The Proper Treatment of Quantification in Ordinary English, Part 2: Intensional Logic

In these notes, we will explore the translation language employed in PTQ, Intensional Logic

(1) **Why Intensional Logic?**

- Our English fragment includes such words as **seek**, **conceive**, **believe that**, **wish to**, **allegedly**, **necessarily**, and **about**
  - Notably, these expressions must take the *intensions* of their complements as argument (rather than the *extensions* of their complements)

- Consequently, we will want the induced semantics for the English fragment to end up mapping some English expressions to *their intensions*.

- Thus, since we're developing an indirect interpretation of the fragment, we'll need a logical language that can represent such intensions.

- In addition, this language will have the ability to represent the semantics of the modal elements **necessarily**, *present perfect*, and *future*.

(2) **Some Conceptual Background to Intensional Logic**

- In LING 620, you learned that in a sentence like (2a) below, <u>the extension</u> of *thinks* takes as argument <u>the intension</u> of *Mary smokes*.

a. John thinks [that Mary smokes]

- You may also have considered an analysis where the finite complentizer *that* has the meaning in (2b), which entails that *that Mary smokes* will have the extension in (2c)

b. $[[\text{ that XP }]]^{w} = [\lambda w' : [[\text{XP}]]^{w'}]$

c. (i) $[[\text{ that Mary smokes }]]^{w} =$
$[\lambda w' : [[\text{Mary smokes}]]^{w'}] =$
$[\lambda w' : \text{Mary smokes in } w']$

- Thus, the **extension** of *that S* is equal to the **intension** of *S*.

- It probably wasn't noted at the time, but observe that *that S* in such a semantics also itself has an intension:

d. $[\lambda w : [[\text{ that Mary smokes }]]^{w}] = [\lambda w : [\lambda w' : \text{Mary smokes in } w']]$

- Thus, the intension of *that S* is a <u>constant function</u> mapping every world to the intension of *S*.

(3) **The Upshot of All This**
The language IL will have an operator (^) that behaves just like *that* in our analysis above

1. The Syntax of Intensional Logic (IL)

To define the meaningful expressions of IL, we begin by defining its vocabulary. And, to define its vocabulary, we must first define our system of (intensional) types.

(4) **The System of (Intensional) Types** The set *Type* is the smallest set such that:

a. e, t $\in$ *Type*
b. If $\sigma, \tau \in$ *Type*, then $<\sigma, \tau> \in$ *Type*
c. If $\tau \in$ *Type*, then $<s, \tau> \in$ *Type*

(5) **The Vocabulary of IL**

a. <u>The Logical Constants</u>

(i)	*Sentence Connectives:*	¬	Negation	'it is not the case that'
		∧	Conjunction	'and'
		∨	Disjunction	'or' (inclusive)
		→	Implication	'if…then'
		↔	Biconditional	'if and only if'
(ii)	*Identity*	=	Identity	'equals'
(iii)	*Quantifiers*	V	Existential	'there is an…'
		Λ	Universal	'for all…'
(iv)	*Lambda*	λ	Lambda	'the function that…'
(v)	*Modal Operators*	□	Necessity	'it is necessary that...'
		W	Future	'it will be the case that...'
		H	Past	'it has been the case that...'
(vi)	*Intension/Extension Ops*	^	Up	'the intension of…'
		˅	Down	'the extension of…'

The important new additions! [(v) and (vi)]

b. <u>The Syntactic Symbols:</u> (,), [,]

c. <u>The Non-Logical Constants</u> For every type $\tau \in$ *Type*

(i) An infinite set of constants of type τ, Con_τ

(ii) A countably infinite set of variables of type τ, Var_τ

$$Var_\tau = \{v_{n,\tau} : n \in \mathbb{N}\}$$

(6) **Remarks**

- The variable $v_{n,\tau}$ will sometimes be referred to as 'the n[th] variable of type τ'

- For some reason, Montague doesn't enumerate the constants (probably because he doesn't have to, unlike the variables).

- Montague also never specifies *how* the constants of IL should look.

