Distributive Numerals and Pluractionals in Kaqchikel and DPlL: Henderson (2011)

1. The Phenomenon of Central Interest

Kaqchikel (Mayan; Guatemala) displays an interesting interaction between its distributive numerals (1) and its pluractional verbal morphology (2).

(1) **Rough Description of the Key Phenomenon**

- A plain numeral cannot 'take scope' below a verbal pluractional marker (2b)
- A distributive numeral *can* 'take scope' below a verbal pluractional (3)

(1) **Distributive Numerals in Kaqchikel**

a. <u>Unmarked Numeral Sentence:</u>
Konojel xkikanoj **jun** wuj.
they.all searched **one** book
They all searched for a book. (Distributive/Collective Possible)

b. <u>Distributive Numeral Sentence:</u>
Konojel xkikanoj **ju-jun** wuj
they.all searched **DIST-one** book
They all searched for one book each. (Distributive Reading Only)

(2) **Pluractional Verbal Morphology in Kaqchikel**

a. <u>Unmarked Sentence:</u>
Xinkanoj jun wuj
1sgS.searched one book
I looked for a book.
- True if I looked only once

b. <u>Marked Sentence:</u>
Xinkan**ala'** jun wuj
1sgS.searched.**PA** one book
I looked for a book (various times)
- True only if I looked for the *same* book multiple times. False otherwise.

(3) **Distributive Numerals with Pluractionals in Kaqchikel**

Xinkan**ala'** **ju-jun** wuj
1sgS.searched.**PA** **DIST-one** book
I looked for books (various times).

- True only if I looked for *several* books on *different* occasions.
- False if I looked for *only one* book on several occasions.

(4) **Key Overarching Question**

a. The Perennial Question:
What is the semantics of distributive numerals and pluractional morphology?

b. The Acute Question:
How does this semantic explain the contrast between (2b) and (3).

- Why can a 'plain numeral' *not* take scope below a pluractional operator, while a distributive numeral *can*?

Note: As Henderson (2011) notes, the contrast between (2b) and (3) poses problems for Farkas's (1997) approach to distributive numerals (*aka* 'narrow scope indefinites'), where they are simply plain indefinites with a requirement that they take narrow scope.

2. The Semantic Framework: Dynamic Predicate Logic with Plurals (DPlL)

Henderson's (2011) account is couched within the framework of 'Dynamic Logic with Plurals' (DPlL), developed by van den Berg (1996), Nouwen (2003), Brasoveanu (2010).

- The key innovation of this framework is the notion of a 'plural information state', a glitzy term for the idea that interpretation is relative to a *set* of assignment functions.

- To build towards this key idea, I'll start off with a 'toy system' close to DPlL that interprets sentences relative to only a *single* assignment function.

(5) **Central Architectural Feature to Bear in Mind**

Like much classic work in Montague Grammar and DRT, this semantic framework involves *indirect* interpretation, rather than *direct* interpretation.

a. Direct Interpretation (Heim & Kratzer 1998):
A semantics is provided for natural language expressions themselves.

[[all]] = [λP : λQ : for all x, if P(x) = T, then Q(x) = T]

b. Indirect Interpretation:
(i) A semantics is provided only for an artificial logical language.
(ii) A translation procedure is specified mapping expressions of natural language to expressions of that logical language.

"all" → $[\lambda P : \lambda Q : \forall x . P(x) \supset Q(x)]$ $[[\forall x . \phi]]^g = T$ iff for all h[x]g, $[[\phi]]^h = T$
$[[\phi \supset \psi]]^g = T$ iff $[[\phi]]^g = F$ or $[[\psi]]^g = T$

In most cases, the difference between (5a,b) is trivial (Montague 1974). However, in some frameworks like DRT and DPlL, intricacies of the logical notation make direct interpretation impractical (or impossible)…

- Therefore, to set up even a 'toy' DPlL system, I need to properly introduce and define the formal language.

2.1 A Dynamic Logic without Plural Assignment Functions

(6) **The Syntax of the Logical Language (Roughly Defined)**

a. The 'Lexicon' (Basic Symbols):

(i) *Variables for Entities, Events, Numbers*
1. $x_1, \dots, x_n$
2. $e_1, \dots, e_n$
3. $n_1, \dots, n_n$

(ii) *N-ary Relations:* All relations are assumed to be cumulative:
1. Predicates: *man, *book, *hug, *jump
2. Binary Relations: *Agent, *Theme

(iii) *The Predicate* 'atom' atom

(iv) *Numeral Predicates:* one, two, three, …

b. Rules for Forming Formulae:

(i) *Basic Formulae:*
If R is an n-ary relation, and $v_1 \dots v_n$ are variables, then '$R(v_1 \dots v_n)$' is a formula.

(ii) *Random Variable Assignment*
If v_i is a variable, then '[v]' is a formula.

(iii) *Conjunction:*
If 'ϕ' is a formula, and 'ψ' is a formula, then 'ϕ & ψ' is a formula.

c. Illustrative Formulas

(i) $[x_1]$ & atom(x_1) & *student(x_1) & $[e_2]$ & *left(e_2) & *Agent(e_2 , x_1)
A student left

(ii) $[x_1]$ & three(x_1) & *student(x_1) & $[x_3]$ & atom(x_3) & *tiger(x_3) & $[e_2]$ & *trap(e_2) & *Agent(e_2 , x_1) & *Theme(e_2 , x_3)
Three students trapped a tiger.

(7) **The Semantics of the Logical Language, Part 1**

The key semantic innovation in interpreting the logical language above: interpretation is relative to a *pair* of assignment functions $<g, h>$

- You should think of this as a kind of 'input / output' pair:
  - Formulae in the logic map an input assignment function g to an output assignment function h
  - Thus, formulae describe *relations* between assignment functions: (this is kind of like a 'context change potential', but different…)

a. The Semantics of Relations (Yawn)

$[[\ R(v_i \ldots v_n)\ ]]^{<g,h>} = T$ *iff* $g = h$ and $[[R]]^{<g,h>}(h(i))\ldots(h(n))$

Illustration:
$[[\ {*}Agent(e_2\ ,\ x_1)\ ]]^{<g,h>} = T$ *iff* $g = h$ and $[[{*}Agent]]^{<g,h>}(h(2)(h(1))$
iff $g = h$ and $h(1)$ is the *Agent of $h(2)$

b. The Semantics of 'Atom' (Yawn)

$[[\ atom(x_i)\ ]]^{<g,h>} = T$ *iff* $g = h$ and $h(i)$ is an atom

c. The Semantics of Numerals (Yawn)

(i) $[[\ one(x_i)\ ]]^{<g,h>} = T$ *iff* $g = h$ and $|\ h(i)\ | = 1$
(ii) $[[\ two(x_i)\ ]]^{<g,h>} = T$ *iff* $g = h$ and $|\ h(i)\ | = 2$, *etc.*

> As shown above, the interpretation of the preceding formulae has no effect upon the 'input' assignment function. However, the interpretation of the following formulae does.

d. The Semantics of '[ ]'

(i) *New Notation:* $g[i]h$
'h is just like g except for what it maps index i to'

(ii) $[[\ [\ x_i\ ]\ ]]^{<g,h>} = T$ *iff* $g[i]h$

e. The Semantics of Conjunction

$[[\ \phi\ \&\ \psi\ ]]^{<g,h>} = T$ *iff* There is an assignment function k such that $[[\ \phi\ ]]^{<g,k>} = T$ and $[[\psi\ ]]^{<k,h>} = T$

(8) **The Semantics of the Logical Language, Part 2**

With the definitions above, we can define a derivative notion of what it means for a formula 'ϕ' to be true relative to a *single* assignment function.

