An Algebraic Approach to Quantification and Lambda Abstraction: Fregean Interpretations [1]

(1) **The Disambiguated Language Politics+λ**

Politics+λ is the disambiguated language $<A, F_\gamma, X_\delta, S, t>_{\gamma \in \{Concat, Not, And, Or, If, \exists, \forall, \lambda\}, \delta \in \Delta}$ such that

a. Non-Logical Vocabulary
   (i) *Constants:*
   CON_e = { **mitt', barack', michelle'** }
   $CON_{<et>}$ = { **smokes', man', president'** }
   $CON_{<e<e,t>>}$ = { **loves'** }
   For all other $\tau \in T$, $CON_\tau = \varnothing$

   (ii) *Variables:*
   For every type $\tau \in T$, a **countably infinite** set of variables of type τ: $VAR_\tau = \{ v_{\tau, n} : n \in \mathbb{N} \}$

b. Syntactic Algebra:
   $<A, F_\gamma>_{\gamma \in \{Concat, Not, And, Or, If, \exists, \forall, \lambda\}}$ is the algebra such that:
   (i) $\{ F_\gamma \}_{\gamma \in \{Concat, Not, And, Or, If, \exists, \forall, \lambda\}}$ are as defined on the previous handout
   (ii) A is the smallest set such that the following holds:
   1. For all $\tau \in T$, $CON_\tau \subseteq A$ and $VAR_\tau \subseteq A$
   2. A is closed under $\{ F_\gamma \}_{\gamma \in \{Concat, Not, And, Or, If, \exists, \forall, \lambda\}}$

c. Syntactic Categories: $\Delta = T \cup \{ <var,\tau> : \tau \in T \}$

d. Basic Expressions: For every type $\tau \in T$:
   (i) $X_{<var,\tau>} = VAR_\tau$
   (ii) $X_\tau = CON_\tau \cup X_{<var,\tau>}$

e. Syntactic Rules:
   S is the (countably) infinite set consisting of:
   (i) $<F_{Not}, <t>, t>$ (ii) $<F_{And}, <t,t>, t>$
   (iii) $<F_{Or}, <t,t>, t>$ (vi) $<F_{If}, <t,t>, t>$
   And, for every $\sigma, \tau \in T$, a triple of the following form:
   (i) $< F_{Concat}, < <\sigma, \tau>, \sigma >, \tau >$ (ii) $< F_\exists, < <var,\sigma>, t >, t >$
   (iii) $< F_\forall, < <var,\sigma>, t >, t >$ (iv) $< F_\lambda, < <var,\sigma>, \tau >, <\sigma, \tau> >$

[1] These notes are based upon material in the following readings: Thomason (1974) Chapter 7 (Montague's "Universal Grammar").

(2) **Our Present Goal:** Define an interpretation for Politics+λ.

(3) **The Crucial Challenge**
To have an interpretation **B** = <B, G_γ, f>$_{\gamma \in \{\text{Concat, Not, And, Or, If, } \exists, \forall, \lambda\}}$ for Politics+λ, we need to state some semantic (algebraic) operations $G_\exists$, $G_\forall$, and G_λ , which will 'correspond' with $F_\exists$, $F_\forall$, and F_λ. ***What in the world could those be?***

(4) **The Plan**

- As we did several weeks ago for FOL, we're going to see our way to a solution by reflecting informally on the meaning of deictic elements (akin to free pronouns).
- This will bring us to Montgue's definition of a **Fregean Interpretation**
- We'll use **part of** this definition to set up an interpretation structure for Politics+λ.

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1. Some Reflections on the Nature of Meaning

(5) **Illustrative Sentence with Free (Deictic) Pronoun:** He smokes.

(6) a. **Stupid Question**
In a context where I'm pointing to Barack, does sentence (5) have a meaning?

b. **Obvious Answer:**
Sure; after all, in such a context, (5) seems to have the intension in (i) and the extension in (ii).
(i) *Intension of (5), Pointing to Barack:* [λw : Barack smokes in w]
(ii) *Extension of (5), Pointing to Barack:* 1 (true)

(7) a. **Difficult Question**
Suppose I don't specify a context for (5). *Do we still say it has a meaning?*

b. **Negative Answer**
Without specifying a context, (5) doesn't have a defined intension or extension. Thus, sentence (5) seems to be meaningless.

c. **Positive Answer**
To say that (5) has no meaning in such circumstances would fail to distinguish between it and gibberish like "snoochie boochies".
- **Thus, its *potential* to have an intension/extension (relative to a context) should be viewed as constitutive of its meaning.**

(8) **A General Picture of Meaning, Fitting the Positive Answer**
The 'meaning' – broadly construed – of an expression is a function from contexts to intensions. (*cf.* Kaplan's notion of "character" vs. "content").

Meaning —— *Context* ——→ Intension (Sense) —— *World/time of evaluation* ——→ Extension (Reference)

- Some expressions have a *meaning* that maps every context to the same intension.
  - We can say that such expressions have a meaning that is 'not context-dependent'

Illustration: Barack smokes.
Meaning: Function that maps every context c to
[λw : Barack smokes in w]

- Some expressions have a *meaning* that maps different contexts to different intensions.
  - We can say that such expressions have a 'context-dependent' meaning.

Illustration: I smoke.
Meaning: Function that maps every context c to
[λw : the speaker of c smokes in w]

Illustration: He smokes.
Meaning: Function that maps every context c to
[λw : the deictic focus of c smokes in w]

(9) **Where We're Going From This**

- We're going now to try to model meanings (in B) as functions from contexts (variable assignments) to intensions (functions from possible worlds to extensions).

- We'll begin by redefining our system of types and denotations, so that they now include objects that can serve as 'intensions'.