  - However, he uses *j, m, b, n* as meta-language labels for constants of type e

  - Similarly, he uses, **run'**, **man', love'** as meta-language labels for constants of predicative types

- We will follow suit, and temporarily assume the following:
  - $j, m, b, n \in Con_e$
  - **smoke'**, **run'**, **man'** $\in Con_{<e,t>}$
  - **like'** $\in Con_{<e,<e,t>>}$
  - **think'** $\in Con_{<<s,t>,<e,t>>}$

(7) **The Syntax of Intensional Logic**

a. Every variable and constant of type τ is in ME_τ

b. If $\varphi \in ME_\tau$ and u is a variable of type σ, then $\lambda u\varphi \in ME_{<\sigma,\tau>}$

c. If $\varphi \in ME_{<\sigma,\tau>}$ and $\psi \in ME_\sigma$, then $\varphi(\psi) \in ME_\tau$

d. If $\varphi,\psi \in ME_\tau$, then $\varphi = \psi \in ME_t$

e. If $\varphi, \psi \in ME_t$ and u is a variable, then
   (i) $\neg\varphi \in ME_t$
   (ii) $[\varphi \wedge \psi] \in ME_t$
   (iii) $[\varphi \vee \psi] \in ME_t$
   (iv) $[\varphi \rightarrow \psi] \in ME_t$
   (v) $[\varphi \leftrightarrow \psi] \in ME_t$
   (vi) $\mathrm{V}u\varphi \in ME_t$
   (vii) $\Lambda u\varphi \in ME_t$
   (viii) $\Box\varphi \in ME_t$
   (ix) $\mathrm{W}\varphi \in ME_t$
   (x) $\mathrm{H}\varphi \in ME_t$

f. If $\varphi \in ME_\tau$, then $[{}^\wedge\varphi] \in ME_{<s,\tau>}$

g. If $\varphi \in ME_{<s,\tau>}$, then $[{}^\vee\varphi] \in ME_\tau$

(8) **Meaningful Expression of IL** φ is a meaningful expression of IL if $\varphi \in \cup_{\tau \in Type} ME_\tau$

(9) **Remarks**

a. Note that in (7c) the notation for a predicate applied to its argument is $\varphi(\psi)$, and not $(\varphi\ \psi)$, as in TL.

b. Note that '=' can appear between any two expressions of the same type. Thus, all the following are meaningful expressions:
   (i) $j = b$
   (ii) **man'** = **run'**
   (iii) **man'**(j) = **run'**(b)
   Our semantics for '=' will entail that **man'**(j) = **run'**(b) is logically equivalent to [**man'**(j) $\leftrightarrow$ **run'**(b)]

c. "The expression [^α] is regarded as denoting (or as having as its *extension*) the *intension* of the expression α" (pp. 23-24)
   - Thus, the operator '^' is akin to English *that* in (2b)

d. "The expression [$^{\vee}\alpha$] is meaningful only if α is an expression that denotes an intension or sense; in such a case [$^{\vee}\alpha$] denotes the corresponding extension."

e. "In the presentation of actual expressions, of intensional logic, square brackets will sometimes for perspicuity be omitted, and sometimes gratuitously inserted."

(10) **Some Illustrative Meaningful Expressions of IL**

a. $\lambda v_{1,e}$ [**smoke'**($v_{1,e}$) $\wedge$ **run'**($v_{1,e}$)]
   Informally: The characteristic function of the set of smokers that run.

b. $\lambda v_{1,<e,t>}\ v_{1,<e,t>}(j)$
   Informally: The characteristic function of the set of properties true of John.
   (≈ (λP_1 (P_1 **john'**)))

c. $\lambda v_{1,<e,t>}\ \mathrm{V} v_{1,e}$ [**man'**($v_{1,e}$) $\wedge$ $v_{1,<e,t>}$ ($v_{1,e}$)]
   Informally: The characteristic function of the set of properties true of a man
   (≈ ($\lambda P_1\ \exists x_1$ ((**man'** x_1) & ($P_1\ x_1$))))

d. **think'**([^**like'**(j)(m)])(b)
   Informally: 'Bill thinks that Mary likes John'

e. $\Box$**run'**(j)
   Informally: 'It is necessary that John runs'

f. $\Lambda v_{1,e}$ H **smoke'**($v_{1,e}$)
   Informally: 'For all x, it has been the case that x smokes.'

