Background on Modality and Choice Functions

1. Modality: A Refresher on the Basics

Overarching Question:
What is the meaning of 'modal auxiliaries' in English (might, must, can, may, etc.)?
What is the core semantic difference between the (a) and (b) sentences below?

(1) **The Modal Auxiliary 'Must'**

a. Dave is 18 years old.
b. Dave must be 18 years old.

(2) **The Modal Auxiliary 'May'**

a. Dave has a hamburger.
b. Dave may have a hamburger.

Classic Intuition:
The non-modal (a)-sentences describe the world *as it is*.
The modal (b)-sentences describe the world *as it could/should be.*

Sentences with modal auxiliaries describe possibilities, things that aren't (necessarily) true, but could be true in certain imaginable situations.

But, how do we more formally express this 'classic intuition'?

(3) **The Ontology of Possible Worlds**

(a) Possible World
A 'possible world' w is (more-or-less) an imaginable universe.

(b) Set of All Possible Worlds
We use the set W to mean the set of all possible worlds (all imaginable situations)

(c) Illustrative Members of W
The real world w_0
Worlds where Al Gore became president in 2001
Worlds where people can fly
Worlds where you eat with your feet
Worlds where Dave has a hamburger

How can we use these objects to analyze the meaning of English modals?

(4) **Possible World Semantics for Modals, Informally Stated**

a. Semantics of "May"

(i) [[May VP]] = There is some possible world where VP holds

(ii) *Example:*
[[Dave may have a hamburger]] = [[May [Dave has hamburger]]] =
There is some possible world where Dave has a hamburger

b. Semantics of "Must"

(i) [[Must VP]] = In all possible worlds, VP holds

(ii) *Example:*
[[Two plus two must equal four]] =[[Must [two plus two equal four]]] =
In all possible worlds, two plus two equals four.

(5) **Possible World Semantics, Formally Stated**

a. Models
A model for English is an ordered pair <D, W>, where D is a set of entities and W is a set of possible worlds.

b. Parameterization of the Interpretation Function
A sentence of English is evaluated with respect to a model *M*, a variable assignment *g*, and **a possible world *w*:** $[[\ ]]^{M,g,w}$

c. Definition of the Interpretation Function

(i) *Predicates (VPs)*
Are of type <e,t>, but value depends upon the world of evaluation

$[[\text{dance}]]^{M,g,w} = \lambda x.\ x \text{ is dancing in } w.$

(ii) *Names*
Are of type e, and value doesn't depend upon world of evaluation

$[[\text{Dave}]]^{M,g,w} = \text{Dave}$

(iii) *Modals*
Are sentential operators that introduce quantification over the possible worlds of evaluation.

$[[\text{must XP}]]^{M,g,w} = \forall w' \in W\ .\ [[\text{XP}]]^{M,g,w'}$

$[[\text{may XP}]]^{M,g,w} = \exists w' \in W\ .\ [[\text{XP}]]^{M,g,w'}$

2. The Appearance of Ambiguity

Fundamental Problem for Basic Semantics in (4) and (5):

The semantics in (4) and (5) is too crude, and doesn't capture certain obvious characteristics of modal statements in English.

Modal statements themselves seem to be ambiguous in a way that the simple semantics in (4) and (5) cannot capture.

2.1 Deontic Readings of Modals

Consider our sentence in (1b) – repeated below as (6b) – uttered in the context in (6a).

(6) **A Context for 'Must'**

a. Context
Dave is only 11, and so he can't vote. So, in order to vote…

b. Dave must be 18 years old.

But now consider what the simple semantics in (4)/(5) predicts the sentence to mean.

(7) **Semantics for (6b) Under the Semantics in (4)/(5)**

[[Dave must be 18 years old]] = [[Must [Dave be 18 years old]]] =

In **all possible (imaginable) worlds,** Dave is 18 years old.
($\forall w' \in W$. Dave is 18 years old in w'.)

PROBLEM:
Under the semantics in (7), the sentence in (6b) is false.
After all, it's clearly not the case that in **every imaginable universe** Dave is 18 years old.

But, let's not throw out the baby with the bathwater…
Let's reflect on what the meaning of (6b) is in the context of (6a).
The following seems to be a fair paraphrase.

(8) **Paraphrase of (6b) in the Context of (6a)**

Given what the law is, if Dave is voting, the Dave must be 18 years old.

If we accept (8) as a fair paraphrase, we can give the following characterization in terms of 'possible worlds':

(9) **A Characterization of Paraphrase (8) in terms of Possible Worlds**

Consider the possible worlds *w* in W that have the following qualities:
  (i) the law is being followed in them
  (ii) Dave is voting in them
In all **those** worlds, Dave is 18 years old.

Conclusion:
In order to really capture the meaning of (6b) in context, all we need to do is modify our possible world semantics in (5) so that we aren't looking at **ALL** the imaginable situations, but rather only a **subset** of them that satisfy some particular property:
the worlds where the law is being followed

(10) **Special Terminology: Deontic Reading of 'Must' / 'May'**

A 'deontic reading' of a modal is one where the modal is understood to quantify over those worlds where **a given set of rules / laws is being followed**

(11) **Example: A Deontic Readings of 'May'**

Now that he is 18, Dave **may** vote.
(In some worlds where the law is being followed, Dave is voting.)

Summary:
So, we've seen that modals in English like 'must' and 'may' can be receive readings where they don't quantify over the full set of possible (imaginable) universes, and that one example of such readings is a 'deontic reading'…
… however, there are numerous other readings of this type…

2.2 Epistemic Readings of Modals

Consider sentence (12b) in the context in (12a).