(10) **The Types (New Definition)**
The set T of types is the smallest set such that:

a.	$e \in T$	(entities)
b.	$t \in T$	(truth-values)
c.	If $\sigma \in T$ and $\tau \in T$, then $\langle \sigma, \tau \rangle \in T$	(functions from σ to τ)
d.	**If $\sigma \in T$, then $\langle s, \sigma \rangle \in T$**	**(functions from world-times to σ)**

(11) **The Denotations (New Definition)**
Let T be the set of types, let E be some non-empty set (of entities), **and let I be some other non-empty set (of 'indices' / world-time pairs).** The set $D_{\tau, E, I}$ of *denotations of type* τ *based on E and I* is defined as follows:

a. $D_{e,E,I}$ = $E \cup \{\textbf{garbage}\}$
b. $D_{t,E,I}$ = $\{ 0, 1 \}$
c. If $\sigma, \tau \in T$ then $D_{<\sigma,\tau>,E,I}$ = the set of functions from $D_{\sigma,E,I}$ to $D_{\tau,E,I}$
d. **If $\sigma \in T$ then $D_{<s,\sigma>,E,I}$ = the set of functions from I to $D_{\sigma,E,I}$**
= $(\mathbf{D}_{\sigma,E,I})^{I}$

(12) **Remark**
The only real change to our earlier definition for the set of 'denotations' comes in (11d), which yields the 'intensional objects', including the following:

a. $D_{<s,e>,E,I}$ = Functions from I (world-time pairs) to E (entities).
= Individual Concepts

b. $D_{<s,t>,E,I}$ = Functions from I (world-time pairs) to {0,1}
= Propositions

c. $D_{<s,<e,t>>,E,I}$ = Functions from I (world-time pairs) to $D_{<e,t>,E,I}$
= Properties

(13) **Some Considerations Towards 'Meanings'**

- Following (8) and (9), we want 'meanings' to be functions from contexts (variable assignments) to intensions (functions from world-times to the denotations in (11)).

- Thus, if J is our set of contexts (variable assignments) and I is our set of world-time pairs, then a meaning should be something from the following set:

$$((\cup_{\tau \in T} D_{\tau, E, I})^{I})^{J}$$

Functions from J to functions from I to the set of all possible denotations

- But recall from our first week that – due to the nature of currying – we can easily shift between the following two objects: $(A^{B})^{C}$ and $A^{B \times C}$

- Thus, we can – and Montague does – state that meanings are elements from the following set:

$$(\cup_{\tau \in T} D_{\tau, E, I})^{I \times J}$$

Functions from the pairs in (I×J) to the set of all possible denotations

(14) **The Set of Possible Meanings**
Let T be the set of types, let E, I and J be non-empty sets (E = entities; I = indices/world-times; J = contexts/variable-assignments). The set $M_{\tau, E, I, J}$ of *meanings of type* τ *based on E, I, and J* is $(D_{\tau, E, I})^{I \times J}$

Note: In UG, Montague sometimes shifts freely between $(D_{\tau, E, I})^{I \times J}$ and $(\,(D_{\tau, E, I})^{I}\,)^{J}$

With this conception of meaning at hand, we can begin to define Montague's notion of a 'Fregean Interpretation'…
A key ingredient in that definition is the notion of a 'type assignment'…..

(15) **Type Assignment**
Let T be the set of types in (10). Let **L** be a language $\langle\langle A, F_\gamma, X_\delta, S, \delta_0\rangle_{\gamma \in \Gamma, \delta \in \Delta}, R\rangle$. A type assignment for **L** is a function $\sigma: \Delta \rightarrow T$ such that $\sigma(\delta_0) = t$.

Note: Thus, a type assignment pairs each syntactic category label δ with exactly one type τ, and it ensures that δ_0 (the category label for 'declarative sentences') is paired with type t.

(16) **The Core Idea Behind a Fregean Interpretation**
Informally speaking, Montague's Fregean interpretations are intended to be interpretations $\mathbf{B} = \langle B, G_\gamma, f\rangle_{\gamma \in \Gamma}$ such that:

a. B is a set of 'meanings', as defined in (14); i.e., $B \subseteq \bigcup_{\tau \in T} M_{\tau, E, I, J}$

b. **Expressions of the same category are assigned meanings of the same type.**
- If g is the meaning assignment determined by **B**, then for each syntactic category C_δ, there is a single type τ such that if $\varphi \in C_\delta$, then $g(\varphi) \in M_{\tau, E, I, J}$

(17) **Question**: Why have this condition in (16b)?

- Right from the start, (16b) was criticized as too restrictive. For example, it entails that "Barack" and "some man" must have the same type of denotation (and so the former cannot directly refer to Barack).

- To my knowledge, the restriction in (16b) doesn't 'do' anything.
  - It isn't necessary for the meaning assignment g to be a homomorphism…
  - It's just stipulated in UG and PTQ; there isn't much independent motivation

(18) **Possible Answer (?)**
It makes the mapping between the syntax and semantics akin to that of a typed logic?

(19) **Official Formal Definition of a Fregean Interpretation**

Let **L** be a language <<A, F_γ, X_δ, S, δ_0>$_{\gamma \in \Gamma, \delta \in \Delta}$, R>. A Fregean interpretation **B** of **L** is an interpretation <B, G_γ, f>$_{\gamma \in \Gamma}$ of **L** such that for some type assignment σ and non-empty sets E, I, J:

a. $B \subseteq \cup_{\tau \in T} M_{\tau, E, I, J}$

Note: Thus, the semantic values in B will all be 'meanings' in the sense of (14). Thus, the meaning assignment based on **B** maps expressions of **L** to
(i) functions from <context,world-time> pairs to $\cup_{\tau \in T} D_{\tau, E, I}$
(ii) functions from contexts to
functions from world-time pairs to $\cup_{\tau \in T} D_{\tau, E, I}$

- This clearly guarantees us property (16a).

b. For all $\delta \in \Delta$, if $\varphi \in X_\delta$, then $f(\varphi) \in M_{\sigma(\delta), E, I, J}$

Note:
That is, if category δ is paired with type τ by the type assignment σ, then the lexical interpretation function f maps every basic expression of category δ to a meaning of type τ.

- This, combined with the next condition, guarantees property (16b).

c. If < F_γ , <δ_1, … δ_n>, δ> ∈ S, and b_1, … b_n are such that for all i, $b_i \in M_{\sigma(\delta i), E, I, J}$, then $G_\gamma(b_1, \dots b_n) \in M_{\sigma(\delta), E, I, J}$.

Note:
That is, if categories δ_1, … δ_n, δ are paired with types τ_1, …, τ_n, τ by σ, then
If the result of applying syntactic operation F_γ to expressions of category δ_1, … δ_n is an expression of C_δ, then
The result of applying the corresponding semantic operation G_γ to meanings of type τ_1, …, τ_n is a meaning of type τ.

- This, combined with the preceding condition, guarantees (16b).