2. The Semantics of Intensional Logic (IL)

In this section, we will develop a formal model-theoretic semantics for IL. As usual, we begin by defining the notion '*denotations of type τ*'

- As the following definition suggests, our model structures will based on a set of entities A, **a set of worlds I, and a set of times J.**

(11) **The Denotations**

Let A, I, and J be non-empty sets (A = set of entities; I = set of possible worlds; J = set of moments of time). If $\tau \in Type$, then the set $D_{\tau, A, I, J}$ of *possible denotations of type τ corresponding to A, I, J* is defined as follows:

a. $D_{e,A,I J}$ = A

b. $D_{t,A,I,J}$ = { 0, 1 }

c. If $\sigma, \tau \in T$ then $D_{<\sigma,\tau>,A,I,J}$ = the set of functions from $D_{\sigma ,A,I,J}$ to $D_{\tau,A,I,J}$

d. If $\sigma \in T$ then $D_{<s,\sigma>,A,I,J}$ = the set of functions from I×J to $D_{\sigma ,A,I,J}$ = $(D_{\sigma ,A,I, J})^{I \times J}$

<u>Notes:</u>

- In PTQ, the set J is the set of times (moments of time); it's not a set of contexts/variable assignments (unlike in UG).
- In PTQ, a denotation of type <s,σ> is a function from *world-time* pairs to denotations of type σ

In addition to the denotations in (11), our model theoretic semantics for IL will make reference to a set of 'senses':

(12) **The Senses**

Let A, I, and J be non-empty sets (A = set of entities; I = set of possible worlds; J = set of moments of time). If $\tau \in Type$, then the set $S_{\tau, A, I, J}$ of *possible senses of type τ corresponding to A, I, J* is equal to $D_{<s, \tau,>,A,I,J}$

With these ingredients, we can now define what a model of IL is…

(13) **A Model of IL**

An intensional model (or interpretation) of IL is a quintuple <A, I, J, ≤, F> such that:

a. A, I, J are non-empty sets

b. ≤ is a linear ordering of J

c. F is a function whose domain is the set of constants of IL

d. Whenever $\tau \in Type$ and $\alpha \in Con_\tau$, $F(\alpha) \in S_{\tau, A, I, J}$

(14) **Remarks**
Thus, an (intensional) model of IL is defined by: (i) a set of entities A, (ii) a set of worlds I, (iii) a set of times J, (iv) a temporal ordering $\leq$, and (v) a 'lexical' interpretation function F.

- Note that the function F maps the constants of IL **to intensions, *not* denotations.**
    - Thus, type e constants are mapped to <s,e> functions (individual concepts)
    - Type <et> constants are mapped to <s,<e,t>> functions (properties)
    - Type <e,<e,t>> constants are mapped to <s,<e,<e,t>>> functions ('relations in intension'), *etc*….

- Models of IL are too complex to specify concretely. Nevertheless, to illustrate key components of the definitions here, we can attempt the following partial description.

(15) **Illustration of an Intensional Model (for IL)**
Let $\mathcal{M}$ be an intensional model <A, I, J, $\leq$, F> such that:

a. A = {Barack, Michele}
b. I = $\{ w_1 , w_2 \}$
c. J = $\{ t_1 , t_2 , t_3 \}$
d. $\leq$ = $\{ <t_1 , t_2> , <t_1 , t_3> , <t_2 , t_3>, <t_1 , t_1> , <t_2 , t_2> , <t_3 , t_3> \}$
e. F comprises at least the following mappings:

F(*b*) =	w_1 , t_1	→	Barack	;	w_2, t_1	→	Barack
	w_1 , t_2	→	Barack	;	w_2 , t_2	→	Barack
	w_1 , t_3	→	Barack	;	w_3 , t_3	→	Barack

F(*m*) =	w_1 , t_1	→	Michelle	;	w_2, t_1	→	Michelle
	w_1 , t_2	→	Michelle	;	w_2 , t_2	→	Michelle
	w_1 , t_3	→	Michelle	;	w_3 , t_3	→	Michelle

F(**smoke'**) =	w_1 , t_1	→	{<Barack,1>, <Michelle,1>}
	w_1 , t_2	→	{<Barack,1>, <Michelle,0>}
	w_1 , t_3	→	{<Barack,0>, <Michelle,0>}
	w_2 , t_1	→	{<Barack,0>, <Michelle,1>}
	w_2 , t_2	→	{<Barack,0>, <Michelle,1>}
	w_2 , t_3	→	{<Barack,0>, <Michelle,1>}

F(**think'**) contains the following mapping:

$$w_1 , t_1 \rightarrow \left(\left\{ \begin{array}{l} << w_1 , t_1 >, 1 > \\ << w_1 , t_2 >, 1 > \\ << w_1 , t_3 >, 0 > \\ << w_2 , t_1 >, 0 > \\ << w_2 , t_2 >, 0 > \\ << w_2 , t_3 >, 0 > \end{array} \right\} \rightarrow \{ <\text{Barack},1>, <\text{Michelle},1> \} \right)$$

(16) **Remarks Regarding the Model in (15)**

- The type-e constant *b* (*m*) is interpreted as an <s,e> function (individual concept) that maps every world-time pair to Barack (Michelle).