(12) **Another Context for 'Must'**

a. Context
We know the following facts:
There was a murder, the weapon was a gun, the only person who owns a gun is Dave, Dave had a grudge against the murder victim, Dave is the only person without an alibi for the night of the murder.

b. Dave must be the murderer.

Observations

- In context (12a), sentence (12b) is again not understood to be quantifying over **all imaginable** situations (what about the ones where Dave *doesn't* own a gun?)
- But, sentence (12b) also is clearly not being given a deontic interpretation either (after all, murder is against the law…)

Let's again reflect on the meaning of (12b) in context.
The following seems to be a fair paraphrase:

(13) **Paraphrase of (12b) in the Context of (12a)**

Given what we know, Dave must be the murderer.

(14) **A Characterization of Paraphrase (13) in terms of Possible Worlds**

Consider the possible worlds *w* in W that have the following quality:
They are consistent with what we know.
(That is, everything that we know to be the case in the actual world, also holds in these other imaginable worlds…)
In all **those** worlds, Dave is the murderer.

Conclusion:
Again, in order to really capture the meaning of (12b) in context, all we need to do is modify our possible world semantics in so that we are looking only a the **subset** of possible worlds that satisfy a given property: **the worlds where what we know is (also) true**

(15) **Special Terminology: Epistemic Reading of 'Must' / 'May'**

An 'epistemic reading' of a modal is one where the modal is understood to quantify over those worlds where **what we know to be true in the actual world also holds**

(16) **Example: An Epistemic Readings of 'May'**

Given what we know, Dave **may** be the murderer.
(In some worlds where what we know also holds, Dave is the murderer.)
(*I.e.,* our knowledge doesn't rule out Dave being the murderer.)

2.3 Circumstantial Readings of Modals

Consider sentence (17b) in the context in (17a).

(17) **Another Context for 'Must'**

a. Context
Dave is a newborn baby. Given their physiology, newborn babies become hungry every two hours or so.

b. Dave must feed every two hours.

Observations

- In context (17a), sentence (17b) is again not understood to be quantifying over **all imaginable** situations (what about worlds where babies get their nutrition from the sun?)
- But, sentence (17b) also is clearly not being given a deontic interpretation
(after all, the law doesn't require you to feed your baby at a given schedule…)
or an epistemic interpretation
(after all, under the targeted reading, the sentence would be true even if we knew that Dave's neglectful mother *doesn't* feed him every two hours.)

Let's again reflect on the meaning of (17b) in context.
The following seems to be a fair paraphrase:

(18) **Paraphrase of (17b) in the Context of (17a)**

Given certain physical facts about the world (*i.e.*, baby physiology), in order to stay alive, Dave must feed every two hours.

(19) **A Characterization of Paraphrase (18) in terms of Possible Worlds**

Consider the possible worlds *w* in W that have the following qualities:
(i) They share certain relevant physical properties with our own world
(*i.e.,* baby physiology)
(ii) They are worlds where Dave stays alive
In all **those** worlds, Dave feeds every two hours.

Conclusion:
Again, in order to really capture the meaning of (17b) in context, all we need to do is modify our possible world semantics in so that we are looking only a the **subset** of possible worlds that satisfy a given property:
the worlds where certain physical facts from our own world also hold

(20) **Special Terminology: Circumstantial Reading of 'Must' / 'Can'**

A 'circumstantial reading' of a modal is one where the modal is understood to quantify over those worlds where **certain (relevant) physical facts from our own world also hold**

(21) **Example: A Circumstantial Readings of 'Can'**

Dave **can** jump 4 feet.
(In some worlds sharing the same physical facts as our own world, Dave jumps 4 feet.)

<u>An Aside on Limits to the Observed Ambiguity</u>
In sentence (21), we saw that the modal 'can' is able to receive a 'circumstantial' reading. *Interestingly, the modal auxiliary 'may' cannot receive a circumstantial reading.*

(22) **No Circumstantial Reading for 'May'**

Now that Dave has trained very hard, he **can / ?? may** jump 4 feet.

Similarly, the modal auxiliary 'can' cannot receive an epistemic reading.

(23) **No Epistemic Reading for 'Can'**

Given what we know, Dave **may / ??can** be the murderer.

<u>General Phenomenon:</u>
Not all modal auxiliaries in English allow all readings. *There are idiosyncratic, lexical restrictions on the permissible 'readings' a given modal can take.*

- 'can' disallows epistemic readings
- 'may' disallows circumstantial readings
- 'must' (typically) disallows circumstantial readings

2.4 Bouletic Readings of Modals

Consider sentence (24b) in the context in (24a).

(24) **Another Context for 'Must'**

a. <u>Context</u>
Dave wants to build a deck in his backyard. Currently, he doesn't have any wood.

b. Dave must buy a pallet of 2x4s.

Observations

- In context (24a), sentence (24b) is again not understood to be quantifying over **all imaginable** situations (what about worlds where Dave doesn't want to build a deck?)
- But, sentence (24b) also is clearly not being given a deontic interpretation
  (after all, the law isn't relevant here)
  or an epistemic interpretation
  and the reading is slightly different from the 'circumstantial reading' (since it's not about Dave's physical abilities or requirements…)

Let's again reflect on the meaning of (24b) in context.
The following seems to be a fair paraphrase:

(25) **Paraphrase of (24b) in the Context of (24a)**

Given the things Dave *wants to do* (i.e., build a deck), Dave must buy a pallet of 2x4s.

(26) **A Characterization of Paraphrase (25) in terms of Possible Worlds**

Consider the possible worlds *w* in W that have the following qualities:
(i) they are all worlds where Dave accomplishes his desires
(ii) they are worlds where certain physical facts from our own world are held constant (e.g. Dave can't just conjure wood out of thin air).
In all **those** worlds, Dave buys a pallet of 2x4s.