$[[\ \phi\ ]]^{g} = T$ *iff* there is an assignment function h such that $[[\ \phi\ ]]^{<g,h>} = T$

(9) **Illustration: 'A Student Left'**

a. Translation of 'A student left' [1]
$[x_1]$ & atom(x_1) & *student(x_1) & $[e_2]$ & *left(e_2) & *Agent(e_2 , x_1)

b. Semantics of the Logical Formula

(i) $[[\ (9a)\ ]]^{g} = T$ *iff* (by (8))

(ii) there is an assignment function h such that $[[\ (9a)\ ]]^{<g,h>} = T$*iff* (by (7e))

(iii) there are assignment functions *a, b, c, d, e, h* such that
$[[\ [\ x_1\ ]\ ]]^{<g,a>} = T$, and
$[[\ \text{atom}(x_1)\ ]]^{<a,b>} = T$, and
$[[\ {*}\text{student}(x_1)\ ]]^{<b,c>} = T$, and
$[[\ [\ e_2\ ]\ ]]^{<c,d>} = T$, and
$[[\ {*}\text{left}(e_2)\ ]]^{<d,e>} = T$, and
$[[\ {*}\text{Agent}(e_2 , x_1)\ ]]^{<e,h>} = T$ *iff* (by (7a-d))

(iv) there are assignment functions *a, b, c, d, e, h* such that
g[1]a, and
a = b and b(1) is an atom, and
b = c and c(1) is a (plurality of) student, and
c[2]d, and
d = e and e(2) is a (plurality of) event of leaving, and
e = h and h(1) is the *Agent of h(2) *iff* (by reasoning)

(v) **there is an assignment function *h* which is just like *g* except in the values it assigns to index 1 and 2, and:**
h(1) is an atom
h(1) is a student
h(2) is an event of leaving, and
h(1) is the agent of h(2).

[1] Note that Henderson (2011) does not provide an explicit translation procedure mapping expressions of Kaqchikel to expressions of DPlL. However, I know of no reason to think that one *couldn't* be specified.

c. Application of the Semantics:

(i) *Scenario:* Bill is a student, and Bill left.
(ii) *Claim:* Let *g* be any assignment function. [[(9a)]]^{g} = T
(iii) *Reasoning:*

- Let *h* be any assignment function just like *g*, except that *h*(1) = Bill, and *h*(2) = the event of Bill's leaving.

- It follows that: h(1) is an atom
  h(1) is a student
  h(2) is an event of leaving
  h(1) is the agent of h(2).

- Thus, there is an *h* such that [[(9a)]]$^{<g,h>}$ = T. And so, by the definition in (8), it follows that [[(9a)]]^{g} = T

(10) **Key Conclusion**

- In a scenario where a student has left, formula (9a) will be true relative to any assignment function.

- Thus, if we take (9a) as the logical translation of the sentence 'A student left', we correctly predict 'A student left' to be true in such a situation.

With the basic logical language in place, we now step into the world of 'Plural Assignments'...

2.2 Dynamic Plural Logic (DPlL)

(11) **The Central Addition**
Interpretation is now relative to *sets* of assignment functions *G* = { g_1, g_2, g_3, … }

A Commonly Used Visual Aid:

Indices (→ the column numbers 1, 2, 3 …)

G =	1	2	3	…
g_1	$g_1(1) = a$	$g_1(2) = e$	$g_1(3) = i$	
g_2	$g_2(1) = b$	$g_2(2) = f$	$g_2(3) = j$	
g_3	$g_3(1) = c$	$g_3(2) = g$	$g_3(3) = k$	
g_4	$g_4(1) = d$	$g_4(2) = h$	$g_4(3) = l$	

Members of set G (brace spanning g_1–g_4)

(12) **An Important Notation: *G*[i]**
If *i* is an index, and *G* is a set of assignment functions, then *G*[i] = { g(i) : $g \in$ G }

Illustration: G[1] = { a, b, c, d }
G[2] = { e, f, g, h }
G[3] = { i, j, k, l }

With this concept in place, we will slightly extend the language of our dynamic logic, and then slightly revise our semantics for it.

(13) **The Syntax of the DPlL Language (Roughly Defined)**

a. The 'Lexicon' (Basic Symbols)
As in (6a), but also including the following logical constants:

(i) *Numbers:* 0, 1, 2, 3, ….

b. Rule for Forming Basic Formulae:
As in (6b), but also including the following additional rule:

(i) *Inequalities Between Variables and Numbers*
If v_i is a variable, and *n* is a number, then all the following are formulae:

1. '$v_i = n$'
2. '$v_i > n$'
3. '$v_i < n$'

c. Illustrative Formulas

(i) '*Some students left*' →

[x_1] & $\mathbf{x_1 = 1}$ & *student(x_1) & [e_2] & $\mathbf{e_2 = 1}$ & *left(e_2) & *Agent(e_2 , x_1)

(ii) '*Three students trapped a tiger*' →

[x_1] & $\mathbf{x_1 = 1}$ & three(x_1) & *student(x_1) & [x_3] & atom(x_3) & *tiger(x_3) & [e_2] & $\mathbf{e_2 = 1}$ & *trap(e_2) & *Agent(e_2 , x_1) & *Theme(e_2 , x_3)

(14) **The Semantics of the Logical Language, Part 1**

The key semantic innovation in interpreting the logical language above: interpretation is relative to a *pair* of *sets* of assignment functions <*G, H*>

a. The Semantics of Relations (Yawn)
$[[\ R(v_i \ldots v_n)\ ]]^{<G,H>} = T$ *iff* G = H and $[[R]]^{<g,h>}(h(i))\ldots(h(n))$ *for all* h ∈ H

Illustration:
$[[\ {*}Agent(e_2, x_1)\ ]]^{<G,H>} = T$ *iff* G = H and h(1) is the agent of h(2) *for all* h ∈ H

b. The Semantics of 'Atom' (Yawn)
$[[\ atom(x_i)\ ]]^{<G,H>} = T$ *iff* G = H and h(i) is an atom *for all* h ∈ H

c. The Semantics of Numerals (Yawn)
(i) $[[\ one(x_i)\ ]]^{<G,H>} = T$ *iff* G = H and | h(i) | = 1 *for all* h ∈ H
(ii) $[[\ two(x_i)\ ]]^{<G,H>} = T$ *iff* G = H and | h(i) | = 2 *for all* h ∈ H, *etc.*

d. The Semantics of Inequality Statements (NEW!)
(i) $[[\ x_i = n\ ]]^{<G,H>} = T$ *iff* G = H and | H[i] | = n
(ii) $[[\ x_i > n\ ]]^{<G,H>} = T$ *iff* G = H and | H[i] | > n
(iii) $[[\ x_i < n\ ]]^{<G,H>} = T$ *iff* G = H and | H[i] | < n

- These new inequality statements *constrain* how many *different* entities the various assignment functions in H can assign to the index *i*
    - '$x_i = 1$' requires that they all map index *i* to the *same* entity.
    - '$x_i > n$' requires that *i* is mapped to at least *n* different entities.

As before, the interpretations of the preceding formulae have no effect upon the 'input' set of assignment functions. However, the interpretations of the following formulae do.

e. The Semantics of Conjunction
$[[\ \phi \,\&\, \psi\ ]]^{<G,H>} = T$ *iff* There is a set of functions *K* such that $[[\ \phi\ ]]^{<G,K>} = T$ and $[[\psi\ ]]^{<K,H>} = T$

f. The Semantics of '[ ]'

(i) *New Notation:* *G*[i]*H*
'for every *g* ∈ *G*, there is an *h* ∈ *H* such that *g*[i]*h*, and for every *h* ∈ *H*, there is a *g* ∈ *G* such that *g*[i]*h*.

(ii) $[[\ [\ x_i\ ]\ ]]^{<G,H>} = T$ *iff* G[i]H

Note: *G*[i]*H* holds *iff* all the assignment functions in *G* and *H* look exactly the same, except in their value for the index *i*.

(15) **The Semantics of the Logical Language, Part 2**

With the definitions above, we can define a derivative notion of what it means for a formula ‘ϕ’ to be true relative to a *single* set of assignment functions.