- The type-<e,t> constant **smoke'** is interpreted as an <s,<e,t>> function (property). Informally speaking, in this model:
  - At world w_1, Barack and Michelle both smoke at time t_1, but then at time t_2, Michelle quits. Finally, at time t_3, Barack also quits.
  - At world w_2, Barack is never a smoker, but Michelle is and never quits.

- The type-<<s,t>,<e,t>> constant **think'** is interpreted as an <s,<<s,t>,<e,t>>> function (the intension of *thinks*). Informally speaking, in this model:
  - At world w_1 and time t_1, both Barack and Michelle think that Barack smokes.

Now that we have the definition of a model, the next step is to define the notion of a 'variable assignment'…

(17) **Definition of an $\mathcal{M}$-Assignment**
Let $\mathcal{M}$ be an intensional model <A, I, J, ≤, F>. The function *g* is an $\mathcal{M}$-assignment if:
a. It has as its domain the set of all variables of IL.
b. If *u* is a variable of type τ, then $g(u) \in D_{\tau, A, I, J}$

<u>Note:</u> Variable assignments maps variables directly to *denotations* (*extensions*), not to *senses* (*intensions*).

With these ingredients, we can now recursively define the central notion 'extension with respect to an intensional model, world, time, and variable assignment'

(18) **Extension With Respect to Model, World, Time, and Variable Assignment**
Let $\mathcal{M}$ be an intensional model <A, I, J, ≤, F> for IL, let $i \in I$ and $j \in J$, and let g be an $\mathcal{M}$-assignment. If α is a meaningful expression of IL, then $[[\alpha]]^{M,i,j,g}$, *the extension of* α *with respect to* $\mathcal{M}$*, i, j, and g*, is defined as follows:

a. If α is a constant, then $[[\alpha]]^{M,i,j,g} = F(\alpha)(<i,j>)$

<u>Note:</u> Thus, we obtain the extension of a constant α by applying the intension $F(\alpha)$ to the world-time pair $<i,j>$.

b. If α is a variable, then $[[\alpha]]^{M,i,j,g} = g(\alpha)$

c. If $\alpha \in ME_\tau$ and u is a variable of type σ, then $[[\lambda u\alpha]]^{M,i,j,g}$ = the function h with domain $D_{\sigma, A,I,J}$ such that whenever x is in that domain, $h(x) = [[\alpha]]^{M,i,j,g(u/x)}$

d. If $\alpha \in ME_{<\sigma,\tau>}$ and $\beta \in ME_\sigma$, then $[[\alpha(\beta)]]^{M,i,j,g} = [[\alpha]]^{M,i,j,g}([[\beta]]^{M,i,j,g})$

e. If $\varphi,\psi \in ME_\tau$, then $[[\varphi = \psi]]^{M,i,j,g} = 1$ *iff* $[[\varphi]]^{M,i,j,g} = [[\psi]]^{M,i,j,g}$

f. If $\varphi,\psi \in ME_t$, then

(i)	$[[\neg\varphi]]^{M,i,j,g}$	$= 1$ *iff* $[[\varphi]]^{M,i,j,g} = 0$
(ii)	$[[[\varphi \wedge \psi]]]^{M,i,j,g}$	$= 1$ *iff* $[[\varphi]]^{M,i,j,g} = 1$ and $[[\psi]]^{M,i,j,g} = 1$
(iii)	$[[[\varphi \vee \psi]]]^{M,i,j,g}$	$= 1$ *iff* $[[\varphi]]^{M,i,j,g} = 1$ or $[[\psi]]^{M,i,j,g} = 1$
(iv)	$[[[\varphi \rightarrow \psi]]]^{M,i,j,g}$	$= 1$ *iff* $[[\varphi]]^{M,i,j,g} = 0$ or $[[\psi]]^{M,i,j,g} = 1$
(v)	$[[[\varphi \leftrightarrow \psi]]]^{M,i,j,g}$	$= 1$ *iff* $[[\varphi]]^{M,i,j,g} = [[\psi]]^{M,i,j,g}$