Note: We'll also want in (19) to ensure that *f* and *g* do not end up mapping any meaningful expressions of **L** to **garbage**, but we'll leave the details of that implicit in what follows…

(20) **Concomitant Definition**

A Fregean interpretation for **L** *connected with E, I, J, σ* is an interpretation **B** such that the conditions in (19a-c) hold.

2. A 'Partly-Fregean' Interpretation for Politics+λ

Now, in Politics+λ there are no 'intensional' environments…

Thus, we won't be making full use of the definition in (19) to interpret Politics+λ...

Let us, then, develop our own notion of a 'Partly-Fregean' interpretation, which will be quite applicable to our language in (1)…

(21) **The Key Ideas Behind a 'Partly-Fregean' Interpretation**

Since we won't be dealing with intensions for now, let us view meanings as functions from contexts (variable assignments) to denotations.

Meaning ———————————→ Extension (Reference)
Context (*Variable Assignment*)

a. <u>The Types (For Politics+λ):</u>
The set T of types is the smallest set such that:
(i) $e \in T$ (entities)
(ii) $t \in T$ (truth-values)
(iii) If $\sigma \in T$ and $\tau \in T$, then $<\sigma, \tau> \in T$ (functions from σ to τ)

b. <u>The Possible Denotations (For Politics+λ):</u>
Let T be the set of types and E be some non-empty set (of entities). The set $D_{\tau, E}$ of *denotations of type τ based on E* is defined as follows:
(i) $D_{e,E} = E \cup \{$**garbage**$\}$
(ii) $D_{t,E} = \{ 0, 1 \}$
(iii) If $\sigma, \tau \in T$, then $D_{<\sigma,\tau>,E}$ = the set of functions from $D_{\sigma,E}$ to $D_{\tau,E}$

c. <u>The Possible Meanings (For Politics+λ):</u>
Let T be the set of types, and let E and J be non-empty sets (E = entities; J = contexts/variable-assignments). The set $M_{\tau, E, J}$ of *meanings of type τ based on E and J* is $(D_{\tau, E})^J$

(22) **Partly-Fregean Interpretation**

Let **L** be a language $<<A, F_\gamma, X_\delta, S, \delta_0>_{\gamma \in \Gamma, \delta \in \Delta}, R>$. A partly-Fregean interpretation **B** of **L** is an interpretation $<B, G_\gamma, f>_{\gamma \in \Gamma}$ of **L** such that for some type assignment σ and non-empty sets E, J:

a. $B \subseteq \cup_{\tau \in T} M_{\tau, E, J}$

b. For all $\delta \in \Delta$, if $\varphi \in X_\delta$, then $f(\varphi) \in M_{\sigma(\delta), E, J}$

c. If $< F_\gamma , <\delta_1, \dots \delta_n>, \delta> \in S$, and $b_1, \dots b_n$ are such that for all i, $b_i \in M_{\sigma(\delta i), E, J}$, then $G_\gamma(b_1, \dots b_n) \in M_{\sigma(\delta), E, J}$.

Note: Our definition of a 'partly-Fregean' interpretation simply takes Montague's definition of a Fregean interpretation (19) and removes the set I (indices / world-time pairs).

(23) **Concomitant Definition**
A partly-Fregean interpretation for **L** *connected with E, J*, σ is an interpretation **B** such that the conditions in (22a-c) hold.

(24) **Goal:** Define a partly-Fregean interpretation for Politics+λ, where J is the set of possible variable assignments.

(25) **Immediate Issue:**
- Up to now, variable assignments have been defined with respect to a given model.
- Thus, we need a new definition of 'variable assignment', one that makes no reference to a model…

(26) **New Definition of Variable Assignment (for Politics+λ)**
Let E be a non-empty set (of entities). A *variable assignment based on E* is a function g satisfying the conditions below:
(i) *Domain of g:* $\cup_{\tau \in T} \text{VAR}_\tau$
(ii) *Range of g:* If $\alpha \in \text{VAR}_\tau$, then $g(\alpha) \in D_{\tau,E}$ (and $g(\alpha) \neq$ **garbage**)

Note: Variable assignment map variables directly to *denotations*, not to *meanings*.

(27) **The Central, Scary Question that We've Been Putting Off Until Now**

How do we build a semantics for Politics+λ, where the semantic values ('meanings') are functions from variable assignments to denotations?

(28) **Some Informal Considerations to Get Us Started**

a. The meaning of a constant maps every variable assignment to the same denotation

- Just like the English name *Barack*, the individual constant **barack'** should map every variable assignment to the same individual, Barack
  - o After all, in our model-theoretic semantics, $[[\textbf{barack'}]]^{M,g}$ doesn't depend on g…

b. The meaning of a variable can map different variable assignments to different denotations.
  - o After all, in our model-theoretic semantics, $[[x_3]]^{M,g}$ *does* depend on g

(29) **Formalizing the Informal Considerations**

a. If $\varphi \in CON_\tau$, then it should be that f(φ)(g) = f(φ)(g'), for all g, g' ∈ J.

b. If $\varphi \in VAR_\tau$, then it should be that f(φ)(g) = g(φ) for all g ∈ J.

Note: If (29) holds, then

(i) The meaning of a constant maps every variable assignment g to the same denotation.
(ii) The meaning of a variable *v* maps a variable assignment to g(*v*).
The meaning of *v* can map different variable assignments to different denotations.

Let's build on the ideas in (28)-(29), to define a special sub-class of partly-Fregean interpretations…

(30) **Logically Possible Partly-Fregean Interpretation of Politics+λ** [2]

A logically possible partly-Fregean interpretation of Politics+λ is a partly-Fregean interpretation **B** = $\langle B, G_\gamma, f\rangle_{\gamma \in \{\text{Concat, Not, And, Or, If}, \exists, \forall, \lambda\}}$ for Politics+λ connected with E, J, σ, such that:

a. J is the set of variable assignments based on E.

b. If $\varphi \in CON_\tau$, then f(φ)(g) = f(φ)(g'), for all g, g' ∈ J.

c. If $\varphi \in VAR_\tau$, then f(φ)(g) = g(φ) for all g ∈ J.

d. …. *To be filled in later* …

Finally, to complete the definition in (30), we will specify what the semantic operations in $\{G_\gamma\}_{\gamma \in \{\text{Concat, Not, And, Or, If}, \exists, \forall, \lambda\}}$ *must be…*

This, of course, is the most challenging part…

- Rather than motivate these operations from the 'ground up', I'm simply going to introduce them and show you how they work…

- **Note that the set B =** $\bigcup_{\tau \in T} M_{\tau, E, J}$ **is closed under each of the following operations** G_γ**.**

[2] The term in (30) and its definition mimic Montague's definition in UG of a 'logically possible Fregean interpretation' of his Intensional Logic.