$[[\phi]]^{G} = T$ *iff* there is a set of assignment functions *H* such that $[[\phi]]^{<G,H>} = T$

(16) **Illustration: ‘Some Students Left’**

a. Translation of ‘Some students left’
$[x_1]$ & $x_1 = 1$ & $*student(x_1)$ & $[e_2]$ & $e_2 = 1$ & $*left(e_2)$ & $*Agent(e_2, x_1)$

b. Semantics of the Logical Formula

(i) $[[(16a)]]^{g} = T$ *iff* (by (15))

(ii) there is a set of assignments *H* such that $[[(16a)]]^{<G,H>} = T$ *iff* (by (14e))

(iii) there are sets of assignment functions *A, B, C, D, E, F, H* such that
$[[[x_1]]]^{<G,A>} = T$, and
$[[x_1 = 1]]^{<A,B>} = T$, and
$[[*student(x_1)]]^{<B,C>} = T$, and
$[[[e_2]]]^{<C,D>} = T$, and
$[[x_2 = 1]]^{<D,E>} = T$, and
$[[*left(e_2)]]^{<E,F>} = T$, and
$[[*Agent(e_2, x_1)]]^{<F,H>} = T$ *iff* (by (14a-d, f))

(iv) there are sets of assignment functions *A, B, C, D, E, F, H* such that
G[1]A, and
A = B and | B[1] | = 1 (*i.e.*, every $b \in$ B maps 1 to the same thing)
B = C and c(1) is a plurality of students for all $c \in$ C, and
C[2]D, and
D = E and | E[2] | = 1 (*i.e.*, every $e \in$ E maps 2 to the same thing)
E = F and f(2) is a plurality of events of leaving, for all f $\in$ F, and
F = H and h(1) is the *Agent of h(2) for all $h \in$ H
iff (by reasoning)

(v) **there is a set of assignment functions *H* which is just like *G* except in the values it assigns to index 1 and 2, and:**
Every $h \in$ H maps indices 1 and 2 to the same thing
h(1) is a plurality of students for all $h \in$ H
h(2) is a plurality events of leaving, for all h $\in$ H
h(1) is the *Agent of h(2), for all h $\in$ H

c. Application of the Semantics:
(i) *Scenario:* Bill and Tom are students, and Bill and Tom left.
(iii) *Claim:* Let *G* be any set of assignment functions. $[[\text{ (16a) }]]^G = T$
(iii) *Reasoning:*

- Let *H* be any set of assignment functions just like *G*, except that H[1] = {Bill+Tom}, and H[2] = { Bill's-leaving+Tom's-leaving }

H =	1	2	3	…
h_1	Bill+Tom	e_1+e_2	*a*	
h_2	Bill+Tom	e_1+e_2	*b*	
h_3	Bill+Tom	e_1+e_2	*c*	

- It follows that: Every h ∈ H maps 1 and 2 to the same thing
  h(1) is a plurality of students for all h ∈ H
  h(2) is a plurality of events of leaving, for all h ∈ H
  h(1) is the *Agent of h(2), for all h ∈ H

- Thus, there is an *H* such that $[[\text{ (16a) }]]^{<G,H>} = T$. And so, by the definition in (15), it follows that $[[\text{ (16a) }]]^G = T$

(17) **Key Conclusion**
- In a scenario where a bunch of students left, formula (16a) will be true relative to any set of assignment functions.
- Thus, if we take (16a) as the logical translation of the sentence 'Some students left', we correctly predict 'Some students left' to be true in such a situation.

Note: At the moment, using *sets* of assignment functions isn't doing any analytic work for us…
…but it will soon!...

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3. The Semantics of Pluractional Morphology in Kaqchikel

(18) **The Interaction Between Pluractionality and Distributivity, Part 1**

a. Xe'inq'etej ri ak'wala
1sgS.hugged the children
I hugged the children. (True if I hugged the children once, all together)
(True if I hugged each child separately)

b. Xe'inq'ete**la'** ri ak'wala
1sgS.hugged.**PA** the children
I hugged the children. (FALSE if I hugged them once, all together)
(FALSE if I hugged them together repeatedly)
(TRUE if I hugged each child separately)

As shown above in (18), if a pluractional verb takes a plural argument, the action must distribute over the individual atoms in the plurality

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- Sentence (18b) does not allow a reading equivalent to 'I hugged the children repeatedly'.

However, as shown below in (19), if a pluractional verb takes a ***singular*** *argument, the action can distribute over separate space/times*

- Sentence (19) does allow a reading equivalent to 'I planted the tree repeatedly'

(19) **The Interaction Between Pluractionality and Distributivity, Part 2**

Xintiki**la'**	jun	che'
1sgS.plant.**PA**	one	tree

I planted a tree (repeatedly) TRUE if I planted the same tree numerous times.

(20) **Obvious, Burning Question**
Given that a 'repetitive' reading for (19) is in principle possible, why is such a reading not also possible for (18b)? Why can't (18b) mean I hugged the same group of children repeatedly?

- If we assume Lasersohn's (1995) semantics, the facts in (18b) suggest that the 'non-overlap' parameter *f* must be set to 'Theme'

- But, this would wrongly predict that (19) should be ungrammatical!

3.1 The Formal Semantic Analysis

(21) **First Important Ingredient: The 'Max' Operator**

$[[\text{ Max}^{\,i,\,\ldots\,,j}\ \phi\ ]]^{<G,H>} = \text{T}$ *iff*

$[[\ [\ x_i\ ]\ \&\ \ldots\ [\ x_j\ ]\ \&\ \phi\ ]]^{<G,H>} = \text{T}$ AND
for all H', if $[[\ [\ x_i\ ]\ \&\ \ldots\ [\ x_j\ ]\ \&\ \phi\ ]]^{<G,H'>} = \text{T}$, then $H'[i] \subseteq H[i]$, and … $H'[j] \subseteq H[j]$

Here's What 'Max' Does:

a. First, it requires that *H* differ from *G* in the values it assigns to indices *i,…,j*

b. Next, it requires that $[[\ \phi\ ]]^{<G,H>}$ hold (and so it puts conditions on those indices)

c. Finally, it requires that, for each index *i*, H[i] is the *largest* set of entities that satisfy the conditions set by ϕ

(22) **Illustration of the 'Max' Operator**

Suppose that the set of boys is { Bill, Tom, Frank }. Let *G* be the set of assignment functions in (22b). Let *H* be as in (22c), and *J* be as in (22d).

a. Propositions:
(i) $[[\text{ Max}^1 \text{ boy}(x_1)]]^{<G,H>} = F$
(ii) $[[\text{ Max}^1 \text{ boy}(x_1)]]^{<G,J>} = T$

b. G

	1	2
g_1	Dino	Mary
g_2	Rex	Sue
g_3	Uranium	Linda

c. H

	1	2
h_1	Bob	Mary
h_2	Tom	Sue
h_3	Bob	Linda

d. J

	1	2
j_1	Bob	Mary
j_2	Tom	Sue
j_3	Frank	Linda

e. Explanation:

(i) Clearly, it's the case that $[[[x_1]]]^{<G,H>} = T$ and $[[[x_1]]]^{<G,J>} = T$

(ii) Also, it's the case that $[[\text{boy}(x_1)]]^{<G,H>} = T$, and $[[\text{boy}(x_1)]]^{<G,J>} = T$

(iii) Now, *H*[1] = { Bob, Tom } and *J*[1] = { Bob, Tom, Frank }.

(iv) Therefore, it's *not* true that for all *H'*, if $[[[x_1] \& \text{boy}(x_1)]]^{<G,H'>} = T$, then $H'[1] \subseteq H[1]$. Therefore $[[\text{ Max}^1 \text{ boy}(x_1)]]^{<G,H>} = F$

(v) However, given that the boys = { Bob, Tom, Frank }, it *is* the case that for all *H'*, if $[[[x_1] \& \text{boy}(x_1)]]^{<G,H'>} = T$, then $H'[1] \subseteq J[1]$. Therefore $[[\text{Max}^1 \text{ boy}(x_1)]]^{<G,J>} = T$

(23) **Another Informal Characterization**

What '$\text{Max}^{i,\dots,j}\ \phi$' does is the following:

- It introduces the variables *i,…,j*
- It 'stores' in those variables *all* the entities that satisfy the condition 'ϕ'

(24) **The Semantics of Pluractional Morphology in Kaqchikel**

The pluractional morphme ***la'*** in Kaqchikel can be translated to the DPlL formula below

- Where the variable *e* is the event variable introduced by the verb itself
- And the variable *x* is the variable introduced by the direct object.

a. $\text{Max}^{3,4} .\ e_3 > n\ \&\ e_3 \leq e\ \&\ \text{atom}(e_3)\ \&\ x_4 \leq x\ \&\ \text{atom}(x_4)\ \&\ {}^*\text{Theme}(e_3 , x_4)$

b. <u>What This Does, Informally:</u>

(i) It requires the index *3* to store **all** the atomic sub-events of the event introduced by the verb.
- This is achieved by 'Max' and the condition '$e_3 \leq e\ \&\ \text{atom}(e_3)$'

(ii) It requires the index *4* to store **all** the atomic sub-parts of the entity introduced by the direct object.
- This is achieved by 'Max' and the condition '$x_4 \leq x\ \&\ \text{atom}(x_4)$'

(iii) It requires that there be *more* than one atomic sub-event of the event introduced by the verb.
- This is achieved by the condition '$e_3 > n$'
- This is akin to Lasersohn's '$|e| > n$'

(iv) It requires that every atomic sub-part of the direct object *x* be *Theme* to some atomic sub-event of the event *e*, and *vice versa*.
- This is achieved by the condition '${}^*\text{Theme}(e_3 , x_4)$'

c. <u>What This Does, More Formally:</u> Where *H* is such that $[[\ (24a)\ ]]^{<G,H>} = T$

(i) $\text{Max}^{3,4}$: *H*[3] will be the biggest set of events fitting this condition:
H[4] will be the biggest set of entities fitting this condition:

(ii) $e_3 > n$: it can't be that every $h \in H$ maps 3 to the same thing.

