Montague's Theory of Translation: Translation Bases and Indirect Interpretations [1]

1. From Translation Functions to Interpretations

(1) **What We Currently Have**

a. Mini-English (and Disambiguated-Mini-English):

Mini-English is $<<E, K_\gamma, X_\delta, S_E, S>_{\gamma \in \{\text{Merge-S, Merge-IV, Not, And, If}\}, \delta \in \Delta}, R>$

(i) $< E, K_\gamma, X_\delta, S_E, S>_{\gamma \in \{\text{Merge-S, Merge-IV, Not, And, If}\}, \delta \in \Delta}$ is Disambiguated Mini-English

(ii) R maps trees in E to the first member of their root node.

b. Politics-NoQ

Politics-NoQ is the language $<< A, F_\gamma, X_\tau, S, t>_{\gamma \in \{\text{Concat, Not, And}\}, \tau \in T}, Id>$

(i) $<A, F_\gamma, X_\tau, S, t>_{\gamma \in \{\text{Concat, Not, And}\}, \tau \in T}$ is the disambiguated language Politics-NoQ.

(ii) Id is the identity function.

c. Interpretation for Politics-NoQ

Let the set S = { Michelle, Barack, Mitt }. Let **B** = $<B, G_\gamma, f>_{\gamma \in \{\text{Concat, Not, And}\}}$ be the Fregean interpretation based on S, such that *f* is as defined before.

d. Translation Base from Mini-English to Politics-NoQ

Let be **T** is the translation base $< g, H_\gamma, j>_{\gamma \in \{\text{Merge-S, Merge-IV, Not, And, If}\}}$, where g, H_γ, and j are as defined before.

Where *k* is the translation function determined by **T** and *h* is the meaning assignment determined by **B**, we have the following homomorphisms.

$< E, K_\gamma >_{\gamma \in \{\text{Merge-S, Merge-IV, Not, And, If}\}}$ → (*k*) → $<A, H_\gamma>_{\gamma \in \{\text{Merge-S, Merge-IV, Not, And, If}\}}$

$< A, F_\gamma >_{\gamma \in \{\text{Concat, Not, And}\}}$ → (*h*) → $< B, G_\gamma >_{\gamma \in \{\text{Concat, Not, And}\}}$

(2) **Key Issue**

- What we want is a homomorphism that maps E to B.
- **But since the operation indices are different in** $<A, H_\gamma>_{\gamma \in \{\text{Merge-S, Merge-IV, Not, And, If}\}}$ **and** $< A, F_\gamma >_{\gamma \in \{\text{Concat, Not, And}\}}$ **we don't have such a homomorphism yet…**

[1] These notes are based upon material in the following readings: Halvorsen & Ladusaw (1979), Dowty *et al.* (1981) Chapter 8, and Thomason (1974) Chapter 7 (Montague's "Universal Grammar").

(3) **The Key Theorem Relating Translation to Interpretation**
If $<A, F_\gamma>_{\gamma \in \Gamma}$ is an algebra, h is a homomorphism from $<A, F_\gamma>_{\gamma \in \Gamma}$ to some algebra ($<B, G_\gamma>_{\gamma \in \Gamma}$), and for each $\gamma \in \Pi$, H_γ is a polynomial operation over $<A, F_\gamma>_{\gamma \in \Gamma}$, **then there is exactly one algebra $<B, G_\gamma>_{\gamma \in \Pi}$ such that h is a homomorphism from $<A, H_\gamma>_{\gamma \in \Pi}$ to $<B, G_\gamma>_{\gamma \in \Pi}$** (Montague 1974: 225)