Note: Thus, if $\varphi,\psi \in ME_t$, then $[[[\varphi \leftrightarrow \psi]]]^{M,i,j,g} = 1$ *iff* $[[\varphi = \psi]]^{M,i,j,g} = 1$
Thus, if $\varphi,\psi \in ME_t$, then $[\varphi \leftrightarrow \psi]$ and $\varphi = \psi$ are logically equivalent.

g. If $\varphi \in ME_t$, and u is a variable of type τ, then

(i) $[[\bigvee u\varphi]]^{M,i,j,g} = 1$ *iff* there is an $x \in D_{\tau, A,I,J}$ such that $[[\varphi]]^{M,i,j,g(u/x)} = 1$

(ii) $[[\bigwedge u\varphi]]^{M,i,j,g} = 1$ *iff* for all $x \in D_{\tau, A,I,J}$, $[[\varphi]]^{M,i,j,g(u/x)} = 1$

h. If $\varphi \in ME_t$, then

(i) $[[\Box\varphi]]^{M,i,j,g} = 1$ *iff* for all $i' \in I$ and $j' \in J$, $[[\varphi]]^{M, i', j', g} = 1$

Note: Thus, '$\Box\varphi$' is more aptly translated as 'necessarily always φ'

(ii) $[[W\varphi]]^{M,i,j,g} = 1$ *iff* for some $j' \in J$ such that $j < j'$, $[[\varphi]]^{M, i, j', g} = 1$

(iii) $[[H\varphi]]^{M,i,j,g} = 1$ *iff* for some $j' \in J$ such that $j' < j$, $[[\varphi]]^{M, i, j', g} = 1$

i. If $\alpha \in ME_\tau$, then $[[[\hat{}\alpha]]]^{M,i,j,g}$ = the function h with domain I×J such that if $<i',j'> \in I\times J$, then $h(<i',j'>) = [[\alpha]]^{M, i',j', g}$

Note: Thus, $[[[\hat{}\alpha]]]^{M,i,j,g}$ is the function mapping a world-time pair to the extension of α at that world-time. Thus, given (19) below, $[[[\hat{}\alpha]]]^{M,i,j,g}$ is *the intension of* α *with respect to* $\mathcal{M}$ *and g*, $[[\alpha]]^{M,g}$

j. If $\alpha \in ME_{<s,\tau>}$, then $[[[^{\vee}\alpha]]]^{M,i,j,g} = [[\alpha]]^{M,i,j,g}(<i,j>)$

Note: Thus, if the extension of α at *i* and *j* is some intension, then the extension of $[^{\vee}\alpha]$ at *i* and *j* is that intension applied to *i* and *j*.

(19) **Intension With Respect to a Model and Variable Assignment**
Let $\mathcal{M}$ be an intensional model <A, I, J, ≤, F> for IL and let g be an $\mathcal{M}$-assignment. If α is a meaningful expression of IL, then $[[\alpha]]^{M,g}$, *the intension of α with respect to* $\mathcal{M}$ *and g*, is the function *h* with domain I×J such that if <i',j'> ∈ I×J, then *h*(<i',j'>) = $[[\alpha]]^{M,i',j',g}$

(20) **Truth With Respect to a Model, World, and Time**
Let $\mathcal{M}$ be an intensional model <A, I, J, ≤, F> for IL and let i ∈ I and j ∈ J. If φ ∈ ME_t , then φ is *true with respect to* $\mathcal{M}$, i, and j *iff* for any $\mathcal{M}$-assignment *g*, $[[\varphi]]^{M,i,j,g} = 1$

Let us now illustrate these definitions by using them to interpret meaningful expressions of IL

(21) **Illustrative Computations, Part 1**
In the computations below, let $\mathcal{M}$ be an intensional model of the kind described in (15). Let g be an arbitrary $\mathcal{M}$-assignment.

a. **smoke'**(*b*)