(iii) $e_3 \leq e$: $h(3)$ is a sub-event of *e*, for all $h \in H$

(iv) $\text{atom}(e_3)$: $h(3)$ is an atom, for all $h \in H$

(v) $x_4 \leq x$: $h(4)$ is a sub-part of *x*, for all $h \in H$

(vi) $\text{atom}(x_4)$: $h(4)$ is an atom, for all $h \in H$

(vii) ${}^*\text{Theme}(e_3 , x_4)$: h(4) is the theme of h(3), for all $h \in H$

We'll now get a sense of how this works by applying it to particular sentences…

3.2 Applying the Formal Analysis

Let's begin by considering how one approaches the semantics of *non*-pluractional sentences.

(25) **A Non-Pluractional Sentence with Plural Direct Object**

Xe'inq'etej	oxi'	ak'wala
1sgS.hugged	three	children

I hugged three children. (True if I hugged the children once, all together)
(True if I hugged each child separately)

(26) **The DPlL Translation of (25)** [2]

$[x_1]$ & $x_1 = 1$ & $*child(x_1)$ & $three(x_1)$ & $[e_2]$ & $e_2 = 1$ & $*hug(e_2)$ & $*Agent(e_2, speaker)$ & $*Theme(e_2, x_1)$

(27) **The Truth of (25) in Situations of Collective Hugging**

a. Situation: Collective Hugging of Three Boys

Hugs	Agent	Theme
e_1	speaker	$kid_1+kid_2+kid_3$

b. Proposition: Formula (26), the translation of (25), is *true* in such a scenario.

c. Explanation:

- Let *G* be any set of assignment functions.
- Clearly, we could create a set of assignment functions *H* such that *G*[1,2]*H*, and *H* appears as follows:

H	1	2	…
h_1	$kid_1+kid_2+kid_3$	e_1	
h_2	$kid_1+kid_2+kid_3$	e_1	
h_3	$kid_1+kid_2+kid_3$	e_1	

- Now, note that:

$\mid H[1] \mid = \mid \{ kid_1+kid_2+kid_3\} \mid = 1$	*Thus,* $[[x_1 = 1]]^H = T$
h(1) is a plurality of children, for all $h \in H$	*Thus,* $[[*child(x_1)]]^H = T$
$\mid h(1) \mid = 3$ *for all* $h \in H$	*Thus*, $[[three(x_1)]]^H = T$
$\mid H[2] \mid = \mid \{ e_1\} \mid = 1$	*Thus,* $[[e_2 = 1]]^H = T$
h(2) is a plurality of huggings, for all $h \in H$	*Thus,* $[[*hug(e_2)]]^H = T$
The agent of h(2) is the speaker, for all $h \in H$	*Thus,* $[[*Agent(e_2, sp)]]^H = T$
The theme of h(2) is h(1), for all $h \in H$	*Thus,* $[[*Theme(e_2, x_1)]]^H = T$

- Therefore, by the definitions in (14)-(15), it follows that:
  (i) $[[(26)]]^{<G,H>}$, and therefore:
  (ii) $[[(26)]]^G$

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[2] As mentioned above, Henderson (2011) does not concretely show how (25) is mapped to the DPlL formula in (26), but it's likely that such a procedure is specifiable.

(28) **The Truth of (25) in Situations of Distributive Hugging**

a. Situation: Distributive Hugging of Three Boys

Hugs	Agent	Theme
e_1	speaker	kid_1
e_2	speaker	kid_2
e_3	speaker	kid_3

b. Proposition: Formula (26), the translation of (25), is *true* in such a scenario.

c. Explanation:

- Let *G* be any set of assignment functions.
- Clearly, we could create a set of assignment functions *H* such that *G*[1,2]*H*, and *H* appears as follows:

H	1	2	…
h_1	$kid_1+kid_2+kid_3$	$e_1+e_2+e_3$	
h_2	$kid_1+kid_2+kid_3$	$e_1+e_2+e_3$	
h_3	$kid_1+kid_2+kid_3$	$e_1+e_2+e_3$	

- Now, note that:

$\mid H[1] \mid = \mid \{ kid_1+kid_2+kid_3 \} \mid = 1$	*Thus,* $[[x_1 = 1]]^H = T$
h(1) is a plurality of children, for all $h \in H$	*Thus,* $[[*child(x_1)]]^H = T$
$\mid h(1) \mid = 3$ *for all* $h \in H$	*Thus*, $[[three(x_1)]]^H = T$
$\mid H[2] \mid = \mid \{ e_1+e_2+e_3 \} \mid = 1$	*Thus,* $[[e_2 = 1]]^H = T$
h(2) is a plurality of huggings, for all $h \in H$	*Thus,* $[[*hug(e_2)]]^H = T$
The *agent of h(2) is the speaker, for all $h \in H$	*Thus,* $[[*Agent(e_2 ,sp)]]^H = T$
The *theme of h(2) is h(1), for all $h \in H$	*Thus,* $[[*Theme(e_2 ,x_1)]]^H = T$

- Therefore, by the definitions in (14)-(15), it follows that:
  (i) $[[(26)]]^{<G,H>}$, and therefore:
  (ii) $[[(26)]]^G$

So far, we've seen that the DPlL translation in (26) of the non-pluractional sentence in (25) correctly predicts that this sentence will be true in situations of collective ***and*** *distributive hugging*

Now, we'll consider the predictions for the pluractional sentence…

(29) **A Pluractional Sentence with Plural Direct Object**

Xe'inq'ete**la**	oxi'	ak'wala
1sgS.hugged.**PA**	three	children
I hugged three children.		(FALSE if I hugged the children once, all together)
		(FALSE if I hugged the same three kids repeatedly)
		(TRUE if I hugged each of the kids separately)

(30) **The DPlL Translation of (29)**

$[x_1]$ & $x_1 = 1$ & $*child(x_1)$ & $three(x_1)$& $[e_2]$ & $e_2 = 1$ & $*hug(e_2)$ & $*Agent(e_2 , speaker)$ & $Max^{3, 4}$. $e_3 > n$ & $e_3 \leq e_2$ & $atom(e_3)$ & $x_4 \leq x_1$ & $atom(x_4)$ & $*Theme(e_3 , x_4)$

(31) **The Truth of (29) in Situations of Distributive Hugging**

a. Situation: Distributive Hugging of Three Boys

Hugs	Agent	Theme
e_1	speaker	kid_1
e_2	speaker	kid_2
e_3	speaker	kid_3

b. Proposition: Formula (30), the translation of (29), is *true* in such a scenario.

c. Explanation:

- Let *G* be any set of assignment functions.
- Clearly, we could create a set of assignment functions *H* such that *G*[1,2,3,4]*H*, and *H* appears as follows:

H	1	2	3	4
h_1	$kid_1+kid_2+kid_3$	$e_1+e_2+e_3$	e_1	kid_1
h_2	$kid_1+kid_2+kid_3$	$e_1+e_2+e_3$	e_2	kid_2
h_3	$kid_1+kid_2+kid_3$	$e_1+e_2+e_3$	e_3	kid_3