(31) **The Operation G_{Concat}**

a. If $\alpha \in M_{<\sigma,\tau>, E, J}$ and $\beta \in M_{\sigma, E, J}$, and $\beta \neq$ **garbage**, then $G_{Concat}(\alpha,\beta)$ =
The function *F-A* such that if $g \in J$, then $F\text{-}A(g) = \alpha(g)(\beta(g))$

b. Otherwise, $G_{Concat}(\alpha,\beta)$ = **garbage**

(32) **How Does This Work?**

- Consider the following two sentences: (**smokes' barack'**) (**smokes'** x_3)

- Since F_{Concat} and G_{Concat} should 'correspond', it follows for meaning assignment h:

h((**smokes' barack'**)) =	h((**smokes'** x_3)) =
h(F_{Concat}(**smokes'**, **barack'**) =	h(F_{Concat}(**smokes'**, x_3) =
G_{Concat}(h(**smokes'**), h(**barack'**)) =	G_{Concat}(h(**smokes'**), h(x_3)) =
G_{Concat}(f(**smokes'**), f(**barack'**)) =	G_{Concat}(f(**smokes'**), f(x_3)) =

- Now, given (22), in a partly-Fregean interpretation of Politics+λ:
f(**smokes'**) $\in M_{<e,t>, E, J}$ f(**barack'**), f(x_3) $\in M_{e,, E, J}$

- Thus, given (31a), it follows that:

G_{Concat}(f(**smokes'**), f(**barack'**)) =
The function *F-A* such that if $g \in J$, then *F-A*(g) = f(**smokes'**)(g)(f(**barack'**)(g))

G_{Concat}(f(**smokes'**), f(x_3)) =
The function *F-A'* such that if $g \in J$, then –*FA'*(g) = f(**smokes'**)(g)(f(x_3)(g))

- Since **smokes'** and **barack'** are constants, and x_3 is a variable, (30b,c) entails that:
  - For any g, g' ∈ J, f(**smokes'**)(g) = f(**smokes'**)(g') = $s \in D_{<e,t>, E}$
  - For any g, g' ∈ J, f(**barack'**)(g) = f(**barack'**)(g') = $b \in D_{e, E}$
  - For any g, ∈ J, f(x_3)(g) = g(x_3)

- Thus, it therefore follows that:

h((**smokes' barack'**)) = G_{Concat}(f(**smokes'**), f(**barack'**)) =
The function *F-A* such that if $g \in J$, then *F-A*(g) = $s(b)$

h((**smokes'** x_3)) = G_{Concat}(f(**smokes'**), f(x_3)) =
The function *F-A'* such that if $g \in J$, then *F-A'*(g) = $s(g(x_3))$

(33) **Remarks**

- Thus, the 'meaning' of (**smokes' barack'**) is a constant function from contexts (variable assignments) to truth-values:
  - It maps every variable assignment to the value that *s* yields for *b*.

- Thus, the 'meaning' of (**smokes'** x_3) is *not* a constant function from contexts (variable assignments) to truth-values:
  - It maps every variable assignment to the value that *s* yields for $g(x_3)$

- **Parallel with Demonstratives:**
  - The 'meaning' of *Barack smokes* maps every context to the same intension
  - The 'meaning' of *He smokes* maps different contexts to different intensions

- **Parallel with Models:**
  - No matter what variable assignment *g* is chosen [[(**smokes' barack'**)]]^{M,g} will be the same value: I(**smokes'**)(I(**barack'**))
  - The value of [[(**smokes'** x_3)]]^{M,g} will equal I(smokes)(g(x_3)), and so will vary with the variable assignment *g*.

(34) **The Operation G_{Not}**

a. If $\alpha \in M_{t, E, J}$ then $G_{Not}(\alpha)$ =
   The function *Neg* such that if g ∈ J, then *Neg*(g) = 1 iff α(g) = 0

b. Otherwise, $G_{Neg}(\alpha)$ = **garbage**

(35) **How Does This Work? (Part 1)**

- Consider the following two sentences: ~(**smokes' barack'**) ~(**smokes'** x_3)

- Since F_{Not} and G_{Not} should 'correspond', it follows for meaning assignment h:

h(~(**smokes' barack'**))	=	h(~(**smokes'** x_3))	=
h(F_{Not}((**smokes' barack'**))	=	h(F_{Not}((**smokes'** x_3))	=
G_{Not}(h(**smokes' barack'**))	=	G_{Concat}(h(**smokes'** x_3))	=

- Now, recall from (32) that h(**smokes' barack'**), h((**smokes'** x_3)) $\in M_{t, E, J}$
  Thus, it follows from (34) that:

  G_{Not}(h(**smokes' barack'**)) =
  The function *Neg* such that if g ∈ J, *Neg*(g) = 1 iff h(**smokes' barack'**)(g) = 0

  G_{Concat}(h(**smokes'** x_3)) =
  The function *Neg'* such that if g ∈ J, *Neg'*(g) = 1 iff h(**smokes'** x_3)(g) = 0

(36) **How Does This Work? (Part 2)**

- Now, also recall from (32) that:
h((**smokes' barack'**)) = The function *F-A* such that if g ∈ J, then *F-A*(g) = *s*(*b*)
h((**smokes'** x_3)) = The function *F-A'* such that if g ∈ J, then *F-A'*(g) = *s*(g(x_3))

- It thus follows that:

h(~(**smokes' barack'**)) = G_{Not}(h(**smokes' barack'**)) =
The function *Neg* such that if g ∈ J, *Neg*(g) = 1 iff *s*(*b*) = 0

h(~(**smokes'** x_3)) = G_{Concat}(h(**smokes'** x_3)) =
The function *Neg'* such that if g ∈ J, *Neg'*(g) = 1 iff *s*(g(x_3)) = 0

(37) **Remarks**

- Thus, the 'meaning' of ~(**smokes' barack'**) is a constant function from contexts (variable assignments) to truth-values:
  - It maps every variable assignment to 1 iff *s*(*b*) = 0

- Thus, the 'meaning' of ~(**smokes'** x_3) is *not* a constant function from contexts (variable assignments) to truth-values:
  - It maps every variable assignment to 1 iff *s*(g(x_3)) = 0

- **Parallel with Demonstratives:**
  - The 'meaning' of *Barack doesn't smoke* maps every context to the same intension
  - The 'meaning' of *He doesn't smoke* maps different contexts to different intensions

- **Parallel with Models:**
  - No matter what variable assignment *g* is chosen [[~(**smokes' barack'**)]]^{M,g} will be the same value: 1 iff I(**smokes'**)(I(**barack'**)) = 0
  - The value of [[~(**smokes'** x_3)]]^{M,g} will equal 1 iff I(smokes)(g(x_3)) = 0, and so will vary with the variable assignment *g*.