Conclusion:
Again, in order to really capture the meaning of (24b) in context, all we need to do is modify our possible world semantics in so that we are looking only a the **subset** of possible worlds that satisfy a given property:
the worlds where a certain agent's desires hold

(27) **Special Terminology: Bouletic Reading of 'Must' / 'Can'**

A 'bouletic reading' of a modal is one where the modal is understood to quantify over those worlds where **a certain (relevant) agent's desires hold**

(28) **Example: A Bouletic Readings of 'Can'**

If Dave wants a car with AWD, Dave **can** buy a Subaru.
(In some of the worlds where Dave accomplishes his desire of owning a car with AWD, Dave buys a Subaru.)

3. Summary Discussion: The Modal Base

We've seen different observable 'readings' for modal auxiliaries can be obtained by varying the exact (sub)set of possible worlds that they quantify over.

(29) **Readings and their Associated Worlds**

a. Denotic Reading
The worlds where some salient set of rules/laws is being followed

b. Epistemic Reading
The worlds where what we know about the actual world also holds

c. Circumstantial Reading
The worlds where certain physical facts from our own world also hold

d. Bouletic Reading
The worlds where some salient agent's desires are realized

Given how crucial this 'subset of possible worlds' is to the meaning of the modal, we should dignify it with a name:

(30) **The Modal Base**

The 'modal base' is the (sub)set of possible worlds that a given modal is understood to quantify over.

Crucial Observation:

In sentences (6b), (12b), (17b), (24b), the **identity** of the modal base isn't **overtly represented** in the sentence.

(6b) Dave **must** be 18 years old.

(12b) Dave **must** be the murderer.

(17b) Dave **must** feed every two hours.

(24b) Dave **must** buy a pallet of 2x4s.

Rather, as we've seen, it's the **context** that determines the modal base in all of these examples.

The context dependency of the identity of the modal base suggests the following picture:

(31) **A Formal Semantics for Modals, Updated**

a. Models

A model for English is an ordered pair <D, W>, where D is a set of entities and W is a set of possible worlds.

b. Parameterization of the Interpretation Function

A sentence of English is evaluated with respect to a model *M*, a variable assignment *g*, a possible world *w*, **and a modal base** $\mathbf{B} \subseteq \mathbf{W}$**:** $[[\ \ ]]^{M,g,w,\mathbf{B}}$

c. Definition of the Interpretation Function

(i) *Predicates (VPs)*
Are of type <e,t>, but value depends upon the world of evaluation

$[[\text{ dance }]]^{M,g,w,\,B}$ = λx. x is dancing in *w*.

(ii) *Names*
Are of type e, and value doesn't depend upon world of evaluation

$[[\text{ Dave }]]^{M,gw,B}$ = Dave

(iii) *Modals*
Are sentential operators that introduce quantification over the possible worlds of evaluation.
Their meaning is partly determined by the value of the contextual parameter B (the modal Base).

$[[\text{ must XP }]]^{M,g,w,\mathbf{B}}$ = $\forall w' \in \mathbf{B}\,.\,[[\text{XP}]]^{M,g,w',B}$

$[[\text{ may XP }]]^{M,g,w,\mathbf{B}}$ = $\exists\, w' \in \mathbf{B}\,.\,[[\text{XP}]]^{M,g,w',B}$

(32) **Example: Epistemic Reading of Sentence (12b)**

EP = { *w* : everything we know in w_0 holds in *w* }

a. $[[\text{ Dave must be the murderer }]]^{M,g,w,\mathbf{EP}}$ = (*via syntax*)
b. $[[\text{ must [Dave be the murderer] }]]^{M,g,w,\mathbf{EP}}$ = (*via* (31ciii))
c. $\forall w' \in \{w\text{: everything we know in } w_0 \text{ holds in } w\}.[[\text{ Dave be the murder }]]^{M,g,w',\mathbf{EP}}$
= (*via regular rules*)

d. $\boldsymbol{\forall w' \in \{w : \text{everything we know in } w_0 \text{ holds in } w\}}$**. Dave is the murder in *w'***
'In every imaginable world where what we know is true, Dave is the murder'

3.1 Some Final Remarks on the 'Ambiguity' of Modals

In our informal discussion in Section 2, we spoke of modals in the following way:

(33) **Informal (but Incorrect) Generalizations Regarding Modals**

- Modals (in English) are 'ambiguous'

- This 'ambiguity' consists in their allowing (in principle) any of a set of four distinct 'readings': *deontic, epistemic, circumstantial, bouletic*

- We explained the existence of these distinct 'readings' via the claim that the modal base can (in principle) fall within any of four distinct categories:

Denotic Base:	The worlds where laws/rules are followed
Epistemic Base:	The worlds where what we know is true
Circumstantial Base:	The worlds where certain physical facts hold
Bouletic Base:	The worlds where someone's desires are realized

PROBLEM:

The line between these distinct 'readings' (and *per force* 'modal bases') is often very blurry.

It is often very difficult to categorize the interpretation of a given modal (in context) in terms of these simple, discrete categories.

(34) **Example 1: The Contrast Between 'Circumstantial' and 'Bouletic' Readings**

Consider the contrast between so-called 'circumstantial' and 'bouletic' readings.

a. Circumstantial Reading (of (17b)):
Given certain facts about infant physiology, *in order for Dave to stay alive*, Dave must feed every two hours.

b. Bouletic Reading (of (24b)):
Given that Dave wants to build a deck, *given certain facts about the world* (i.e., that Dave doesn't have any wood, and can't conjure wood out of thin air), Dave must buy a pallet of 2x4s.