- Now, note that:

$\mid H[1] \mid = \mid \{ kid_1+kid_2+kid_3\} \mid = 1$	*Thus,* $[[x_1 = 1]]^H = T$
h(1) is a plurality of children, for all $h \in H$	*Thus,* $[[*child(x_1)]]^H = T$
$\mid h(1) \mid = 3$ *for all* $h \in H$	*Thus*, $[[three(x_1)]]^H = T$
$\mid H[2] \mid = \mid \{ e_1+e_2+e_3\} \mid = 1$	*Thus,* $[[e_2 = 1]]^H = T$
h(2) is a plurality of huggings, for all $h \in H$	*Thus,* $[[*hug(e_2)]]^H = T$
The *agent of h(2) is the speaker, for all $h \in H$	*Thus,* $[[*Agent(e_2 ,sp)]]^H = T$
$\mid H[3] \mid = \mid \{e_1 , e_2 , e_3\} \mid = 3 > n$	*Thus,* $[[e_3 > n]]^H = T$
$h(3) \leq h(2)$, for all $h \in H$	*Thus,* $[[e_3 \leq e_2]]^H = T$
h(3) is an atom, for all $h \in H$	*Thus,* $[[atom(e_3)]]^H = T$
$h(4) \leq h(1)$, for all $h \in H$	*Thus,* $[[x_4 \leq x_1]]^H = T$
h(4) is an atom, for all $h \in H$	*Thus,* $[[atom(x_4)]]^H = T$
h(4) is the theme of h(3) for all $h \in H$	*Thus,* $[[*Theme(e_3, x_4)]]^H = T$

… *and*, since *H*[3] contains <u>all</u> the events of hugging, and *H*[4] contains <u>all</u> the kids hugged, it follows that any other set *H'* that satisfies the conditions above, will be such that *H'*[3] ⊆ *H*[3] and *H'*[4] ⊆ *H*[4]

- Therefore, by the definitions above, it follows that:
  (i) $[[(30)]]^{<G,H>}$, and therefore:
  (ii) $[[(30)]]^G$

(32) **The Falsity of (29) in Situations of a Single Collective Hugging**

a. Situation: Single Collective Hugging of Three Boys

Hugs	Agent	Theme
e_1	speaker	$kid_1+kid_2+kid_3$

b. Proposition: Formula (30), the translation of (29), is *false* in such a scenario.

$[x_1]$ & $x_1 = 1$ & $*child(x_1)$ & $three(x_1)$& $[e_2]$ & $e_2 = 1$ & $*hug(e_2)$ & $*Agent(e_2 , speaker)$ & $Max^{3,4}$. $e_3 > n$ & $e_3 \leq e_2$ & $atom(e_3)$ & $x_4 \leq x_1$ & $atom(x_4)$ & $*Theme(e_3 , x_4)$

c. Explanation:

- Let *G* be any set of assignment functions.
- It will *not* be possible to construct a set of assignment functions *H* such that $[[(30)]]^{<G,H>}$
  - In order for $[[*hug(e_2)]]^{<G,H>}$ = T, it must be that h(2) is an event of hugging for all $h \in H$
    - But, since there is only one event of hugging – e_1 – it follows that h(2) = e_1 for all $h \in H$
  - In order for $[[e_3 \leq e_2$ & $atom(e_3)]]^{<G,H>}$ = T, it must be that h(3) is an atomic sub-event of h(2) for all $h \in H$
    - Thus, it must be that h(3) is an atomic sub-event of e_1 for all $h \in H$
    - Thus, it must be that h(3) = e_1 for all $h \in H$
  - In order for $[[*Theme(e_3 , x_4)]]^{<G,H>}$ = T, it must be that h(4) is the theme of h(2) for all $h \in H$
    - Thus, it must be that h(4) is the theme of e_1 for all $h \in H$
    - Thus, it must be that h(4) = $kid_1+kid_2+kid_3$
  - **Finally, in order for $[[atom(x_4)]]^{<G,H>}$ = T, it must be that h(4) is an atom, for all $h \in H$**
    - **But, h(4) = $kid_1+kid_2+kid_3$ for all $h \in H$, and so we see that these conditions cannot be simultaneously satisfied within scenario (32a)**

(33) **The Falsity of (29) in Situations of a Multiple Collective Huggings**

a. Situation: Single Collective Hugging of Three Boys

Hugs	Agent	Theme
e_1	speaker	$kid_1+kid_2+kid_3$
e_2	speaker	$kid_1+kid_2+kid_3$
e_3	speaker	$kid_1+kid_2+kid_3$

b. Proposition: Formula (30), the translation of (29), is *false* in such a scenario.

$[x_1]$ & $x_1 = 1$ & $*child(x_1)$ & $three(x_1)$& $[e_2]$ & $e_2 = 1$ & $*hug(e_2)$ & $*Agent(e_2 , speaker)$ & $Max^{3,4}$. $e_3 > n$ & $e_3 \leq e_2$ & $atom(e_3)$ & $x_4 \leq x_1$ & $atom(x_4)$ & $*Theme(e_3 , x_4)$

d. Explanation:

- Let *G* be any set of assignment functions.
- It will *not* be possible to construct a set of assignment functions *H* such that $[[(30)]]^{<G,H>}$
  - In order for $[[*hug(e_2)]]^{<G,H>}$ = T, it must be that h(2) is an event of hugging for all $h \in H$
  - In order for $[[e_3 \leq e_2$ & $atom(e_3)]]^{<G,H>}$ = T, it must be that h(3) is an atomic sub-event of h(2) for all $h \in H$
  - In order for $[[*Theme(e_3 , x_4)]]^{<G,H>}$ = T, it must be that h(4) is the theme of h(2) for all $h \in H$
  - **Finally, in order for $[[atom(x_4)]]^{<G,H>}$ = T, it must be that h(4) is an atom, for all $h \in H$**
    - **But, there is no (sub-)event of hugging in (33a) with an atomic theme. Any such theme h(4) will be the plurality $kid_1+kid_2+kid_3$**
    - **Thus, we see that these conditions cannot be simultaneously satisfied within scenario (33a)**

(34) **Interim Summary**
We've just seen that the semantics in (24) predicts that pluractional sentences like (29) – which have a plural direct object – must receive distributive readings.
- Such sentences are true when each child was hugged individually.
- Such sentences are false if the children are hugged collectively, *even if they are hugged collectively multiple times.*

(35) **A Pluractional Sentence with Singular Direct Object**

Xintiki**la'**	jun	che'
1sgS.plant.**PA**	one	tree

I planted a tree (repeatedly) TRUE if I planted the same tree numerous times.

(36) **The DPlL Translation of (35)**

$[x_1]$ & $x_1 = 1$ & *tree(x_1) & atom(x_1)& $[e_2]$ & $e_2 = 1$ & *plant(e_2) & *Agent(e_2 , speaker) & Max ^{3,4} . $e_3 > n$ & $e_3 \leq e_2$ & atom(e_3) & $x_4 \leq x_1$ & atom(x_4) & *Theme(e_3 , x_4)

(37) **The Truth of (36) in Situations of Repeated Planting of the Same Tree**

a. Situation: Multiple Plantings of the Same Tree

Plantings	Agent	Theme
e_1	speaker	tree_1
e_2	speaker	tree_1
e_3	speaker	tree_1

b. Proposition: Formula (36), the translation of (35), is *true* in such a scenario.

c. Explanation:

- Let *G* be any set of assignment functions.
- Clearly, we could create a set of assignment functions *H* such that *G*[1,2,3,4]*H*, and *H* appears as follows:

H	1	2	3	4
h_1	tree_1	$e_1+e_2+e_3$	e_1	tree_1
h_2	tree_1	$e_1+e_2+e_3$	e_2	tree_1
h_3	tree_1	$e_1+e_2+e_3$	e_3	tree_1

- Now, note that:

$\mid H[1] \mid = \mid \{ \text{tree}_1 \} \mid = 1$	*Thus,* $[[x_1 = 1]]^H = T$
h(1) is a plurality of trees, for all $h \in H$	*Thus,* $[[\text{*tree}(x_1)]]^H = T$
h(1) is an atom for all $h \in H$	*Thus*, $[[\text{atom}(x_1)]]^H = T$
$\mid H[2] \mid = \mid \{ e_1+e_2+e_3 \} \mid = 1$	*Thus,* $[[e_2 = 1]]^H = T$
h(2) is a plurality of plantings, for all $h \in H$	*Thus,* $[[\text{*plant}(e_2)]]^H = T$
The *agent of h(2) is the speaker, for all $h \in H$	*Thus,* $[[\text{*Agent}(e_2 ,\text{sp})]]^H = T$
$\mid H[3] \mid = \mid \{e_1 , e_2 , e_3\} \mid = 3 > n$	*Thus,* $[[e_3 > n]]^H = T$
$h(3) \leq h(2)$, for all $h \in H$	*Thus,* $[[e_3 \leq e_2]]^H = T$
h(3) is an atom, for all $h \in H$	*Thus,* $[[\text{atom}(e_3)]]^H = T$
$h(4) \leq h(1)$, for all $h \in H$	***Thus,* $[[x_4 \leq x_1]]^H = T$**
h(4) is an atom, for all $h \in H$	***Thus,* $[[\text{atom}(x_4)]]^H = T$**
h(4) is the theme of h(3) for all $h \in H$	*Thus,* $[[\text{*Theme}(e_3, x_4)]]^H = T$

… *and*, since *H*[3] contains all the events of planting, and *H*[4] contains all the trees planted, it follows that any other set *H'* that satisfies the conditions above, will be such that *H'*[3] $\subseteq$ *H*[3] and *H'*[4] $\subseteq$ *H*[4]

- Therefore, by the definitions above, it follows that:
  (i) $[[(36)]]^{<G,H>}$, and therefore:
  (ii) $[[(36)]]^{G}$

(38) **Interim Summary**

We've just seen that – unlike sentences like (29) – sentences like (35), which have a *singular* direct object, *can* get 'repetitive' readings.

- Such sentences are true when the *same* tree is planted repatedly.

Thus, when combined with the result in (34), we see that the account captures Henderson's (2011:) generalization that…

> "…the distributivity entailments of the pluractional are conditionalized. The object of a transitive verb must be interpreted distributively if possible, otherwise the only requirement is that a simple plurality of events satisfies the predicate…"

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4. The Semantics of Distributive Numerals in Kaqchikel

Recall the following key facts regarding Kaqchikel distributive numerals and their interaction with pluractional morphology.

- As shown in (39b), distributive numerals can appear to 'scope below' pluractional operators;
  - o Sentences like (39b) require a reading where there are three (different) tortillas in each event of touching.

- As shown in (39a), plain numerals *cannot* 'scope below' pluractional operators;
  - o Sentences like (39a) require a reading where the *same* tortillas are touched in each event.

- As shown in (40), distributive numerals need a quantificational 'licenser'. They cannot appear on their own in a sentence.
  - o This shows that Kaqchikel distributive numerals do not on their own yield a 'pluractional' reading (unlike Korean, German; like English)

(39) **Scope With Respect to the Pluractional Operator**

a. Xinchap**ala'** oxi' way
1sgS.touch.**PA** three tortilla
I touched three tortillas individually.
- o TRUE if speaker touched exactly three tortillas separately.

b. Xinchap**ala'** **ox-ox** way
1sgS.touch.**PA** **DIST-three** tortilla
I touched tortillas in threes.
- o TRUE only if there are multiple events of the speaker touching three tortillas
- o FALSE if the speaker touched exactly three tortillas (separately or no)

(40) **No Distributive Numerals Without a Quantificational Licenser**

*Xinchap	**ox-ox**	way
1sgS.touch	**DIST-three**	tortilla

4.1 The Formal Semantic Analysis

(41) **First Important Ingredient: The Concept of a 'Test'**

a. So far, we've been interpreting formulae relative to pairs of sets of assignment functions (*e.g.* <G, H>)

b. Now, we're going to interpret formulae relative to pairs of *pairs* consisting of:
(i) A set of assignment functions *G, H, J*
(ii) A set of formulae, called **tests** ζ , ζ' , ζ''

c. <u>The Interpretation Function:</u> $[[\ \varphi\ ]]^{< G[\zeta\ ]\ ,\ H[\zeta']\ >}$

(42) **Introducing Tests into The Valuation**

a. The introduction of 'tests' in (41) will not affect our definitions in (14).
- The formulae in (14) will not be sensitive to the 'tests' parameters at all.

b. The only 'test'-sensitive formulae in our language will the ones in (42c) below.
- We will add the rule in (42ci) to our syntactic rules in (13b)
- We will add the rule in (42cii) to our semantic rules in (14)

c. <u>Introducing Tests into the Valuation</u>

(i) *Formulas for Tests*
If 'ϕ' is a formula, then '$^{\phi}$' is also a formula

(ii) *Interpreting 'Test Formulas'*
$[[\ ^{\phi}\ ]]^{< G[\zeta\ ]\ ,\ H[\zeta']\ >}$ = T *iff* G = H and $\zeta' = \zeta \cup \{ \phi \}$

(43) **The Effect of 'Tests' Upon the Definition of Truth**

- 'Tests' will not be relevant for truth relative to a *pair* of assignment sets.
  - Rather, as noted in (42) truth relative to a pair of assignment sets is essentially the same as before.
  - The only change is that 'test formulae' like '$^{\phi}$' add additional formulae to the set of tests.
- 'Tests' *will* be relevant for truth relative to a *single* assignment, as shown below.

(44) **Definition of Truth Relative to a Set of Assignment functions**

Let G be a set of assignment functions. Then $[[\phi]]^{G}$ = T *iff*

a. There is a set of assignment functions H, and

b. There is a set of 'tests' $\zeta = \{ \psi_1 , \ldots , \psi_n \}$ such that:

c. $[[\phi]]^{< G[\varnothing] , H[\zeta] >}$ = T, and
- That is, for all the sub-formulae φ that don't introduce tests, $[[\varphi]]^{< G, H >} = T$
- And, ζ contains all those (superscripted) formulae that *do* introduce tests.

d. $[[\psi_1 \& \ldots \& \psi_n]]^{< H[\varnothing] , H[\varnothing]>}$
- That is, all the 'test' formulae are true in the final assignment set *H*

(45) **Summary: The Concept of a 'Test'**

- A 'test' is a formula which is not *immediately* interpreted, but is basically 'passed on' until the very final stage of interpretation.
  - Thus, they are conditions that the 'output context' has to satisfy.
- In this sense, they are kind of the inverse of presuppositions (conditions that the 'input context' has to satisfy)
  - For this reason, Brasoveanu (2010) refers to 'tests' as 'post-suppositions'

(46) **The Semantics of Distributive Numerals in Kaqchikel**

Distributive numerals introduce a special 'test', requiring that the 'output' set of assignments map the index of the numeral to more than one thing…

a. *ju-jun* 'one.DIST' → $[x_i]$ & $^{xi > n}$ & atom(x_i),
b. *ox-ox* 'three.DIST' → $[x_i]$ & $^{xi > n}$ & three(x_i), *etc.*

4.2 Applying the Formal Analysis

(47) **Pluractional Verb with Distributive Numeral Object**

Xinpiskoli**la'**	**ju-jun**	way
1sgS.flip.**PA**	**DIST-one**	tortilla

I flipped tortillas one by one.
- TRUE only if there are multiple events of the speaker flipping different tortillas
- FALSE if the speaker flipped only the same tortilla multiple times

(48) **The DPlL Translation of (47)**

$[x_1]$ & $^{x1 > n}$ & atom(x_1) & *tortilla(x_1) & $[e_2]$ & $e_2 = 1$ & *flip(e_2) & *Agent(e_2, speaker) & Max $^{3, 4}$. $e_3 > n$ & $e_3 \leq e_2$ & atom(e_3) & $x_4 \leq x_1$ & atom(x_4) & *Theme(e_3 , x_4)

(49) **The Truth of (47) in Situations of Multiple Flippings of Different Tortillas**

a. Situation: Multiple Flippings of Different Tortillas

Flippings	Agent	Theme
e_1	speaker	tortilla$_1$
e_2	speaker	tortilla$_2$
e_3	speaker	tortilla$_3$

b. Proposition: Formula (48), the translation of (47), is *true* in such a scenario.