(38) **The Operation G_{And}**

a. If α ∈ $M_{t, E, J}$ and β ∈ $M_{t, E, J}$, then G_{And}(α,β) =
The function *Conj* such that if g ∈ J, then *Conj*(g) = 1 iff α(g) = β(g) = 1

b. Otherwise, G_{And}(α) = **garbage**

(39) **How Does This Work?**

- Consider the following two conjunctions:
  ((**smokes' barack'**) & (**smokes' mitt'**)) ((**smokes' barack'**) & (**smokes'** x_3))

- Since F_{And} and G_{And} should 'correspond', it follows for meaning assignment h:

  h(((**smokes' barack'**) & (**smokes' mitt'**))) =
  G_{And}(h((**smokes' barack'**)), h((**smokes' mitt'**))

  h(((**smokes' barack'**) & (**smokes'** x_3))) =
  G_{And}(h((**smokes' barack'**)), h((**smokes'** x_3))

- Now, recall that h(**smokes' barack'**), h(**smokes' mitt'**), h((**smokes'** x_3)) $\in M_{t, E, J}$
  Thus, it follows from (38) that:

  G_{And}(h((**smokes' barack'**)), h((**smokes' mitt'**)) =
  The function *Conj* such that if g ∈ J, then *Conj*(g) = 1 iff
  h((**smokes' barack'**))(g) = h((**smokes' mitt'**)(g) = 1

  G_{And}(h((**smokes' barack'**)), h((**smokes'** x_3)) =
  The function *Conj* such that if g ∈ J, then *Conj*(g) = 1 iff
  h((**smokes' barack'**))(g) = h((**smokes'** x_3)(g) = 1

- Next, recall that from (32) – and a parallel computation – that:
  h((**smokes' barack'**)) = The function *F-A* such that if g ∈ J, then *F-A*(g) = *s*(*b*)
  h((**smokes' mitt'**)) = The function *F-A* such that if g ∈ J, then *F-A*(g) = *s*(*m*)
  h((**smokes'** x_3)) = The function *F-A'* such that if g ∈ J, then *F-A'*(g) = *s*(g(x_3))

- Consequently:
  h(((**smokes' barack'**) & (**smokes' mitt'**))) =
  The function *Conj* such that if g ∈ J, then *Conj*(g) = 1 iff *s*(*b*) = *s*(*m*) = 1

  h(((**smokes' barack'**) & (**smokes'** x_3))) =
  The function *Conj* such that if g ∈ J, then *Conj*(g) = 1 iff *s*(*b*) = *s*(g(x_3)) = 1

(40) **Remarks**

- Again, we have it that h(((**smokes' barack'**) & (**smokes' mitt'**))) is a constant function from variable assignment to truth-values.
  - Just as [[((**smokes' barack'**) & (**smokes' mitt'**))]]^{M,g} doesn't vary with g

- Again, we have it that h(((**smokes' barack'**) & (**smokes'** x_3))) is *not* a constant function from variable assignment to truth-values.
  - Just as [[((**smokes' barack'**) & (**smokes'** x_3))]]^{M,g} *does* vary with g

Given the discussion in (39)-(40), it can easily be seen that the following operations will similarly serve as semantic correlates of F_{Or} and F_{If}

(41) **The Operation G_{Or}**

a. If $\alpha \in M_{t, E, J}$ and $\beta \in M_{t, E, J}$, then $G_{Or}(\alpha,\beta)$ =
The function *Disj* such that if g ∈ J, then *Disj*(g) = 1 iff α(g) =1 or β(g) = 1

b. Otherwise, $G_{Or}(\alpha)$ = **garbage**

(42) **The Operation G_{If}**

a. If $\alpha \in M_{t, E, J}$ and $\beta \in M_{t, E, J}$, then $G_{If}(\alpha,\beta)$ =
The function *Cond* such that if g ∈ J, then *Cond*(g) = 1 iff α(g) =0 or β(g) = 1

b. Otherwise, $G_{If}(\alpha)$ = **garbage**

Now, we finally come to the central problem of how to treat the variable-binding operators…

(43) **The Operation $G_{\exists}$**

a. If there is a type τ ∈ T and a variable *v* ∈ VAR_{τ} such that for all g ∈ J,
α(g) = g(*v*), and $\beta \in M_{t, E, J}$, then
$G_{\exists}(\alpha,\beta)$ = The function *E* such that if g ∈ J, *E*(g) = 1 iff
there is an x ∈ $D_{\tau,E}$ such that β(g(*v*/x)) = 1

b. Otherwise, $G_{\exists}(\alpha,\beta)$ = **garbage**

(44) **How Does This Work? (Part 1)**

- First, note the requirement that there be a type τ ∈ T and a variable *v* ∈ VAR_{τ} such that for all g ∈ J, α(g) = g(*v*).

- Given our conditions in (30b,c), this will only ever hold (in a logically possible partly-Fregean interpretation) if α = f(*v*), i.e., the 'meaning' of the variable *v*.