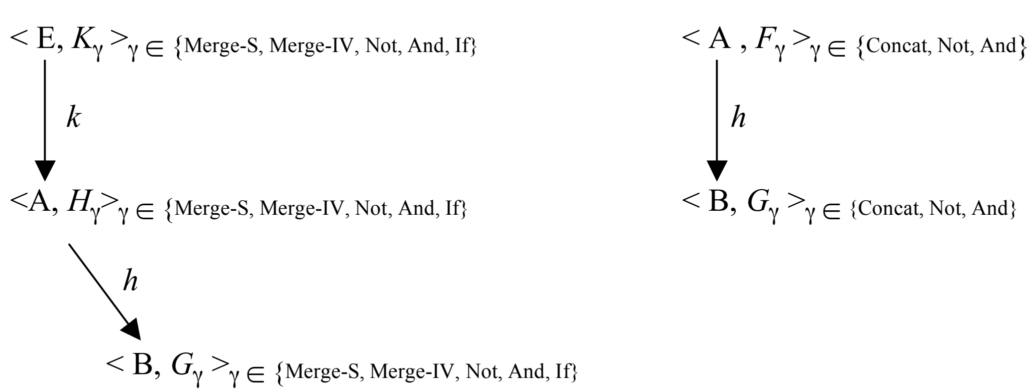
(4) **Restatement of the Key Theorem**
Suppose the following conditions all hold:
a. There are two algebras **A** = $<A, F_\gamma>_{\gamma \in \Gamma}$ and **B** = $<B, G_\gamma>_{\gamma \in \Gamma}$
b. There is a homomorphism h from **A** to **B**
c. There is a an algebra **A'** = $<A, H_\gamma>_{\gamma \in \Pi}$ where $\{ H_\gamma \}_{\gamma \in \Pi}$ are all polynomial operations over $<A, F_\gamma>_{\gamma \in \Gamma}$

There is therefore one (exactly one) algebra **B'** = $<B, G_\gamma>_{\gamma \in \Pi}$ such that h is also a homomorphism from **A'** to **B'**.

(5) **The Importance of the Key Theorem**

- Conditions (4a-c) are exactly what we have in (1)!
  a. $<A, F_\gamma>_{\gamma \in \{\text{Concat, Not, And}\}}$ and $<B, G_\gamma>_{\gamma \in \{\text{Concat, Not, And}\}}$ are algebras.
  b. Meaning assignment h is a homomorphism from **A** $<A, F_\gamma>_{\gamma \in \{\text{Concat, Not, And}\}}$ to **B** $<B, G_\gamma>_{\gamma \in \{\text{Concat, Not, And}\}}$
  c. $<A, H_\gamma>_{\gamma \in \{\text{Merge-S, Merge-IV, Not, And, If}\}}$ is an algebra where the operations are all polynomial operations over $<A, F_\gamma>_{\gamma \in \{\text{Concat, Not, And}\}}$

- Therefore, the key theorem in (3)/(4) guarantees us the following:
  There is an algebra $<B, G_\gamma>_{\gamma \in \{\text{Merge-S, Merge-IV, Not, And, If}\}}$ such that h is also a homomorphism to it from $<A, H_\gamma>_{\gamma \in \{\text{Merge-S, Merge-IV, Not, And, If}\}}$



(6) **Remarks**

- Note that in the algebra $< B, G_\gamma >_{\gamma \in \{\text{Merge-S, Merge-IV, Not, And, If}\}}$ the set B is the set of meanings in our interpretation of Politics-NoQ

- Although Montague doesn't say it, the operations $\{ G_\gamma \}_{\gamma \in \{\text{Merge-S, Merge-IV, Not, And, If}\}}$ are all polynomial operations over $<B, G_\gamma>_{\gamma \in \{\text{Concat, Not, And}\}}$ whose definitions mirror those of the operations in our translation base $\{ H_\gamma \}_{\gamma \in \{\text{Merge-S, Merge-IV, Not, And, If}\}}$

- As desired, $h^o k$ is a homomorphism from $<E, K_\gamma>_{\gamma \in \{\text{Merge-S, Merge-IV, Not, And, If}\}}$ (Disambiguated-Mini-English) to $< B, G_\gamma >_{\gamma \in \{\text{Merge-S, Merge-IV, Not, And, If}\}}$ (our derived semantic algebra).

- **Thus, thanks to our translation base T, we now have an interpretation for Mini-English.**

  <u>The Interpretation for Mini-English:</u> $< B, G_\gamma , h^o j >_{\gamma \in \{\text{Merge-S, Merge-IV, Not, And, If}\}}$
  (i) $<B, G_\gamma >_{\gamma \in \{\text{Merge-S, Merge-IV, Not, And, If}\}}$ is the derived algebra guaranteed by (3)/(4),
  (ii) h is the meaning assignment determined by $< B, G_\gamma >_{\gamma \in \{\text{Concat, Not, And}\}}$
  (iii) j is the lexical translation function in our translation base **T**.

  - Note that this structure will satisfy our general definition of an interpretation:

    - Because $h^o k$ is a homomorphism, we know that for all $\gamma \in$ {Merge-S, Merge-IV, Not, And, If}, K_γ and G_γ are of the same arity.

    - $h^o j$ is a function from the basic expressions of Mini-English into B.