Both the 'circumstantial' reading and the 'bouletic' reading make reference both to (i) facts about the world and (ii) the desires of some (implicit) agent.

Thus, the line between these seems rather artificial and 'taxonomic'

(35) **Example 2: The Contrast Between 'Epistemic' and 'Deontic' Readings**

Even the apparently clear line between 'epistemic' and 'denotic' readings can get blurry when we consider contexts where we are considering our knowledge *about the law*.

a. Context:
My daughter is a US Citizen. By law, the US Social Security Administration has a file for every US citizen.

b. The Social Security Administration **must** have a file for my daughter.

Question:
Is the reading of 'must' in (35b), given context (35a), 'epistemic' or 'deontic'?

Conclusion 1:
Our classification of readings and modal bases for modals is only vague and 'taxonomic'.
Thus, our formal semantics in (31) is correct to allow the 'modal base parameter' B to be any old subset of the set of possible worlds W (rather than one of a finite set of antecedently defined bases, like 'denotic', 'epistemic', etc.)

Conclusion 2:
Given that the observed variation in the meaning for modals can be so subtle:
Our formal semantics in (31) is correct to attribute that variation to an ***infinitely variable*** *contextual parameter (rather than claim that modals are, say, 'lexically ambiguous')*

(36) **The Meaning of an English Modal, Schematized**

A modal auxiliary in English will receive a meaning that matches the following schema:

$$\lambda p_{<st>}.\ \underbrace{\underline{\{\ \forall\ ,\ \exists\ \}}}_{\textit{determined by } \textbf{\textit{lexical entry}}} w \in \underbrace{\underline{\text{BASE}}}_{\textit{determined by } \textbf{\textit{context}}}\ .\ p(w)$$

(37) **The Meaning of a Lillooet Modal (Preview of Tuesday)**

Rullmann *et al.* (2007):
A modal auxiliary in Lillooet will receive a meaning that matches the following schema:

$$\lambda p_{<st>}.\ \underbrace{\underline{\{\ \forall\ ,\ \exists\ \}}}_{\textit{determined by } \textbf{\textit{context}}} w \in \underbrace{\underline{\text{BASE}}}_{\textit{determined by } \textbf{\textit{lexical entry}}}\ .\ p(w)$$

…but, given our discussion above, what would it mean if the Base were lexically determined?

4. Choice Functions (in the Semantics of Indefinites)

Rullmann *et al.* (2007) employ 'choice functions' in their analysis of Lillooet modals.

For this reason, I will briefly review what choice functions are, and what (originally) motivated their introduction to linguistic theory…

4.1 Basic Background: The 'QR' Theory of Scope Assignment

Since time immemorial, people have noticed that the sentence in (38a) allows the reading characterized in (38b,c).

(38) **Objects Scoping Over Subjects**

a. A professor advises most of the students

b. Most of the students are such that there exists a professor who mentors them.

c. **most'** (λx.student(x)) (λy. ∃z. professor(z) & mentor(z,y))

So, we need a theory of grammar that predicts the possibility of such 'inverse scope readings'.

(39) **A Classic Solution: Quantifier Raising (QR)**

The grammar of English (and other languages) includes a movement rule called 'Quantifier Raising' (QR), which can covertly move a quantificational DP to a position higher than the subject (39a).

Directly interpreting the structures generated by such movement yields the 'inverse scope' readings (39b).

a. *Quantifier Raising (Covert Movement)*

[Most of the students] [λ1. a professor advises t_1]

b. **most'** (λx.student(x)) (λy. ∃z. professor(z) & mentor(z,y))

Some Important Initial Evidence For the QR Account in (39)

If a **movement** operation is what is responsible for the assignment of scope…
… then since there are certain well-known constraints on movement…

We predict that the assignment of scope should be constrained in a similar fashion!

(40) **One (Initially) Exciting Parallel: Coordinate Structure Constraint (CSC)**

a. No Movement Out of a Coordinate Structure

* Who$_1$ did a professor advise [Bill and t_1].

b. No Scope Out of a Coordinate Stucture

(i) A professor advises Bill and most of the students.

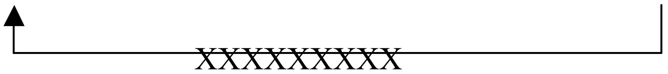
(ii) Unavailable Reading:

'Most students are such that a professor advises Bill and them.'

most'(λx.student(x))(λy. ∃z. professor(z) & mentor(z,Bill) & mentor(z,y))

(iii) Unavailability of Reading Predicted by QR

[Most of the students] [λ1. a professor advises [Bill and t_1]]



(41) **Serious Problem for the QR Analysis: Exceptional Scope of Indefinites**

While the scope of essentially quantificational DPs ('most boys', 'every dog', *etc.*) does seem to be limited in a fashion akin to the observable constraints on movement…

…the scope of indefinite DPs ('some dog', 'three men', etc.) does not appear to be as limited…

a. Most professors advise Bill and some undergrad.

b. Available Reading

∃x. undergrad(x) & [**most'** (λy.professor(y)) (λz. mentor(z,Bill) & mentor(z,x))]

Moreover, if QR is what is responsible for the assignment of scope, then the possibility of reading (41b) would entail that QR can apply as in (41c)

c. Island-Violating QR?

[some undergrad] [λ1. most professors advise [Bill and t_1]]

4.2 Problems with an Island-Free QR

In light of the facts in (41), maybe we could simply emend the 'classic' QR theory in (39) to the following:

(42) **The Theory of 'Island-Free' QR**

When applying to indefinites, the operation of QR is (for some reason) no longer subject to movement constraints.