- Thus, this condition ensures that if the first argument of $G_{\exists}$ is not a variable-meaning, then the output will be **garbage.**

  - Thus, our system will map syntactic garbage like '**∃mitt'** (**smokes' mitt'**) to **garbage**

(45) **How Does This Work? (Part 2)**

- Now, consider the simple existential sentence: $\exists x_3$ (**smokes'** x_3)

- Since $F_\exists$ and $G_\exists$ should 'correspond', it follows that for meaning assignment h:
  h($\exists x_3$ (**smokes'** x_3)) = h($F_\exists(x_3$, (**smokes'** x_3)) = $G_\exists$(h(x_3), h((**smokes'** x_3)))

- Next, note that h((**smokes'** x_3)) $\in M_{t, E, J}$ and for any g $\in$ J, h(x_3)(g) = f(x_3)(g) = g(x_3)

- Thus: $G_\exists$(h(x_3), h((**smokes'** x_3))) =
  The function *E* such that if g $\in$ J, *E*(g) = 1 iff
  there is an x $\in D_{e,E}$ such that h((**smokes'** x_3))(g(x_3/x)) = 1

- Now, recall that for any g $\in$ J, h((**smokes'** x_3))(g) = *s*(g(x_3))

- Thus: $G_\exists$(h(x_3), h((**smokes'** x_3))) =
  The function *E* such that if g $\in$ J, *E*(g) = 1 iff
  there is an x $\in D_{e,E}$ such that *s*(g(x_3/x)(x_3)) = 1

- Finally, by definition of 'g(x_3/x)', we have it that:
  h($\exists x_3$ (**smokes'** x_3)) = The function *E* such that if g $\in$ J, *E*(g) = 1 iff
  there is an x $\in D_{e,E}$ such that *s*(x) = 1

(46) **Remarks**

- Thus, the meaning of $\exists x_3$(**smokes'** x_3) is a constant function on variable assignments
  - It maps any variable assignment g to 1 iff there is an x $\in D_{e,E}$ such that the 'denotation of smokes' (*s*) maps x to 1

- **Parallel with Models:**
  For any variable assignment g, $[[\exists x_3(\textbf{smokes'}\ x_3)]]^{M,g}$ = 1 iff there is an x $\in D_{e,E}$ such that I(**smokes'**)(x) = 1.

Given the discussion in (44)-(46), it can easily be seen that the following operations will similarly serve as semantic correlates of $F_\forall$

(47) **The Operation $G_\forall$**

a. If there is a type $\tau \in$ T and a variable $v \in VAR_\tau$ such that for all g $\in$ J,
  α(g) = g(v), and $\beta \in M_{t, E, J}$, then
  $G_\forall(\alpha,\beta)$ = The function *A* such that if g $\in$ J, *A*(g) = 1 iff
  for all x $\in D_{\tau,E}$, β(g(v/x)) = 1

b. Otherwise, $G_\forall(\alpha,\beta)$ = **garbage**

And, last but not least, how do we handle the semantics of lambda abstraction?...

(48) **The Operation G_λ**

a. If there is a type $\sigma \in T$ and a variable $v \in VAR_\sigma$ such that for all $g \in J$,
$\alpha(g) = g(v)$, and $\beta \in M_{\tau, E, J}$, then
$G_\lambda(\alpha,\beta)$ = The function L such that if $g \in J$, $L(g)$ =
The function p with domain $D_{\sigma,E}$ such that for any $x \in D_{\sigma,E}$, $p(x) = \beta(g(v/x))$

b. Otherwise, $G_\lambda(\alpha,\beta)$ = **garbage**

(49) **How Does This Work?**

- Again, the requirement that 'for all $g \in J$, $\alpha(g) = g(v)$' ensures that if the first argument α is *not* the meaning of a variable, then $G_\lambda(\alpha,\beta)$ = **garbage**
  - Thus, '(λ**mitt'** (**smokes' mitt'**))' will end up being interpreted as **garbage**

- Now, consider the simple formula: (λx_3 (**smokes'** x_3))

- Since F_λ and G_λ should 'correspond', it follows that for meaning assignment h:
h((λx_3 (**smokes'** x_3))) = h(F_λ(x_3 , (**smokes'** x_3)) = G_λ(h(x_3), h((**smokes'** x_3)))

- Next, note that h((**smokes'** x_3)) $\in M_{t, E, J}$ and for any $g \in J$, $h(x_3)(g) = f(x_3)(g) = g(x_3)$

- Thus: G_λ(h(x_3), h((**smokes'** x_3))) =
The function L such that if $g \in J$, $L(g)$ =
The function p with domain $D_{e,E}$ such that for any $x \in D_{e,E}$,
$p(x)$ = h((**smokes'** x_3))($g(x_3/x)$)

- Now, recall that for any $g \in J$, h((**smokes'** x_3))(g) = $s(g(x_3))$

- Thus: G_λ(h(x_3), h((**smokes'** x_3))) =
The function L such that if $g \in J$, $L(g)$ =
The function p with domain $D_{e,E}$ such that for any $x \in D_{e,E}$,
$p(x) = s(g(x_3/x)(x_3))$

- Finally, by definition of '$g(x_3/x)$', we have it that:
h((λx_3 (**smokes'** x_3))) =
The function L such that if $g \in J$, $L(g)$ = the function p with domain $D_{e,E}$ such that for any $x \in D_{e,E}$, $p(x) = s(x)$

(50) **Remarks**

- So, the meaning of (λx_3(**smokes'** x_3)) is a constant function on variable assignments
  - It maps any variable assignment g to the function *p* which takes an $x \in D_{e,E}$ and returns the value that the 'denotation of smokes' (*s*) maps x to.

- **Parallel with Models:**
  For any variable assignment g, $[[(\lambda x_3(\textbf{smokes'}\ x_3))]]^{M,g}$ =
  The function *p* whose domain is $D_{e,E}$ and whose range is $D_{t,E}$, and for all $x \in D_{e,E}$
  $p(x) = I(\textbf{smokes})(x)$.

(51) **Summary**
It seems, then, that the operations $\{G_\gamma\}_{\gamma \in \{Concat, Not, And, Or, If, \exists, \forall, \lambda\}}$ will serve nicely as 'semantic correlates' of our syntactic operations $\{F_\gamma\}_{\gamma \in \{Concat, Not, And, Or, If, \exists, \forall, \lambda\}}$

Consequently, let us wrap up our definition in (30) as follows…

(52) **Logically Possible Partly-Fregean Interpretation of Politics+λ**

A logically possible partly-Fregean interpretation of Politics+λ connected with E, J, σ is a partly-Fregean interpretation **B** = $\langle B, G_\gamma, f\rangle_{\gamma \in \{Concat, Not, And, Or, If, \exists, \forall, \lambda\}}$ for Politics+λ connected with E, J, σ, such that:

a. J is the set of variable assignments based on E.

b. If $\varphi \in CON_\tau$, then $f(\varphi)(g) = f(\varphi)(g')$, for all $g, g' \in J$.

c. If $\varphi \in VAR_\tau$, then $f(\varphi)(g) = g(\varphi)$ for all $g \in J$.

d. The operations $\{G_\gamma\}_{\gamma \in \{Concat, Not, And, Or, If, \exists, \forall, \lambda\}}$ are as defined in (31)-(48)

3. A Logically Possible Partly-Fregean Interpretation for Politics+λ

Let us now illustrate the key notion in (52) by concretely spelling out such an interpretation.

