

Montague's General Theory of Semantics¹

1. Revising Our Algebraic Perspective on 'Interpretation'

Last time, we motivated and illustrated the following general definition of what a 'language' is:

(1) The Montagovian Definition of 'Language' (To Be Slightly Revised)

A language L is a structure $\langle A, F_\gamma, X_\delta, S, \delta_0 \rangle_{\gamma \in \Gamma, \delta \in \Delta}$ such that:

- a. $\langle A, F_\gamma \rangle_{\gamma \in \Gamma}$ is an algebra.
- b. A is the smallest set such that:
 - (i) For all $\delta \in \Delta$, $X_\delta \subseteq A$.
 - (ii) A is closed under the operations F_γ for all $\gamma \in \Gamma$
- c. S is a set of sequences of the form $\langle F_\gamma, \langle \delta_1, \dots, \delta_n \rangle, \delta \rangle$, where $\gamma \in \Gamma$, F_γ is an n -ary operation, and $\delta_1, \dots, \delta_n, \delta \in \Delta$
- b. $\delta_0 \in \Delta$

Given this (re)definition of what a 'language' is, we must now slightly revise our earlier definition of what an 'interpretation' of a language is.

(2) The Montagovian Definition of 'Interpretation' (To Be Slightly Revised)

Let L be a language $\langle A, F_\gamma, X_\delta, S, \delta_0 \rangle_{\gamma \in \Gamma, \delta \in \Delta}$. An *interpretation* for L is a structure $\langle B, G_\gamma, f \rangle_{\gamma \in \Gamma}$ such that:

- a. $\langle B, G_\gamma \rangle_{\gamma \in \Gamma}$ is an algebra with the same number of operations as $\langle A, F_\gamma \rangle_{\gamma \in \Gamma}$

Note: This is basically already represented in the fact that that the operations G_γ and F_γ are indexed by the same set Γ

- b. G_γ is an operation of the same arity as F_γ , for all $\gamma \in \Gamma$

Note: As we'd expect, each G_γ is the 'semantic operation' corresponding to the syntactic operation F_γ

- c. f is a function from $\cup_{\delta \in \Delta} X_\delta$ into B .

Note: f is a function from the basic expressions (lexical items) to some meanings. Thus, f represents the 'lexical semantics' of our language, and so is like the function 'I' in a model $\langle D, I \rangle$.

¹ These notes are based upon material in the following readings: Halvorsen & Ladusaw (1979), Dowty *et al.* (1981) Chapter 8, and Thomason (1974) Chapter 7 (Montague's "Universal Grammar").

(3) **The Definition of ‘Meaning Assignment’**

Let L be a language $\langle A, F_\gamma, X_\delta, S, \delta_0 \rangle_{\gamma \in \Gamma, \delta \in \Delta}$. Let $\mathbf{B} = \langle B, G_\gamma, f \rangle_{\gamma \in \Gamma}$ be an interpretation for L .

The *meaning assignment* for L determined by \mathbf{B} is the unique homomorphism g from the syntactic algebra $\langle A, F_\gamma \rangle_{\gamma \in \Gamma}$ to the semantic algebra $\langle B, G_\gamma \rangle_{\gamma \in \Gamma}$ such that $f \subseteq g$.

(4) **Some Remarks**

- a. The fact that an interpretation \mathbf{B} does indeed determine such a (unique) homomorphism g is not obvious, but follows from conditions (2a,b).
- b. Under the definitions in (2) and (4), we still conceive of ‘meaning’ (i.e., semantics) as a homomorphism from a syntactic algebra to a semantic one.
- c. Under these new definitions, however, a meaning assignment g will *also* map all the ‘syntactic garbage’ in A to some kind of meanings in B .
 - **This consequence is definitely surprising. We’ll see in a bit, however, that it’s basically ‘harmless’.**
 - **Note, too, that it’s a necessary consequence of our general conception of semantics as ‘homomorphism between algebras’.**
 - Since the set of all and only the well-formed expressions of a language *don’t generally form an algebra*...
 - In the general case, the ‘syntactic algebra’ for a language will contain a bunch of ‘syntactic garbage’.
 - Consequently, a semantics for the language will also end up interpreting the ‘syntactic garbage’ in the algebra...

We’ll now illustrate the definitions in (2) and (3) by describing a general class of interpretations for FOL-NoQ, as well as some specific instances of this class...

2. Towards The Notion of a ‘Fregean Interpretation’

In his classic paper ‘Universal Grammar’, Montague introduces the term ‘Fregean Interpretation’ as a label for a specific *kind* of ‘interpretation’, as defined in (2).