(43) **Major Problems with the Hypothesis in (42)**

a. Syntactic
  (i) It's a bizarre and stipulative weakening of the theory of movement

  (ii) (As we'll see later), there is evidence that *QR of indefinites* **is** subject to movement constraints.

b. Semantic
  (i) The hypothesis in (42) *over-generates readings* (*esp.* with specific plural indefinites)

4.2.1 Island-Free QR of Indefinites Over-Generates Readings

(44) **Background Assumptions, Part 1: The Semantics of Plurals**

a. Plural NPs
Plural NPs denote 'pluralities' (sets of objects)

[[boys]] = 'the set of all possible sets of boys'
= $P(\lambda x.\text{boy}(x))$
= { {Dave}, … , {Dave, Tom }, {Tom, Bill, Frank}, …}

b. Singular NPs
Singlular NPs denote singleton sets

[[boy]] = 'those sets of boys that have only one member'
= $\{ X \subseteq P(\lambda x.\text{boy}(x)) : |X| = 1 \}$
= { {Dave}, {Bill}, {Frank}, {Tom}, … }

c. VPs
All VPs (whether singular or plural) are predicates of sets

[[dance]] = $\lambda X_{<et,t>}.\ \text{dance}(X)$

d. A Model Where Dave and Bill Danced Together:

[[dance]] = { {Dave}, {Dave, Bill}, {Bill} }

(45) **Background Assumptions, Part 2: The Semantics of Plural Indefinites**

Plural indefinites involve existential quantification of ‘pluralities’ (sets of objects)

a. Example DP

[[three boys]] = $\lambda Q_{<et,t>}.\ \exists X_{<et>}.\ \text{boys}(X)\ \&\ |X| = 3\ \&\ Q(X)$

b. Example Sentence

[[three boys ran]] = $\exists X_{<et>}.\ \text{boys}(X)\ \&\ |X| = 3\ \&\ \text{ran}(X)$
‘there is a set of boys whose cardinality is 3, and that set of boys ran (together)’

c. A Model Where (45b) is True

[[ran]] = { … { Dave, Bill, Tom }, … }

(46) **Central Background Assumption: The Connection between QR and Distributivity**

Generalization:
When an indefinite [D NP] undergoes QR to a position sister to YP, YP is understood to *distribute* over all the members of NP.

a. Example Sentence
A professor mentored three students.

b. Wide-Scope Reading of ‘Three Students’
‘There is a set of three students, and **for each of those three students**, there is a (possibly distinct) professor that mentored them.’

$\exists X_{<et>}.\text{student}(X)\ \&\ |X| = 3\ \&\ [\ \underline{\mathbf{\forall y \in X}}\ .\ \exists Z_{<et>}.\ \text{professor}(Z)\ \&\ \text{mentor}(Z,y)\ ]$

Side-note (Exercise to the Reader):

Without the distributivity underlined in (46b), the predicted reading of a ‘wide scope’ plural indefinite is no different from the reading assigned to a ‘narrow scope’ plural indefinite. That is, the expressions in (47a) and (47b) are logically equivalent.

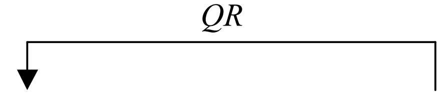
(47) **Semantic Vacuity of QR without Distributivity**

a. $\exists X_{<et>}.\text{student}(X)\ \&\ |X| = 3\ \&\ [\ \exists Z_{<et>}.\ \text{professor}(Z)\ \&\ \text{mentor}(Z,Y)\ ]$
b. $\exists X_{<et>}.\text{professor}(X)\ \&\ [\ \exists Z_{<et>}.\ \text{student}(Z)\ \&\ |X| = 3\ \&\ \ \&\ \text{mentor}(Z,Y)\ ]$

… thus, the very existence of a distributive reading is our sign that QR has occurred…

(48) **The Distributive Semantics of QR**

a. Generalization: QR Brings Distributivity



$[\![\text{ XP } [\lambda 1 \quad [_{YP} \ldots t_1 \ldots]]]\!]^g \quad =$

$[\![\text{XP}]\!]^g \; [\lambda X \; . \; \underline{\mathbf{\forall y \in X}} . \;\; [\![\text{YP}]\!]^{g(1 \rightarrow y)}$

b. Example: Sentence (46a)

$[\![\quad$ [three students] $\quad [\lambda 1 \quad$ [a professor mentored t_1]]]

$\exists X_{<et>}.\text{student}(X) \;\&\; |X| = 3 \;\&\; [\; \underline{\mathbf{\forall y \in X}}. \; \exists Z_{<et>}. \text{ professor}(Z) \;\&\; \text{mentor}(Z,y) \;]$

The Semantic Problem with ‘Island-Free QR’ (Ruys 1992, Reinhart 1997):

If QR were permitted to move an indefinite outside of a syntactic island, then we would wrongly predict *distributive readings* where they don’t seem to actually occur.

(49) **An Indefinite Scoping Out of the Antecedent of a Conditional**

a. Sentence:
If three of my relatives die, I’ll inherit this house.
(Context: Amongst my relatives, I’m ‘fourth in line’ to inherit this house.)

b. Meanings:

(i) *Possible Reading (though false in context (49a)*

‘If **any three** of my relatives die, I’ll inherit this house’

IF $[\exists X_{<et>}.\text{relatives}(X) \;\&\; |X| = 3 \;\&\; \text{die}(X)]$ [I inherit this house]

(ii) *Observed, True Reading of (49a)* ***(Collective Reading)***

‘There are these three relatives of mine, and if **all three together die**, I’ll inherit this house’

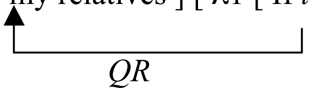
$\exists X_{<et>}.\text{relatives}(X) \;\&\; |X| = 3 \;\&\;$ [**IF** [die(X)][I inherit this house]]

Interim Observations

- Under the (salient) reading in (49bii), the existential force of the indefinite takes scope above the *entire conditional*.