(53) **A Logically Possible Partly-Fregean Interpretation for Politics+λ**

Let E be the set {Barack, Michelle, Mitt}. Let J be the set of variable assignments based on E. Let σ be the type assignment for Politics+λ such that for all $\tau \in T$, $\sigma(\tau) = \sigma(\langle var,\tau \rangle) = \tau$. Let **B** = $\langle B, G_\gamma, f \rangle_{\gamma \in \{\text{Concat, Not, And, Or, If, } \exists, \forall, \lambda\}}$ be the logically possible partly-Fregean interpretation of Politics+λ connected with E, J, σ such that f consists of the following mappings.

a. f(**barack’**) = The function *b* such that for all g ∈ J, *b*(g) = Barack.
b. f(**michelle’**) = The function *m* such that for all g ∈ J, *m*(g) = Michelle
c. f(**mitt’**) = The function *mt* such that for all g ∈ J, *mt*(g) = Mitt

d. f(**smokes’**) = The function *s* such that for all g ∈ J, *s*(g) =
o = { <Michelle,0>, <Barack,1>, <Mitt,0> }

e. f(**man’**) = The function *mn* such that for all g ∈ J, *mn*(g) =
i = { <Michelle,0>, <Barack,1>, <Mitt,1> }

f. f(**president’**) = The function *pr* such that for all g ∈ J, *pr*(g) =
k = { <Michelle,0>, <Barack,1>, <Mitt,0> }

g. f(**loves’**) = The function *l* such that for all all g ∈ J, *l*(g) =

$$j = \left(\begin{array}{ll} \text{Michelle} \rightarrow & \left\{ \begin{array}{l} \text{Michelle} \rightarrow 1 \\ \text{Barack} \rightarrow 1 \\ \text{Mitt} \rightarrow 0 \end{array} \right\} \\ \text{Barack} \rightarrow & \left\{ \begin{array}{l} \text{Michelle} \rightarrow 1 \\ \text{Barack} \rightarrow 1 \\ \text{Mitt} \rightarrow 0 \end{array} \right\} \\ \text{Mitt} \rightarrow & \left\{ \begin{array}{l} \text{Michelle} \rightarrow 0 \\ \text{Barack} \rightarrow 0 \\ \text{Mitt} \rightarrow 1 \end{array} \right\} \end{array} \right)$$

Finally, as we’ve done so many times before, we can use the meaning assignment h *determined by **B** to assign meanings to the expressions of* Politics+λ

- To save space, I’ve not includes all the calculation steps below. Students are encouraged to work out all the steps for themselves.

In the calculations below, let h be the meaning assignment determined by the interpretation **B** defined in (53).

(54) $\forall x_3$ (**man'** x_3)

- h($\forall x_3$ (**man'** x_3)) =
- The function *A* such that if g ∈ J, *A*(g) = 1 iff for all x ∈ $D_{e,E}$, *i*(x) = 1 =
- The function *A* such that if g ∈ J, *A*(g) = 0

Compare:
Relative to the model $\mathcal{M}$ defined in (13) on the previous handout, for any variable assignment g, $[[\forall x_3$ (**man'** $x_3)]]^{M,g} = 0$

(55) $\exists P_4$ (P_4 **michelle'**)

- h($\exists P_4$ (P_4 **michelle'**)) =
- The function *E* such that if g ∈ J, *E*(g) = 1 iff
  there is an x ∈ $D_{<e,t>,E}$ such that x(Michelle) = 1 =
- The function *E* such that if g ∈ J, *E*(g) = 1

Compare:
Relative to the model $\mathcal{M}$ defined in (13) on the previous handout, for any variable assignment g, $[[\exists P_4$ (P_4 **michelle'**)$]]^{M,g} = 1$

(56) (λx_3 ((**man'** x_3) & (**smokes'** x_3)))

- h((λx_3 ((**man'** x_3) & (**smokes'** x_3)))) =
- The function *L* such that if g ∈ J, *L*(g) = the function *p* with domain $D_{e,E}$ such that for any x ∈ $D_{e,E}$, *p*(x) = 1 *iff* *i*(x) = 1 and *o*(x) = 1. =
- The function *L* such that if g ∈ J, *L*(g) = { <Michelle, 0>, <Barack,1>, <Mitt,0> }

Compare:
Relative to the model $\mathcal{M}$ defined in (13) on the previous handout, for any variable assignment g, $[[(\lambda x_3$ ((**man'** x_3) & (**smokes'** x_3)))$]]^{M,g}$ =
{ <Michelle, 0>, <Barack,1>, <Mitt,0> }

(57) $(\lambda P_4\ (P_4\ \textbf{mitt'}))$

- h($(\lambda P_4\ (P_4\ \textbf{mitt'}))$) =

- The function *L* such that if g ∈ J, *L*(g) = the function *p* with domain $D_{<e,t>,E}$ such that for any x ∈ $D_{<e,t>,E}$, *p*(x) = x(Mitt)

<u>Compare:</u>
Relative to the model $\mathcal{M}$ defined in (13) on the previous handout, for any variable assignment g, $[[(\lambda P_4\ (P_4\ \textbf{mitt'}))]]^{M,g}$ =
The function *p* with domain $D_{<et>,E}$, range $D_{t,E}$ and for all x ∈ $D_{<et>,E}$, *p*(x) = x(Mitt)

(58) $(\lambda P_4\ \forall x_3\ (\ (\textbf{man'}\ x_3) \rightarrow (P_4\ x_3)\ )\ )$

- h($(\lambda P_4\ \forall x_3\ ((\textbf{man'}\ x_3) \rightarrow (P_4\ x_3)))$) =