Clearly, this result in (5)/(6) generalizes to all languages, allowing us to state the following general theorem…

(7) **General Theorem on Indirect Interpretation**
Let **L** and **L'** be languages such that there is an interpretation **B** for **L'** and a translation base **T** from **L** to **L'**. There is an interpretation **B**' for **L**.

You can no doubt already see how (7) follows from what we've seen so far…
For those who are interested, we can give a more explicit proof of it as in (8).

(8) **Proof of the General Theorem on Indirect Interpretation**

- Let h be the meaning assignment determined by **B**. Let k be the translation function determined by **T**. Let j be the lexical translation function in **T**.

- By definition, h is a homomorphism from the syntactic algebra of **L'** $<A, F_\gamma>_{\gamma \in \Gamma}$ to the semantic algebra of **B** $<B, G_\gamma>_{\gamma \in \Gamma}$

- By definition, k is a homomorphism from the syntactic algebra of **L** $<E, K_\gamma>_{\gamma \in \Pi}$ to the algebra $<A, H_\gamma>_{\gamma \in \Pi}$ where $\{H_\gamma\}_{\gamma \in \Pi}$ are all polynomial operations over $<A, F_\gamma>_{\gamma \in \Gamma}$

- Therefore, by the theorem in (3)/(4), there is an algebra $<B, G_\gamma>_{\gamma \in \Pi}$ such that h is also a homomorphism from $<A, H_\gamma>_{\gamma \in \Pi}$ to $<B, G_\gamma>_{\gamma \in \Pi}$

- Consequently, $h^o k$ is a homomorphism from $<E, K_\gamma>_{\gamma \in \Pi}$ to $<B, G_\gamma>_{\gamma \in \Pi}$

- Therefore, we know that for all $\gamma \in \Pi$, K_γ and G_γ have the same arity.

- Moreover, $h^o j$ is a function from the basic categories in **L** to B.

- Therefore, the structure $<B, G_\gamma, h^o j>_{\gamma \in \Pi}$ is an interpretation for **L**.

(9) **Remark**

Furthermore, the meaning assignment determined by the interpretation $<B, G_\gamma, h^o j>_{\gamma \in \Pi}$ will be $h^o k$ (proof left as exercise for the student)

(10) **The Big Upshot**

a. <u>Our Initial Question:</u>
Given our background theory of language and meaning, under what conditions can we guarantee that translating from one language *L* into another language *L'* gives us a compositional semantics for *L*.

b. <u>Answer:</u>
If we can provide an interpretation for *L'* and the translation from *L* to *L'* satisfy the conditions of a **translation base**, then we are guaranteed a compositional semantics for *L*.

Thus, a new viable path to providing a semantics for a (natural) language is to provide a **translation base** from that language to a logical language whose semantics is already defined.

2. Illustration: Mini-English and Politics-NoQ

(11) **First Observation**
When we compose together k and h in (1), we get a mapping from sentences of Disambiguated-Mini-English to meanings in B.

<u>Illustration:</u> Let T be the tree such that R(T) = *Barack loves Michelle*.

a. $h^{o}k(T)$ = (by definition of function composition)

b. $h(k(T))$ = (by definition of Mini-English)

c. $h(k(K_{\text{Merge-S}}(\ \langle \textit{Barack}, \varnothing \rangle,\ K_{\text{Merge-IV}}(\ \langle \textit{loves}\ \varnothing \rangle,\ \langle \textit{Michelle}, \varnothing \rangle))))$ =
(by homomorphism prop.)

d. $h(H_{\text{Merge-S}}(\ k(\langle \textit{Barack}, \varnothing \rangle),\ H_{\text{Merge-IV}}(\ k(\langle \textit{loves}\ \varnothing \rangle), k(\langle \textit{Michelle}, \varnothing \rangle))))$ =
(by definition of k)

e. $h(H_{\text{Merge-S}}(\ \mathbf{barack'},\ H_{\text{Merge-IV}}(\ \mathbf{loves'}, \mathbf{michelle'}\ )))$ = (by def. of $H_{\text{Merge-IV}}$)

f. $h(H_{\text{Merge-S}}(\ \mathbf{barack'}\ ,\ (\mathbf{loves'\ michelle'})))$ = (by definition of $H_{\text{Merge-S}}$)

g. $h(((\mathbf{loves'\ michelle'})\ \mathbf{barack'}))$ = (by definition of Politics-NoQ)

h. $h(\ F_{\text{Concat}}(\ F_{\text{Concat}}(\mathbf{loves'}, \mathbf{michelle'}), \mathbf{barack'}))$ = (by homomorphism prop.)