- Under a QR approach to scope, this would entail that the existential has been moved *from inside the 'if-clause'*, as illustrated in (50).

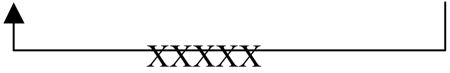
(50) [three of my relatives] [λ1 [If t_1 die, I'll inherit this house]]



- As sentences like (51) show, however, such movement is generally impossible.

(51) **No Movement From Inside an 'If-Clause'**

* Who will you inherit the house [if t_1 dies]?



So, maybe the possibility of reading (49bii) is just more proof that QR of indefinites is (somehow) not subject to the more general constraints on movement??

NO!
Given our semantics for QR in (48), the syntactic analysis in (50) would actually predict that (49a) receives – *not the observed reading in (49bii)* – but a reading that doesn't even seem to exist for (49a)…

(52) **Impossible Reading for Sentence (49a) (Distributive Reading)**

'There are three relatives of mine, and for each of those three relatives, if that relative dies, then I'll inherit this house.'

$\exists X_{<et>}.\text{relatives}(X) \;\&\; |X| = 3 \;\&\; [\ \underline{\mathbf{\forall y \in X}}.\ [\ \mathbf{IF}\ [\text{die}(y)][\text{ I inherit this house }]\ ]\ ]$

The impossible reading in (52) is exactly the reading we'd generate if we were to allow QR to move the indefinite out side of the 'if-clause'…

(53) **Generating the Impossible Reading in (50) via QR**

[three of my relatives] [λ1 [If t_1 die, I'll inherit this house]] = (*via* (48a))

$\exists X_{<et>}.\text{relatives}(X) \;\&\; |X| = 3 \;\&\; [\ \underline{\mathbf{\forall y \in X}}\ .\ [\ \mathbf{IF}\ [\text{die}(y)][\text{ I inherit this house }]\ ]\ ]$

Immediate Conclusions:

- Contrary to the hypothesis of 'island-free QR of indefinites' (in (42)), it seems that QR *isn't* able to move an indefinite like *three of my relatives* outside of syntactic islands (otherwise, the distributive reading in (52) would be a possible reading of (49a))

- **Thus, something *other than QR* must account for the ability of the indefinite in (49a) to take scope outside the antecedent of the conditional…**

(54) **Some General Conclusions**

(a) There *does* exist a movement operation called QR that has the following qualities:

(i) *Qua* movement operation, it is uniformly subject to movement constraints

(ii) It uniformly generates 'distributive' readings

(iii) It can apply to DPs (including indefinites), to (partly) determine the scope of the DPs quantificational force.

(b) Aside from QR, there *also* exists some additional mechanism that can determine the scope of indefinites. **Unlike QR, this mechanism is not sensitive to syntactic movement constraints, and doesn't generate distributive readings (it only generates 'collective' readings).**

The three conclusions under (54a) capture (I) the *possibility* of wide-scope distributive readings of indefinites in simple sentences like (46a), and (II) the *impossibility* of wide-scope distributive readings in 'island' sentences like (49a).

The conclusion under (54b) captures the ability for (49a) to have the wide scope collective reading in (49bii)

Task: Develop a theory of what this 'additional mechanism' is…

4.3 Solution: The Theory of Choice-Functions

In work beginning during the early 1990's, Reinhart hit upon an answer…

Choice Functional Theory of Indefinites (Reinhart 1997)

The 'additional mechanism' described in (54b) is ***the ability for indefinite Ds to be interpreted as variables over choice functions.***

4.3.1 The Basics of a Choice-Functional Semantics for Indefinites

(55) **Choice Function**

A choice function is a function that, when given any set as an argument, returns a member of that set.

(56) **Examples of Choice Functions**

a. *The Function f Defined as Follows*

f({Dave, Frank, Tom}) = Dave
f({Dave, Bill, Tom}) = Bill
f({Willy, Fran}) = Fran

b. *The Function g Defined as Follows*

g({Dave, Frank, Tom}) = Tom
g({Dave, Bill, Tom}) = Dave
g({Willy, Fran}) = Willy

c. *The Function h Defined as Follows*

h({ x : woman(x) }) = Madeline Albright
h({ x: worked-at-Pizza-Hut(x) }) = Seth Cable
h({ x: genius(x)}) = Tanya Reinhart

(57) **Example of Things that Aren't Choice Functions**

a. *The Function j Defined as Follows*

f({Dave, Frank, Tom}) = Dave
f({Dave, Bill, Tom}) = Du Bois Library
f({Willy, Fran}) = Fran

b. $\lambda x.dance(x)$

(58) **Choice-Functional Semantics for Indefinites, Part 1**

Indefinite Determiners Have both a GQ and a CF Interpretation

a. GQ Interpretation

$[[\text{ some }]]^g = \lambda P_{<et,t>}\, \lambda Q_{<et,t>}\, \exists X_{<et>}.\ P(X)\ \&\ Q(X)$

b. Choice-Functional Interpretation

$[[\text{ some}_i\ ]]^g = g(i)$ if g(i) is a choice function
undefined otherwise

(59) **Example: 'Some dog'**

a. GQ Interpretation

$[[\text{ some dog }]] = \lambda Q_{<et,t>}\, \exists X_{<et>}.\ dog(X)\ \&\ Q(X)$

b. Choice-Functional Interpretation

$[[\text{ some}_1\text{ dog }]]^{g(1\rightarrow f)} = f(\{X : dog(X)\}) =$ *some particular dog Z*

Important Fact:

Under the GQ interpretation in (59a), 'some dog' must undergo QR in order to be interpreted.

