatments can be used in any design: completely randomized, randomized block, Latin square, etc.

e.g., consider the wearing quality of a new paint tested under 4 conditions:
1. Hard wood, dry climate  
2. Hard wood, wet climate  
3. Soft wood, dry climate  
4. Soft wood, wet climate

You might expect that wood characteristics and climate interact. This experiment has 2 factors (wood hard or soft) and 2 levels (climate wet or dry) and is thus a 2 x 2 factorial experiment.

e.g., consider the effects of storage temperature and length on genetic material.

You could test under 2 temperatures (say -10°C and -20°C) and 4 storage times (say 1-4 months). This would be a 2 x 4 factorial design with 8 treatments.

**Split-plot design**

In split-plot designs, the levels of one factor (say seedbed preparation, $S$, for planting pines) are assigned at random to large experimental units within blocks; the large units are then divided into smaller units and the levels of the second factor (say application of varying levels of nitrogen fertilizer, $N$) are assigned at random to the small units with the larger units.

The large units are called whole plots or main plots, while the smaller units are called split-plots or subplots.

In this example, we have 2 blocks (I and II) and 3 whole plots ($S_{1,3}$), and 4 split- or subplots ($N_{0,3}$):

<table>
<thead>
<tr>
<th>I</th>
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<tbody>
<tr>
<td>$S_3$</td>
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<tr>
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<td>$N_0$</td>
<td>$N_1$</td>
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**ADVANCED DESIGNS**

**Multifactor Experiments** – often used in large experiments to identify those factors which affect response and to separate these important factors from unimportant factors. Approaches such as the $2^k$ factorial series are useful in the exploratory stages of an investigation because they permit examination of a large number of factors and their interactions in a trial of reasonable size.

**Change-over Trails** – involve the sequential application of 2 or more treatments (e.g., drug A followed by drug B) to the same experimental unit.

**Incomplete Block Designs** – when a large number of treatments is involved, experimental units can be grouped into blocks which are smaller than a complete replication of the treatments.

**Cluster Sample** – a sample in which each sampling unit is a collection, or cluster, of elements (e.g., select city blocks at random and then census everyone on each of those blocks).