i. $G_{\text{Concat}}(\ G_{\text{Concat}}(h(\mathbf{loves'}), h(\mathbf{michelle'})), h(\mathbf{barack'})))$ = (by def. of h)

j. $G_{\text{Concat}}(\ G_{\text{Concat}}(\ j$, Michelle), Barack)) = (by def. of G_{Concat})

k. $G_{\text{Concat}}(\ j$(Michelle) , Barack)) = (by def. of G_{Concat})

l. j(Michelle)(Barack) = (by def. of j)

m. 1

(12) **Second Observation**
Observing the behavior of $h^{o}k$ over a range of examples, we can directly construct an interpretation for Disambiguated-Mini-English, which will mirror the behavior of $h^{o}k$

(13) **The Interpretation of Disambiguated Mini-English**

Let $< \mathrm{B}, G_{\gamma}, l >_{\gamma \in \{\text{Merge-S, Merge-IV, Not, And, If}\}}$ be the structure defined as follows:

a. The Definition of the Set B

The set B is the same set as the set B in $\mathbf{B} = <\mathrm{B}, G_{\gamma}, f>_{\gamma \in \{\text{Concat, Not, And}\}}$, the interpretation of Politics-NoQ we had defined previously.

- That is $\mathrm{B} = \cup_{\tau \in \mathrm{T}} D_{\tau}$, $_{\{\text{Michelle, Barack, Mitt}\}}$

b. The Definition of the Semantic Operations

The operations $\{G_{\gamma}\}_{\gamma \in \{\text{Merge-S, Merge-IV, Not, And, If}\}}$ are defined as follows:

(i)	G_{Not}	=	G_{Not} (defined previously)
(ii)	G_{And}	=	G_{And} (defined previously)
(iii)	G_{If}	=	$G_{Not}<G_{And}<Id_{1,2}, G_{Not}<Id_{2,2}>>>$
(iv)	$G_{Merge\text{-}IV}$	=	G_{Concat} (defined previously)
(v)	$G_{Merge\text{-}S}$	=	$G_{Concat}<Id_{2,2}, Id_{1,2}>$

Note:

- The definitions of the operations $\{G_{\gamma}\}_{\gamma \in \{\text{Merge-S, Merge-IV, Not, And, If}\}}$ mirror the definitions of the operations $\{H_{\gamma}\}_{\gamma \in \{\text{Merge-S, Merge-IV, Not, And, If}\}}$ in our translation base.

- Moreover, $\{G_{\gamma}\}_{\gamma \in \{\text{Merge-S, Merge-IV, Not, And, If}\}}$ are all polynomial operations over the semantic algebra $<\mathrm{B}, G_{\gamma}>_{\gamma \in \{\text{Concat, Not, And}\}}$

- Consequently, $<\mathrm{B}, G_{\gamma}>_{\gamma \in \{\text{Merge-S, Merge-IV, Not, And, If}\}}$ is an algebra.

c. The Definition of the Lexical Interpretation Function

The lexical interpretation function l is defined as follows:

(i)	$l(<$ *Barack* , $\varnothing >)$	=	Barack
(ii)	$l(<$ *Michelle* , $\varnothing >)$	=	Michelle
(iii)	$l(<$ *Mitt* , $\varnothing >)$	=	Mitt
(iv)	$l(<$ *smokes* , $\varnothing >)$	=	the function h equal to f(**smokes'**)
(v)	$l(<$ *loves* , $\varnothing >)$	=	the function j equal to f(**loves'**)

Note: Where g is the meaning assignment determined by **B**, $l = g^{o}j$

(14) **Remark** $< \mathrm{B}, G_{\gamma}, l >_{\gamma \in \{\text{Merge-S, Merge-IV, Not, And, If}\}}$ is an interpretation of Mini-English

(15) **Remark**

The meaning assignment g determined by $<B, G_\gamma, I>_{\gamma \in \{\text{Merge-S, Merge-IV, Not, And, If}\}}$ is equal to $h^\circ k$

(16) **Illustration**

Let g be the meaning assignment determined by $<B, G_\gamma \;\; I>_{\gamma \in \{\text{Merge-S, Merge-IV, Not, And, If}\}}$.
Let T be the tree such that R(T) = *Barack loves Michelle*.

a. $g(T)$ = (by definition of Mini-English)

c. $g(\, K_{\text{Merge-S}}(\; <\textit{Barack}, \varnothing>,$
$K_{\text{Merge-IV}}(\; <\textit{loves}\; \varnothing>, \; <\textit{Michelle}, \varnothing>)\,)\,)$ =
(by homomorphism prop.)