However, under the CF interpretation in (59b), 'some dog' is some particular dog Z, and so it can combine directly with a predicate like 'bark'.

(60) **Example: 'Some dog barked'**

a. $[[\text{ bark }]]^g = \lambda X_{<et>}.\ bark(X)$

b. $[[\text{ some}_1\text{ dog barked }]]^g =$

$[[barked]]^g\ (\ [[some_1\ dog]]^g\ ) =$

$[\lambda X_{<et>}.bark(X)](\ g(1)(\{X : dog(X)\})\ ) =$

$bark(f(\{X : dog(X)\})\ )$ (where $f = g(1)$) $=$

$bark(Z)$ (where Z is the dog 'chosen' by f)

(61) **The Final Ingredient to the Picture: Existential Closure**

- Principles of the syntax/semantic interface do not permit there to be at LF any 'unbound' choice-function variables.
- Thus, prior to LF, an operation of 'existential closure' (EC) inserts an existential operator which (i) has scope over the entire CP, and (ii) binds any previously unbound choice-function variables.

a. Illustration of EC

Structure Prior to LF: Some$_1$ dog barked

Application of EC $\exists_1$ [Some$_1$ dog barked]

b. Interpretation of EC Structures

$[[\ \exists_i \text{ XP }]]^g \;=\; \exists f\ [[\text{ XP }]]^{g(i\rightarrow f)}$

(62) **Example: 'Some Dog Barked'**

(i) $[[\ \exists_1 [\text{ Some}_1 \text{ dog barked }]\ ]]^g$ = *(via (61b))*

(ii) $\exists f\ [[\text{ Some}_1 \text{ dog barked }]]^{g(1\rightarrow f)}$ = *(via Function Application)*

(iii) $\exists f\ [[\text{ barked }]]^{g(1\rightarrow f)}(\ [[\text{ some}_1 \text{ dog }]]^{g(1\rightarrow f)}\ )$ = *(via (60a), (59b))*

(iv) $\exists f\ [\lambda X_{<et>}.\text{bark}(X)](f(\{X : \text{dog}(X)\})\ )$ = *(via Lambda Conv.)*

(v) $\boldsymbol{\exists f\ .\ \text{bark}(f(\{X : \text{dog}(X)\})\ )}$

'There is some choice function *f* such that when you apply it to the set of dogs, gives you an entity that barked'

Important Observation:

Given the nature of choice-functions (as defined in (55)), a choice function taking the set of dogs as argument will necessarily return a dog.

Thus, the truth conditions computed in (62) are necessarily equivalent to:
'There exists some dog x such that x barked'

Thus, the choice-functional semantics in (61) and (62) predict the intuitively correct truth conditions for simple indefinites sentences like (62).

(63) **A More Complex Example: Indefinite Plurals**

a. Interpretation of Plural Numeral Indefinites

$[[\text{ three}_i \text{ NP }]]^g \quad = \quad g(i)(\{X : |X| = 3 \;\&\; [[\text{NP}]]^g(X)\})$
if g(i) is a choice-function
undefined otherwise

b. Example: 'Three dogs barked'

(i) $[[\, \exists_1 \,[\text{ Three}_1 \text{ dogs barked }]\,]]^g \quad = \quad$ *(via (61b))*

(ii) $\exists f \; [[\text{Three}_1 \text{ dogs barked }]]^{g(1\to f)} \quad = \quad$ *(via Function application)*

(iii) $\exists f \; [[\text{ barked }]]^{g(1\to f)} \,(\, [[\text{ three}_1 \text{ dogs }]]^{g(1\to f)} \,) \quad = \quad$ *(via (60a))*

(iv) $\exists f \; [\lambda X_{<et>}.\text{bark}(X)](\; [[\text{ three}_1 \text{ dogs }]]^{g(1\to f)}) \quad = \quad$ *(via (63a))*

(v) $\exists f \; [\lambda X_{<et>}.\text{bark}(X)](f(\{X : |X| = 3 \;\&\; \text{dogs}(X)\})\,) \quad = \quad$ *(via LC)*

(vi) $\boldsymbol{\exists f \;.\; \text{bark}(\; f(\{X : |X| = 3 \;\&\; \text{dogs}(X)\})\;)}$

'There is some choice function f such that when you apply it to the following set – the set of all dog-trios – gives you something that barked.'

4.3.2 Choice Functions and Wide Scope Indefinites

So far, we've seen that accepting the choice-functional interpretation of indefinites in (58b) will make the correct predictions for simple sentences containing indefinites…

IMPORTANT OBSERVATION

The choice-functional semantics also predicts that:
Sentences where an indefinite is contained *inside* an island
should allow readings where the existential force of the indefinite has scope *outside* the island

That is, our account predicts the 'wide-scope reading' of (64a), repeated under (64b).

(64) **Exceptional Wide Scope of Indefinites**

a. Most professors advise Bill and some undergrad.

b. $\underline{\exists x.\ \text{undergrad}(x)}$ & [**most'** $(\lambda y.\text{professor}(y))\ (\lambda z.\ \text{mentor}(z,\text{Bill}) \;\&\; \text{mentor}(z,x))$]

(65) **Derivation of the 'Wide-Scope Reading' in (64b)**

a. Syntactic Structure of (64a):
Given our assumptions in (58b) and (61), we predict that the sentence in (64a) can have the following syntactic structure:

$\exists_1$ [Most professors advise Bill and some_1 undergrad]

b. Semantics of the Choice-Functional Structure
Given our semantics, (65a) is assigned the following interpretation:

$\exists \boldsymbol{f}$. [MOST (λX.professors(X))
(λY. Y advise Bill and [$\boldsymbol{f}$({Z : undergrad(Z)})])]

'There is a choice-function *f* such that
when you apply it to the set of undergrads,
you get some entity x for which the following is true:
most professors are such that they advise Bill and x'

c. Equivalence to the 'Wide-Scope' Interpretation in (64b)
Given the nature of choice-functions, the reading above is equivalent to the following:

'There is some undergrad x such that the following is true:
most professors are such that they advise Bill and x'

MOST IMPORTANT OBSERVATION

The choice-functional semantics also predicts that:
In sentences where an indefinite seems to 'scope out of' an island,
***only a 'collective' interpretation will be possible*!**

That is, our account predicts that sentence (66a) will only have the 'collective' interpretation in (66b), and not the 'distributive' interpretation in (66c).