(i) [[**smoke'**(*b*)]]^{M,w1,t1,g} = (by (18d))
(ii) [[**smoke'**]]^{M,w1,t1,g}([[*b*]]^{M,w1,t1,g}) = (by (18a))
(iii) F(**smoke'**)(<w_1,t_1>)(F(*b*)(<w_1,t_1>)) = (by (15))
(iv) {<Barack,1>, <Michelle,1>}(Barack) =
(v) 1

b. [^**smoke'**(*b*)]

(i) [[[^**smoke'**(*b*)]]]^{M,w1,t1,g} = (by (18i))

(ii) the function *h* with domain I×J such that
if <i',j'> ∈ I×J, then *h*(<i',j'>) = [[**smoke'**(*b*)]]$^{M,i',j',g}$ = (by (18a,d))

(iii) the function *h* with domain I×J such that
if <i',j'> ∈ I×J, then *h*(<i',j'>) = F(**smoke'**)(<i',j'>)(F(*b*)(<i',j'>))
= (by (15))

(iv) the function *h* with domain I×J such that
if <i',j'> ∈ I×J, then *h*(<i',j'>) = F(**smoke'**)(<i',j'>)(Barack) = (by (15))

(v) $\left\{ \begin{array}{l} \langle\langle w_1, t_1\rangle,1\rangle, \langle\langle w_1, t_2\rangle,1\rangle, \langle\langle w_1, t_3\rangle,0\rangle, \\ \langle\langle w_2, t_1\rangle,0\rangle, \langle\langle w_2, t_2\rangle,0\rangle, \langle\langle w_2, t_3\rangle,0\rangle \end{array} \right\}$

(22) **Illustrative Computations, Part 2**

In the computations below, let $\mathcal{M}$ be an intensional model of the kind described in (15). Let g be an arbitrary $\mathcal{M}$-assignment.

a. $\Box\, b = b$

(i)	$[[\ \Box\, b = b\ ]]^{M,w1,t1,g} = 1$	*iff*	(by (18h))
(ii)	for all $i' \in I$ and $j' \in J$, $[[\ b = b\ ]]^{M,\, i',\, j',\, g} = 1$	*iff*	(by (18e))
(iii)	for all $i' \in I$ and $j' \in J$, $[[b]]^{M,i',j',g} = [[b]]^{M,i',j',g}$	*iff*	(by (18a))
(iv)	for all $i' \in I$ and $j' \in J$, $F(b)(<i',j'>) = F(b)(<i',j'>)$		
(v)	Thus, $[[\ \Box\, b = b\ ]]^{M,w1,t1,g} = 1$		

b. H **smoke'**(b)

(i) [[H **smoke'**(b)]] $^{M,w1,t2,g} = 1$ *iff* (by (18h))
(ii) for some $j' \in J$ such that $j' < t_2$, [[**smoke'**(b)]]$^{M,\, w1,\, j',\, g} = 1$
iff (by (18a,d))
(iii) for some $j' \in J$ such that $j' < t_2$, F(**smoke'**)($<w_1, j'>$)(F(b)($<w_1, j'>$)) = 1
iff (by (15))
(iv) for some $j' \in J$ such that $j' < t_2$, F(**smoke'**)($<w_1, j'>$)(Barack) = 1
(v) Given that F(**smoke'**)($<w_1, t_1>$)(Barack) = 1, [[H **smoke'**(b)]]$^{M,w1,t2,g} = 1$

c. W **smoke'**(b)

(i) [[W **smoke'**(b)]] $^{M,w1,t2,g} = 1$ *iff* (by (18h))
(ii) for some $j' \in J$ such that $t_2 < j'$, [[**smoke'**(b)]]$^{M,\, w1,\, j',\, g} = 1$
iff (by (18a,d))
(iii) for some $j' \in J$ such that $t_2 < j'$, F(**smoke'**)($<w_1, j'>$)(F(b)($<w_1, j'>$)) = 1
iff (by (15))
(iv) for some $j' \in J$ such that $t_2 < j'$, F(**smoke'**)($<w_1, j'>$)(Barack) = 1
(v) Given that F(**smoke'**)($<w_1, t_3>$)(Barack) = 0, [[W **smoke'**(b)]]$^{M,w1,t2,g} = 0$

d. **think'**([^**smoke'**(b)])(m) ('Michelle thinks that Barack smokes')