d. $G_{\text{Merge-S}}(\; g(<\textit{Barack}, \varnothing>),$
$G_{\text{Merge-IV}}(\; g(<\textit{loves}\; \varnothing>), g(<\textit{Michelle}, \varnothing>)\,)\,)$ =
(by definition of g)

e. $G_{\text{Merge-S}}($ Barack, $G_{\text{Merge-IV}}(\, j,$ Michelle)) = (by def. of $G_{\text{Merge-IV}}$)

f. $G_{\text{Merge-S}}($ Barack, j(Michelle)) = (by def. of $G_{\text{Merge-S}}$)

g. j(Michelle)(Barack) = (by def. of j)

m. 1

---

3. Indirect Interpretation: A Summary

(17) **Direct Interpretation**
Let **L** be a language. **Direct interpretation of L** is the specification of an interpretation **B** of **L**.

(18) **Indirect Interpreation**
Let **L** be a language. **Indirect interpretation of L** is the specification of a language **L'**, an interpretation **B'** of **L'** and a translation base **T** from **L** to **L'**.

(19) **Indirect Interpretation Always Yields Direct Interpretation**

- Suppose that we have indirectly interpreted the language **L**.
  - That is, we have defined a language **L'** an interpretation **B'** of **L'** and a translation base **T** from **L** to **L'**
- **Given the result in (7)/(8) it is thus trivial to construct an interpretation B of L.**

(20) **Some Choice Quotes**

- From Halvorsen & Ladusaw (1979), p. 210:
  "An understanding of [the eliminability of indirect interpretation] is necessary to understand the use of…logics of PTQ and most other analyses within Montague grammar. As has been stated elsewhere, the [logical language] is an ***expository device and is in no way a necessary part of an analysis of any language offered within this theory.*** By using the easily interpreted [logical language] as a mediator, natural languages can be analyzed syntactically and then provided with a translation…from them to [the logical language] to induce their interpretation….***This method of analysis amounts to direct interpretation of natural language.***" (emphasis mine)

- From Dowty *et al.* (1981), p. 263:
  "Translating English into [a logical language] was therefore not essential to interpreting the English phrases we generated; ***it was simply a convenient intermediate step in assigning them meanings. This step could have been eliminated had we chosen to describe the interpretation of English directly…*** This point is important, because anyone who does not appreciate it may misunderstand the role of [logical languages] in applications of Montague's descriptive framework to natural languages." (emphasis mine)

(21) **Why Do Indirect Interpretation?**

It's Just Sometimes Conceptually 'Easier'
If the logical language is well-designed and familiar to readers, then it can provide a more 'perspicuous' representation (statement / name) of the meanings that we wish to be assigned to the (natural) language expressions.

Illustration (From 610):
'[λx_e : x smokes]' vs. 'The function f from D_e to D_t such that for all $x \in D_e$, $f(x) = 1$ *iff* x smokes.'

- From Halvorsen & Ladusaw (1979), p. 216:
  "Since the translation process is less involved than interpretation, presentation of fragments of the languages becomes clearer."

- From Dowty *et al.* (1981), p. 264:
  "[The purpose of indirect interpretation] was to have a convenient, compact notation for giving a briefer statement of semantic rules than we were able to give in earlier chapters of this book, where semantic rules were formulated rather long-windedly in English… [The logical language] could provide us with names for meanings…"

(21) **Road Map of Where We've Been and Where We're Going**

- We've now covered the conceptual core of the Montague Grammar architecture, as well as the key sections of Montague's "Universal Grammar".
  - Sections of UG Covered: 1, 2, 3, 5

- What we *haven't* covered from UG is:
  - The general theory of Fregean interpretations
  - Montague's presentation of the syntax/semantics of his Intensional Logic
  - Montague's presentation of the syntax/translation of a fragment of English

- **However, those sections of UG are largely superceded by Montague's paper "The Proper Treatment of Quantification in Ordinary English" (PTQ)**
  - If we had all the time in the world, we would cover *both* systems and compare them (hint for final paper).
  - Given our limited time, however, I'd like to now move us from UG to PTQ.

- <u>What's Next on the Agenda:</u>
  Extending this system to handle quantification in FOL and English.
  - An algebraic treatment of FOL (with quantification)
  - An translation base from a fragment of English to FOL.

- *Following that, we'll examine Montague's Intensional Logic and its applications to English (and various puzzles therein).*