(66) **No Distributive Readings of Wide-Scope Indefinites**

a. If three of my relatives die, I will inherit this house.
(Context: Of my relatives, I am fourth-in-line to inherit the house)

b. $\exists X_{<et>}$.relatives(X) & |X| = 3 & [**IF** [die(X)][I inherit this house]]

c. * $\exists X_{<et>}$.relatives(X) & |X| = 3 & [$\underline{\boldsymbol{\forall y \in X}}$. [**IF** [die(y)][I inherit this house]]]

To see this, let us consider the possible meanings our semantics predicts for sentence (66a)…

(67) **Readings Predicted for Sentence (66a)**

a. <u>GQ Interpretation</u>

(i) Let us first assume that the indefinite ‘three’ in (66a) is interpreted as a GQ (58a)

(ii) Given that it is interpreted as a GQ, the indefinite must undergo QR. ***However, since QR is a movement operation, it is predicted to be unable to move the indefinite outside the antecedent of the conditional (51), (54)***

(iii) Thus, the only structure that (66a) could have (if the indefinite were interpreted as a GQ) is the following:

$[_{CP1}$ $\mathbf{[_{CP2}}$ **If [three of my relatives] [λ1** $\boldsymbol{t_1}$ **die]]** I’ll inherit this house]

(iv) Given our semantics for GQ indefinites and QR, we predict that this structure will be interpreted as follows (48), (58a)

IF $[\exists X.\text{relatives}(X) \ \&\ |X| = 3\ \forall y \in X\ \text{die}(y)]$ [I inherit this house]

‘If there exists a trio of my relatives such that all members of that trio have died, then I will inherit this house’

‘If <u>any three</u> of my relatives die, I will inherit this house’

b. <u>Choice-Functional Interpretation</u>

(i) Let us now assume that the indefinite ‘three of my relatives’ in (66a) is interpreted as a a variable over choice functions (58b)

(ii) Following previous reasoning (65a), it follows that the structure of (66a) is the following:

$\exists_1$ $[_{CP1}$ $\mathbf{[_{CP2}}$ **If [three$_1$ of my relatives die]]** I’ll inherit this house]

(iii) Given our semantics, this structure will receive the following meaning:

$\exists f$. **IF** [die(f({X : |X| = 3 & relatives(X)}))] [I inherit this house]

‘There is a choice function f such that if you apply it to the set of all trios of my relatives, you receive some trio X for this the following is true:
If X dies, I will inherit this house’

Observation:

The reading in (67biii) is equivalent to the following:

‘There exists some trio X of my relatives for which the following true:
If X dies, I will inherit this house.’

Thus, the predicted reading in (67b) is the observed ‘collective’ reading in (66b)!

More generally…

Our total semantic system assigns only the readings in (68b,c) for sentence (66a).

(68) **The Predicted Readings of Sentence (66a)**

a. Sentence
If three of my relatives die, I’ll inherit this house.

b. Narrow-Scope (Distributive) Reading

$\textbf{IF}$ [$\exists$X.relatives(X) & |X| = 3 $\forall$Y $\subseteq$ X die(Y)] [I inherit this house]

c. Wide-Scope Collective Reading

$\exists f$. $\textbf{IF}$ [die(f({X : |X| = 3 & relatives(X)}))] [I inherit this house]

Thus, we correctly predict the absence of the ‘wide-scope distributive’ reading in (66c).

These considerations highlight the following generalizations that our account derives.

(69) **Generalization 1**

Informally:
An indefinite ‘scoping out of an island’ cannot get a ‘distributive reading’.

More Precisely:
Suppose that:

(a) We have a sentence S = [$_{YP}$ … [$_{DP}$ D NP] …], where D is an indefinite.
(b) Movement of DP to a position sister to YP is impossible
(c) S receives an interpretation where the ‘existential force’ associated with D has scope over YP

It follows that

(i) D must be interpreted as a choice function variable.
(if it were a GQ, movement constraints would prevent its existential force from having scope above YP)

(ii) **YP is not understood to distribute over some set in [[NP]]**
(such a reading is not generated by the choice-functional semantics)

(70) **Generalization II**

Informally:
Conversely, an indefinite getting a 'distributive reading' cannot take scope outside of an island.

More Precisely:
Suppose that:
- (a) We have a sentence S = [$_{YP}$ … [$_{DP}$ D NP] …], where D is an indefinite.
- (b) S receives an interpretation where YP is understood to distribute over some set in [[NP]]

It follows that:
- (i) DP underwent QR to a position sister to YP
  (given our semantics for QR)
- (ii) **Movement of DP to a position sister to YP is generally possible.**

A Quick Wrap-Up:

Choice functions were originally introduced into semantic theory by Reinhart (2007) to provide an analysis of specific plural indefinites in English.

This analysis allows us to capture the fact that
- (a) while such indefinites are free to 'scope out of islands'
- (b) they yield only 'collective' interpretations of the resulting sentence