c. Explanation:

- Let *G* be any set of assignment functions.
- Clearly, we could create a set of assignment functions *H* such that *G*[1,2,3,4]*H*, and *H* appears as follows:

H	1	2	3	4
h_1	tortilla$_1$	e_1+e_2+e_3	e_1	tortilla$_1$
h_2	tortilla$_2$	e_1+e_2+e_3	e_2	tortilla$_2$
h_3	tortilla$_3$	e_1+e_2+e_3	e_3	tortilla$_3$

- Now, note that $[[(48)]]^{< G[\varnothing], H[x1 > n] >} = T$

$\{ x_1 > n \} = \varnothing \cup \{ x_1 > n \}$	*Thus,* $[[^{x1>n}]]^{< G[\varnothing], H[x1>n] >} = T$
h(1) is an atom *for all* h ∈ H	*Thus*, $[[\text{atom}(x_1)]]^{H} = T$
h(1) is a plurality of tortillas, for all *h* ∈ H	*Thus,* $[[\text{*tortilla}(x_1)]]^{H} = T$
$\mid H[2] \mid = \mid \{ e_1+e_2+e_3 \} \mid = 1$	*Thus,* $[[e_2 = 1]]^{H} = T$
h(2) is a plurality of flippings, for all *h* ∈ H	*Thus,* $[[\text{*flip}(e_2)]]^{H} = T$
The *agent of h(2) is the speaker, for all *h* ∈ H	*Thus,* $[[\text{*Agent}(e_2, sp)]]^{H} = T$
$\mid H[3] \mid = \mid \{e_1, e_2, e_3\} \mid = 3 > n$	*Thus,* $[[e_3 > n]]^{H} = T$
h(3) ≤ h(2), for all *h* ∈ H	*Thus,* $[[e_3 \leq e_2]]^{H} = T$
h(3) is an atom, for all *h* ∈ H	*Thus,* $[[\text{atom}(e_3)]]^{H} = T$
h(4) ≤ h(1), for all *h* ∈ H	*Thus,* $[[x_4 \leq x_1]]^{H} = T$
h(4) is an atom, for all *h* ∈ H	*Thus,* $[[\text{atom}(x_4)]]^{H} = T$
h(4) is the theme of h(3) for all *h* ∈ H	*Thus,* $[[\text{*Theme}(e_3, x_4)]]^{H} = T$

… *and*, since *H*[3] contains all the events of flipping, and *H*[4] contains all the tortillas flipped, it follows that any other set *H'* that satisfies the conditions above, will be such that *H'*[3] ⊆ *H*[3] and *H'*[4] ⊆ *H*[4]

- Finally, note that the test $[[x_1 > n]]^{<H,H>} = T$. **Thus, by the definitions above,** $[[(48)]]^{G} = T$

(50) **The Falsity of (47) in Situations of Multiple Flippings of the Same Tortilla**

a. <u>Situation: Multiple Flippings of Different Tortillas</u>

Flippings	Agent	Theme
e_1	speaker	tortilla$_1$
e_2	speaker	tortilla$_1$
e_3	speaker	tortilla$_1$

b. <u>Proposition:</u> Formula (48), the translation of (47), is *false* in such a scenario.

$[x_1]$ & $^{x1 > n}$ & atom(x_1) & *tortilla(x_1) & $[e_2]$ & $e_2 = 1$ & *flip(e_2) & *Agent(e_2, speaker) & Max $^{3, 4}$. $e_3 > n$ & $e_3 \leq e_2$ & atom(e_3) & $x_4 \leq x_1$ & atom(x_4) & *Theme(e_3 , x_4)

e. <u>Explanation:</u>

- Let *G* be any set of assignment functions.

- It will *not* be possible to construct a set of assignment functions *H* such that $[[(48)]]^{< G[\varnothing], H[x1>n] >} = T$ and $[[x_1 > n]]]]^{< H[\varnothing], H[\varnothing] >} = T$

    - In order for $[[\text{atom}(x_1) \,\&\, {*}\text{tortilla}(x_1)]]^{<G,H>} = T$, it must be that h(1) is a single tortilla for all $h \in H$

    - In order for $[[{*}\text{flip}(e_2)]]^{<G,H>} = T$, it must be that h(2) is an event of flipping for all $h \in H$

    - In order for $[[e_3 \leq e_2 \,\&\, \text{atom}(e_3)]]^{<G,H>} = T$, it must be that h(3) is an atomic sub-event of the flipping event h(2) for all $h \in H$

    - In order for $[[{*}\text{Theme}(e_3 , x_4)]]^{<G,H>} = T$, it must be that h(4) is the theme for h(3) for all $h \in H$
        - Therefore, h(4) must be that tortilla I flipped (tortilla$_1$) in all $h \in H$

    - **Finally,** in order for $[[x_4 \leq x_1 \,\&\, \text{atom}(x_4)]]^{<G,H>} = T$, it must be that h(4) is an atomic sub-part of the tortilla h(1) for all $h \in H$

    - **Therefore, it follows that h(1) must be that same tortilla that I flipped (tortilla$_1$) for all $h \in H$**

    - **Therefore, it follows that H[1] = 1, and so $[[x_1 > n]]^{<H[\varnothing],H[\varnothing]>} = F$**

    - **Therefore, by the definitions in (44), it follows that $[[(48)]]^{G} = F$ in the scenario in (50a)**

We've just seen how the analysis of distributive numerals and pluractionals works together to predict the facts in (47)…

Now let's turn to the facts in (40), repeated below…

(51) **No Distributive Numerals Without a Quantificational Licenser**

*Xinchap	**ox-ox**	way
1sgS.touch	**DIST-three**	tortilla

(52) **The DPlL Translation of (51)**

$[x_1]$ & $^{x1 > n}$ & three(x_1) & *tortilla(x_1) & $[e_2]$ & $e_2 = 1$ & *touch(e_2) & *Agent(e_2, speaker) & *Theme(e_2 , x_1)

(53) **The Contradictory Status of (51)**

a. <u>Proposition:</u> Formula (52), the translation of (51) can *never* be satisfied.

b. <u>Explanation:</u>

- Let *G* be any set of assignment functions.

- It will not be possible to construct a set of assignment functions *H* such that $[[(52)]]^{< G[\varnothing], H[x1>n] >} = T$ and $[[\, x_1 > n \,]]^{< H[\varnothing], H[\varnothing] >} = T$

    - In order for $[[e_2 = 1$ & *touch(e_2)$]]^{<G,H>}$ = T, it must be that every h ∈ H maps 2 to the same touching event.

    - In order for [[*Theme(e_2 , x_1)$]]^{<G,H>}$ = T, it must be that h(1) is the theme of h(2) for all *h* ∈ H

    - Since there is only one theme for any event ('*Theme' is a function)…
        - And since every *h* ∈ H maps 2 to the same touching event…
            - It follows that every *h* ∈ H maps 1 to the same entity

    - **Therefore, it follows that H[1] = 1, and so $[[\, x_1 > n \,]]^{<H[\varnothing],H[\varnothing]>} = F$**

    - **Therefore, by the definitions in (44), it follows that $[[(52)]]^G = F$, *regardless of the situation.***

(54) **Interim Summary**
The 'post-suppositional' semantics for distributive numerals in (46) predicts that:

a. Sentences like (39b) and (47) – which contain a pluractional affix and a distributive numeral – receive an obligatory 'narrow' scope interpretation.
- Such sentences are only true if each event contains a *different* theme.

b. Sentences like (40) – which contain only a distributive numeral and no 'licensing' quantifier – will be ill-formed.
- Given that the unmarked V introduces only a *single* event ('$e_2 = 1$'), it follows that the 'test' introduced by the distributive numeral will be unsatisfiable.

4.3 A Potential Empirical Problem: Distributive Numerals Higher than One

(55) **Sentence Containing 'Distributive Three'**

Xinchap**ala'**	**ox-ox**	way
1sgS.touch.**PA**	**DIST-three**	tortilla

I touched tortillas in threes.

a. TRUE only if there are multiple events of the speaker touching three tortillas
b. FALSE if the speaker touched exactly three tortillas (separately or no)

(56) **The DPlL Translation of (55) [Based Upon (48)]**

$[x_1]$ & $^{x1 > n}$ & three(x_1) & *tortilla(x_1) & $[e_2]$ & $e_2 = 1$ & *touch(e_2) & *Agent(e_2, speaker) & Max ^{3,4} . $e_3 > n$ & $e_3 \leq e_2$ & atom(e_3) & $x_4 \leq x_1$ & atom(x_4) & *Theme(e_3 , x_4)

(57) **The Potential Problem**

a. As argued below, the formula in (56) will be *false* in a scenario like the following, where I *collectively* touched three different tortillas each time.