(i) [[**think'**([^**smoke'**(b)])(m)]]^{M,w1,t1,g} = (by (18d))
(ii) [[**think'**]]^{M,w1,t1,g}([[^**smoke'**(b)]]]^{M,w1,t1,g})([[m]]^{M,w1,t1,g}) = (by (18a))
(iii) F(**think'**)($<w_1,t_1>$)([[^**smoke'**(b)]]]^{M,w1,t1,g})(F(m))($<w_1,t_1>$)) = (by (15))
(iv) F(**think'**)($<w_1,t_1>$)([[^**smoke'**(b)]]]^{M,w1,t1,g})(Michelle) = (by (21b))

(v) F(**think'**)($<w_1,t_1>$)
({$<<w_1 , t_1>,1>$, $<<w_1 , t_2>,1>$, $<<w_1 , t_3>,0>$,
$<<w_2 , t_1>,0>$, $<<w_2 , t_2>,0>$, $<<w_2 , t_3>,0>$})(Michelle) = (by (15))

(vi) 1

(23) **Optional Exercise for the Student**
Compute the value of $[[\ \lambda v_{1,<e,t>}\ \mathrm{V} v_{1,e}\ [\mathbf{smoke'}(v_{1,e}) \wedge v_{1,<e,t>}(v_{1,e})]\ ]]^{M,w1,t1,g}$
Show that it is the characteristic function of <et>-functions that are true of some smoker.

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3. Some Key Validities of Intensional Logic

In this section, we'll make note of some key validities in IL, which we will use to 'convert' the translations of English expressions to simpler, logically equivalent formulae.

(24) **Alpha Conversion**
If variable v is bound in φ, and variable v' appears nowhere in φ, then φ is logically equivalent to $[v/v']\varphi$

Although alpha-conversion is just the same as before, lambda-conversion is now subject to an important new restriction.

(25) **Lambda Conversion**
Let $(\lambda v \psi)$ and φ be meaningful expressions with no variables in common, and let $\varphi \in ME_\tau$ and $v \in Var_\tau$. If (a) or (b) hold, then the expressions in (c) are logically equivalent.

a. The variable v in ψ does not appear in the scope of ^, □, W, F
b. The denotation of φ is the same for every world and time (in every model $\mathcal{M}$)
(That is, for all $\mathcal{M}$, $i,i' \in I$ and $j,j' \in J$, $[[\varphi]]^{M,i,j,g} = [[\varphi]]^{M,i',j',g}$)

c. (i) $[\lambda v\ \psi](\varphi)$ (ii) $[\varphi/v]\psi$

(26) **Why This New Restriction on Lambda Conversion?**

- Consider, for example, the formulae $[\lambda v\ \mathrm{W}\chi](\varphi)$ and $[\varphi/v]\mathrm{W}\chi$

- Consider their extensions at a given world and time:
$[[\ [\lambda v\ \mathrm{W}\chi](\varphi)\ ]]^{M,i,j,g}$ and $[[\ [\varphi/v]\mathrm{W}\chi\ ]]^{M,i,j,g}$

- When we compute $[[\ [\lambda v\ \mathrm{W}\chi](\varphi)\ ]]^{M,i,j,g}$, we will compute $[[\ [\lambda v\ \mathrm{W}\chi]\ ]]^{M,i,j,g}$ and apply it to the value $[[\varphi]]^{M,i,j,g}$

- However, when we compute $[[\ [\varphi/v]\mathrm{W}\chi\ ]]^{M,i,j,g}$, we *don't* compute $[[\varphi]]^{M,i,j,g}$. Rather, we will end up computing $[[\varphi]]^{M,i,j',g}$ for some $j' > j$.

- Therefore, **unless φ has the same extension for every world and time**, we can't guarantee that $[[\ [\lambda v\ \mathrm{W}\chi](\varphi)\ ]]^{M,i,j,g}$ will be the same value as $[[\ [\varphi/v]\mathrm{W}\chi\ ]]^{M,i,j,g}$

(27) **An Analogy to Maybe Make (25)-(26) More Intuitive**

- In LING 620, we learned that (a) is ambiguous, and has both the readings in (b).

a. John thinks that the president smokes.

b. (i) *De Dicto Reading:*
In all of John's 'belief worlds' *w'*, the president in *w'* smokes in *w'*.

(ii) *De Re Reading:*
In all of John's 'believe worlds' *w'*, the president **in** $\mathbf{w_0}$ smokes in *w'*.