- The function *L* such that if g ∈ J, *L*(g) = the function *p* with domain $D_{<e,t>,E}$ such that for any x ∈ $D_{<e,t>,E}$, *p*(x) = 1 iff for all y ∈ $D_{e,E}$, *i*(y) = 0 or x(y) = 1

- The function *L* such that if g ∈ J, *L*(g) = the function *p* with domain $D_{<e,t>,E}$ such that for any x ∈ $D_{<e,t>,E}$, *p*(x) = 1 iff for all y ∈ $D_{e,E}$, if *i*(y) = 1 then x(y) = 1

<u>Compare:</u>
Relative to the model $\mathcal{M}$ defined in (13) on the previous handout, for any variable assignment g, $[[(\lambda P_4\ \forall x_3\ ((\textbf{man'}\ x_3) \rightarrow (P_4\ x_3)))]]^{M,g}$ =
The function *p* with domain $D_{<et>,E}$, range $D_{t,E}$ and for all x ∈ $D_{<et>,E}$, *p*(x) = 1 iff for all y ∈ $D_{e,E}$, if *i*(y) = 1 then x(y) = 1

(59) $(\ (\lambda x_3\ ((\textbf{man'}\ x_3)\ \&\ (\textbf{smokes'}\ x_3)))\ \textbf{mitt'}\ )$

- h($((\lambda x_3\ ((\textbf{man'}\ x_3)\ \&\ (\textbf{smokes'}\ x_3)))\ \textbf{mitt'})$) =

- The function *F-A* such that if g ∈ J, then *F-A*(g) =
h($(\lambda x_3\ ((\textbf{man'}\ x_3)\ \&\ (\textbf{smokes'}\ x_3)))$)(g)(h(**mitt'**)(g)) = (by (53), (56))

- The function *F-A* such that if g ∈ J, then *F-A*(g) = *L*(g)(*mt*(g)) = (by (53), (56))

- The function *F-A* such that if g ∈ J, then *F-A*(g) =
{ <Michelle, 0>, <Barack,1>, <Mitt,0> }(Mitt) =

- The function *F-A* such that if g ∈ J, then *F-A*(g) = 0

<u>Compare:</u> Relative to the model $\mathcal{M}$ defined in (13) on the previous handout, for any variable assignment g, $[[(\ (\lambda x_3\ ((\textbf{man'}\ x_3)\ \&\ (\textbf{smokes'}\ x_3)))\ \textbf{mitt'}\ )]]^{M,g}$ = 0

4. Relationship Between Models and Logically Possible Partly-Fregean Interpretation

We now have two ways of providing Politics+λ a formal semantics:
(i) The model-theoretic semantics from the handout 'Preliminaries'
(ii) The logically possible partly-Fregean interpretation in (53)

(60) **Fun Fact:** There is (again) an important equivalence between these two systems!

(61) **The Relationship Between Models and Interpretations (Part 1)**
Let $\mathcal{M}$ be a model <E,I> for Politics+λ. Let J be the set of variable assignments based on $\mathcal{M}$. Let $B = \cup_{\tau \in T} M_{\tau, E, J}$.

Finally, let f be the function whose domain is $\cup_{\delta \in \Delta} X_\delta$ and for any $\varphi \in \cup_{\delta \in \Delta} X_\delta$, $f(\varphi)$ = the function m whose domain is J and for any g ∈ J, $m(g) = [[\varphi]]^{M,g}$

a. Claim 1:
The structure **B** = $\langle B, G_\gamma, f\rangle_{\gamma \in \{Concat,\ Not,\ And,\ Or,\ If,\ \exists,\ \forall,\ \lambda\}}$ is a logically-possible partly-Fregean interpretation of Politics+λ.

b. Claim 2:
Let h be the meaning assignment determined by **B**. Every meaningful expression φ of Politics+λ is such that, for any variable assignment g ∈ J:

$$h(\varphi)(g) \quad = \quad [[\varphi]]^{M,g}$$

(62) **Remark**
Thus, if we are given a model $\mathcal{M}$ for Politics+λ, we can construct a logically possible partly-Fregean intepretation **B** that assigns equivalent values to the meaningful expressions of Politics+λ.

(63) **The Relationship Between Models and Interpretations (Part 1)**
Let **B** = $\langle B, G_\gamma, f\rangle_{\gamma \in \{Concat,\ Not,\ And,\ Or,\ If,\ \exists,\ \forall,\ \lambda\}}$ be a logically possible partly-Fregean interpetation of Politics+λ connected with E, J, σ. Let I be the function whose domain is $\cup_{\tau \in T} CON_\tau$ and for any $\varphi \in \cup_{\tau \in T} CON_\tau$, $I(\varphi) = f(\varphi)(g)$, for an arbitrary g.

a. Claim 1: The structure $\mathcal{M}$ = <E, I> is a model of Politics+λ.

b. Claim 2:
Let h be the meaning assignment determined by **B**. Every meaningful expression φ of Politics+λ is such that, for any variable assignment g ∈ J:

$$h(\varphi)(g) \quad = \quad [[\varphi]]^{M,g}$$

(63) **Remark**
Thus, if we are given a logically possible partly-Fregean interpretation **B** of Politics+λ, we can construct a model $\mathcal{M}$ for Politics+λ that assigns equivalent values to the meaningful expressions of Politics+λ.

(64) **Summary**

- Given this relationship between models and logically possible partly-Fregean interpretations of Politics+λ, we can freely shift between the two (*cf.* sets and their characteristic functions).

- Similarly, we can view these two systems as being *in essence* 'the same thing' (even though they are *very* different set theoretic objects).

- **Thus, even when we're presenting our semantics in model-theoretic terms, we are also thereby providing a (Montagovian) interpretation for the language.**

(65) **The Importance of This Point**

- As you can probably tell now, once we've got variable binding in the language, the model-theoretic presentation of the semantics is *much simpler and more comprehensible* than its presentation as a (Montagovian) 'interpretation.'

- **The difference in comprehensibility is even more profound for Montague's system of 'Intensional Logic', used in PTQ.**

- For this reason, in PTQ (but not UG), Montague presents the semantics for IL in strictly model-theoretic terms (no 'interpretations').

    - I will be following suit, **but do know that there is straight-forward way of converting Montague's model structures for IL into 'interpretations'…** (just see UG, if you're interested!...)