Touchings	Agent	Theme
e_1	speaker	tortilla$_1$+tortilla$_2$+tortilla$_3$
e_2	speaker	tortilla$_4$+tortilla$_5$+tortilla$_6$
e_3	speaker	tortilla$_7$+tortilla$_8$+tortilla$_9$

b. Henderson (2011) doesn't explicitly say whether (55) is false in such scenarios, but sentences comparable to (55) in other languages with distributive numerals are *true* in such scenarios (Gil 1982; Oh 2001, 2005; Zimmermann 2002)[3]

(58) **The Falsity of Formula (56) in Situation (57a)**

Let *G* be any set of assignment functions. It will not be possible to construct a set of assignment functions *H* such that

- $[[(56)]]^{< G[\varnothing], H[x1>n]>} = T$ and
- $[[x_1 > n]]]]^{< H[\varnothing], H[\varnothing]>} = T$

a. In order for $[[\text{*touch}(e_2)]]^{<G,H>} = T$, it must be that h(2) is an event of touching for all $h \in H$.

b. In order for $[[e_3 \leq e_2 \text{ \& atom}(e_3) \text{ \& *Theme}(e_3, x_4)]]^{<G,H>} = T$, it must be that h(4) is the theme of some flipping event for all $h \in H$.

[3] Henderson (p.c.) also reports a sentence parallel to (55) but containing the verb for 'eat' *must* describe scenarios where there are multiple *collective* eatings of three tortillas ('speakers are adamant that (such sentences) mean you are holding a stack of tortillas in your hand and eating them like a tortilla sandwich'). In his dissertation work, Henderson is developing a revised analysis of pluractionals that can handle these facts.

c. **Finally, in order for $[[\text{atom}(x_4)]]^{<G,H>}$ = T, it must be that h(4) is an atom for all $h \in$ H.**

d. **However, in the scenario under (57a), no theme of a touching event is an atom.**

e. **Therefore, the formula (56) cannot be satisfied in scenario (57a)**

The General Issue:

- According to Henderson's (2011) analysis, a pluractional affix requires each atomic sub-event of the plural event to have an *atomic* theme.
  - This assumption was essential for deriving the fact that a pluractional verb with a plural direct object *necessitates* a distributive reading (29)-(33)
- Thus, regardless of whether the direct object bears a distributive numeral, this semantics predicts that a sentence with a pluractional verb will be *false* in scenarios like (57a)

(59) **Fun Exercise for the Reader**
Show that the semantics in (56) predicts that (55) *will* be true in scenarios like the following, where the speaker *distributively* touched various triplets of tortillas

Touchings	Agent	Theme
e_1	speaker	tortilla_1
e_2	speaker	tortilla_2
e_3	speaker	tortilla_3
e_4	speaker	tortilla_4
e_5	speaker	tortilla_5
e_6	speaker	tortilla_6
e_7	speaker	tortilla_7
e_8	speaker	tortilla_8
e_9	speaker	tortilla_9

(60) **General Empirical Question**
Is the Kaqchikel sentence in (55) true only in scenarios like (59)? Or, can it also be true in scenarios like (57)?

- As noted in footnote 3 above, the answer seems to be that (55) *is* true *only* in 'collective' scenarios like (57), and not in 'distributive' ones like (59).
- In his dissertation work, Henderson is exploring a slight revision to the semantics in (24), one that could avoid the problematic predictions noted above.

5. Summary of the Account

Let us wrap up by reviewing the facts that we sought to explain and how they were captured.

(61) **Fact 1: The Interaction Between Pluractionals and Distributive Numerals**

- Sentence (61a), with a plain numeral, requires that there be at least one book that the speaker looked for repeatedly.

- Sentence (61b), with a distributive numeral, requires that the speaker looked for different books.

a. Xinkan**ala'** jun wuj
1sgS.searched.**PA** one book
I looked for a book (various times)

b. Xinkan**ala'** **ju-jun** wuj
1sgS.searched.**PA** **DIST-one** book
I looked for books (various times).

(62) **Explanation of Fact 1: The Introduction of 'Test' Conditions**

a. The distributive numeral *ju-jun* in (61b) introduces the *test* '$^{xi > n}$', which requires that it be possible to map variable introduced by *ju-jun* to *multiple* entities.
  - This in turn entails that there must be more than book that was sought.

b. The plain numeral *jun* in (61) introduces the condition 'x_i = n', which requires that the variable introduced by *jun* be mapped to the *same* entity.
  - Given the presence of the pluractional affix, in turn entails that the same book was sought multiple times.

(63) **Fact 2: The Interaction Between Pluractionals and Distributivity**

- Sentence (63a), with an unmarked verb, allows that the 'hugging' was either collective or distributive.

- Sentence (63b), with a pluractional verb, requires that the 'hugging' have been distributive.

a. Xe'inq'etej ri ak'wala
1sgS.hugged the children
I hugged the children. (separately or together)

b. Xe'inq'ete**la'** ri ak'wala
1sgS.hugged.**PA** the children
I hugged the children. (separately, not together)

(64) **Explanation of Fact 2: The Obligatory Distributivity of Pluractional Operators**

- The meaning of the pluractional affix requires that the atomic sub-events of the plural event denoted by the verb have atomic themes.

- It also requires that each of these atomic themes be sub-parts of the direct object of the pluractional verb.

- Consequently, a pluractional sentence like (63b) will only be true if the plural direct object 'children' can be broken into atomic sub-parts, each of which is theme to an atomic event of hugging.

Key Consequence: If the direct object of the pluractional verb is *already* atomic, then a purely repetitive reading should be possible.

a. Xintiki**la'** jun che'
1sgS.plant.**PA** one tree
I planted a tree (repeatedly) [TRUE if I planted the same tree numerous times]

(65) **Possible Problematic Consequence**

- As noted above, the semantics given for the pluractional suffix ***la'*** entails that each sub-event of the plural event have an atomic theme.

- This entails that even the sentence in (65a) – whose direct object contains a distributive numeral – will not be able to describe cases of repeated *collective* action.

a. Xinchap**ala'** **ox-ox** way
1sgS.touch.**PA** **DIST-three** tortilla
I touched tortillas in threes.
Prediction: False if I touched each triplet collectively.
Ture if I touched each triplet distributively.

- This seems to be a false prediction for Kaqchikel, but in his dissertation work, Henderson (p.c.) is working on a possible solution…

(66) **Fact 3: The Licensing of Distributive Numerals**

Distributive numerals cannot appear 'on their own'. They must be in the scope of some kind of quantificational 'licenser'.

a. *Xinchap **ox-ox** way
1sgS.touch **DIST-three** tortilla
(NOT: *I touched tortillas in threes.*)

(67) **Explanation of Fact 3: The 'Singularity' of the Event Predicate**

- In the absence of pluractional morphology, the direct object in (66a) must be theme to the event introduced by the verb.

- The main verb introduces the condition '$e_2 = 1$', and so it requires that variable introduced by the verb be mapped to the same event.

- This, in turn, requires the variable introduced by the direct object to be mapped to the same entity, *and so the 'test' condition introduced by the distributive numeral cannot be satisfied.*

(68) **A Minor Point of Criticism**

- The key facts in (63) and (66) (and possibly (65)) seem rather specific to Kaqchikel.

    - From what we've seen, they don't seem to hold in other languages with pluractionals or distributive numerals.

    - Though, as Henderson (p.c.) mentions, the facts in (66) do indeed hold in other languages, such as Hungarian and Romanian.

- Since the prediction of those facts is indelibly tied to the semantics given for pluractional morphemes and distributive numerals, one may rightly worry about the 'exportability' of this analysis to other languages…

    - Though Henderson (p.c.) does have specific ideas regarding the cross-linguistic typology, which he is developing as part of his dissertation research