- We also saw that we can get reading (i) from an LF where the phrase *the president* is in the scope of 'believes', while we get (ii) from and LF where it's moved above the scope of *believes*.

c. (i) *De Dicto LF:* [John [believes [**the president** smokes]]]

(ii) *De Re LF:* [**the president** [1 [John [believes [t_1 smokes] …]

- This is because in LF (i), *the president* ends up semantically evaluated with respect to the belief worlds, while in LF (ii), it is evaluated with respect to the actual world.

- Consequently, we saw that for proper names, the two LFs in (c) end up mapped to the same interpretation, **because proper names have the same value in every world**

d. <u>Two LFs Mapped to the Same Truth-Conditions</u>
(i) [John [believes [Mary smokes]]]
(ii) [Mary [1 [John [believes [t_1 smokes] …]

- The same general issue here is also at play for IL in (25)-(26).

  - The IL operators ^, $\Box$, H, W *shift* the evaluation worlds and time.

  - Thus, if φ is in the scope of ^, $\Box$, H, W, we *won't* interpret φ with respect to the 'actual' world/time

  - Consequently, if the variable *v* in ψ is in the scope of ^, $\Box$, W, F, then in (i) φ is interpreted with respect to the initial, 'matrix' evaluation world/time, while in (ii), it is not.
    (i) $[\lambda v\ \psi](\varphi)$
    (ii) $[\varphi/v]\psi$

  - However, **if φ has the same extension in *every world and time*, then this difference won't matter for the resulting interpretation, and (i) and (ii) *will* be logically equivalent.**

(28) **One Last Important Note**

- Given the definition in (18b), it follows that if φ is a variable, then for all i, i' ∈ I and j, j' ∈ J, $[[\varphi]]^{M,i,j,g} = [[\varphi]]^{M,i',j',g} = g(\varphi)$

- Therefore, if φ is a variable, then as long as $[\lambda v\ \psi](\varphi)$ and φ have no variables in common, then the following *will always* be logically equivalent (even in IL):
  (i) $[\lambda v\ \psi](\varphi)$ (ii) $[\varphi/v]\psi$

In addition to alpha-conversion and lambda-conversion, our translations in PTQ will also make use of the key logical equivalence in (29)…

(29) **Down-Up Cancellation (Dowty *et al.* 1981)**
The following two expressions are logically equivalent:
a. α
b. $[{}^{\vee}[{}^{\wedge}\alpha]]$

- That is, put informally, the extension of α at world i and time j will always be equal to the intension of α applied to the world i and the time j.

(30) **Optional Exercise for Students**
Let $\mathcal{M}$ be an intensional model <A, I, J, ≤, F>, i ∈ I, j ∈ J, and g be an arbitrary $\mathcal{M}$-assignment. Show that $[[\alpha]]^{M,i,j,g} = [[\ [{}^{\vee}[{}^{\wedge}\alpha]]\ ]]^{M,i,j,g}$

4. Some Useful Abbreviations in PTQ

In PTQ, Montague makes use of a large number of meta-linguistic abbreviatory conventions.

(31)	**Relational Notation:**	γ(α,β)	<u>abbreviates</u>	γ(α)(β)
(32)	**Curly Bracket Notation 1:**	γ{α}	<u>abbreviates</u>	$[{}^{\vee}\gamma](\alpha)$
(33)	**Curly Bracket Notation 2:**	γ{α,β}	<u>abbreviates</u>	$[{}^{\vee}\gamma](\alpha,\beta)$

(34) **Key Illustration:** Let *P* be a variable of type <s,<e,t>>

[λ*P P*{*b*}](^**smoke'**) ⇔ (by λ-conversion)
[^**smoke']**{b} = (by Curly Bracket Notation (CBN))
$[{}^{\vee}$[^**smoke']**(b) ⇔ (by Down-Up Cancellation (DUC))
smoke'(b)

(34) **The Curvy Hat**

- Montague puts a curvy 'hat' over a variable to indicate lambda abstraction over that variable.
- I will follow Dowty *et al.* 1981 in not making use of this notation
  - After all, 'λ' is clear and easy enough…

(35) **The Sharp Hat** ûφ <u>abbreviates</u> [^λuφ]

- I will not make much use of this abbreviation.
- However, do familiarize yourself with it, as Montague uses it *a lot*.