- It’s a very complex definition, and we’ll begin to get a handle on it by introducing a core sub-part of it for our language of **FOL-NoQ**

(5) The Language FOL-NoQ

The language FOL-NoQ is the structure $\langle A, F_\gamma, X_\tau, S, t \rangle_{\gamma \in \{\text{Concat, Not, And}\}, \tau \in T}$ where:

a. $\langle A, F_\gamma \rangle_{\gamma \in \{\text{Concat, Not, And}\}}$ is the algebra such that

- (i) $F_{\text{Concat}} =$ Concat (from previous notes)
 $F_{\text{Not}} =$ Not (from previous notes)
 $F_{\text{And}} =$ And (from previous notes)

- (ii) A is the smallest set such that:
1. It contains every individual constant and predicate letter.
 2. It is closed under $F_{\text{Concat}}, F_{\text{Not}}, F_{\text{And}}$

b. The basic categories X_τ are such that:

- (i) $X_e =$ The set of individual constants, CONS
- (ii) $X_{\langle e, \dots, t \rangle} =$ The set of n -ary predicate letters, where $n =$ the number of times e appears in the category label $X_{\langle e, \dots, t \rangle}$
- (iii) For all other types $\tau \in T$, $X_\tau = \emptyset$.

c. The set S is the following (infinite) set of syntactic rules:

- (i) $\langle F_{\text{Not}}, \langle t \rangle, t \rangle$
(ii) $\langle F_{\text{And}}, \langle t, t \rangle, t \rangle$
(iii) $\langle F_{\text{Concat}}, \langle \langle \sigma, \tau \rangle, \sigma \rangle, \tau \rangle$, for all $\sigma, \tau \in T$

Illustrative Members of C_t :

$((((Qa)b) \& \sim(Rc)) \sim(\sim(Ab) \& \sim((Sc)d))) \sim(((Tg)b) \& \sim(Lg))$

We’ll begin by defining a family of sets that will serve as the semantic values in a ‘Fregean interpretation’

(6) **The Set of ‘Possible Denotations’ (To be Revised)**

Let T be the set of types and E be some non-empty set (of entities). The set $D_{\tau, E}$ of denotations of type τ based on E is defined as follows:

- (i) $D_{e, E} = E$
- (ii) $D_{t, E} = \{0, 1\}$
- (iii) If $\sigma, \tau \in T$, then $D_{\langle \sigma, \tau \rangle, E} =$ the set of functions from $D_{\sigma, E}$ to $D_{\tau, E}$
 $= D_{\tau, E}^{D_{\sigma, E}}$

Illustration: Let $E = \{ \text{Michelle, Barack, Mitt} \}$.

$$D_{e, E} = \{ \text{Michelle, Barack, Mitt} \}$$

$$D_{t, E} = \{0, 1\}$$

$$D_{\langle e, t \rangle, E} = \begin{array}{l} \text{All the functions from } \{ \text{Michelle, Barack, Mitt} \} \text{ to } \{0, 1\} \\ \text{All the characteristic functions of members of } P(E). \end{array}$$

$$D_{\langle e, \langle e, t \rangle \rangle, E} = \begin{array}{l} \text{All the functions from } \{ \text{Michelle, Barack, Mitt} \} \text{ to} \\ \text{some function from } \{ \text{Michelle, Barack, Mitt} \} \text{ to} \\ \{1, 0\} \\ = \text{All the characteristic functions of members of } E \times E \end{array}$$

The family of sets defined in (6) will be a key ingredient in the definition of a ‘Fregean Interpretation’ of FOL-NoQ...

The second key ingredient will be the semantic operations defined below...

(7) **The Semantic Operations (To Be Revised)**

- a. The Operation ‘ G_{Not} ’
 $G_{\text{Not}} = \{ \langle 1, 0 \rangle, \langle 0, 1 \rangle \}$
- b. The Operation ‘ G_{And} ’
 $G_{\text{And}} = \{ \langle \langle 1, 1 \rangle, 1 \rangle, \langle \langle 1, 0 \rangle, 0 \rangle, \langle \langle 0, 1 \rangle, 0 \rangle, \langle \langle 0, 0 \rangle, 0 \rangle \}$
- c. The Operation ‘ G_{Concat} ’²
 If $\alpha \in D_{\langle \sigma, \tau \rangle, E}$ and $\beta \in D_{\sigma, E}$, then $G_{\text{Concat}}(\alpha, \beta) = \alpha(\beta)$

With these ingredients, we can now make a first attempt at defining what a ‘Fregean Interpretation’ of FOL-NoQ is...

² Note that G_{Concat} is basically an operation of ‘function application’.

(8) **Fregean Interpretation of FOL-NoQ (To Be *Substantially Revised*)**

Let E be a set (of entities). A *Fregean Interpretation of FOL-NoQ based on E* is an interpretation of $\text{FOL-NoQ} \langle B, G_{\gamma}, f \rangle_{\gamma \in \{\text{Concat}, \text{Not}, \text{And}\}}$ such that

(i) $B = \bigcup_{\tau \in T} D_{\tau, E}$

Note: The set B of semantic values is the union of *all* the denotations of type τ based on E .

(ii) The operations $G_{\text{Concat}}, G_{\text{Not}}, G_{\text{And}}$ are as defined in (7).

(9) **An Immediate Problem for the Definition in (8)**

- According to our definition in (2), in order for $\langle B, G_{\gamma}, f \rangle_{\gamma \in \{\text{Concat}, \text{Not}, \text{And}\}}$ to be an ‘interpretation’, $\langle B, G_{\gamma} \rangle_{\gamma \in \{\text{Concat}, \text{Not}, \text{And}\}}$ must be an *algebra*.
- However, if $B = \bigcup_{\tau \in T} D_{\tau, E}$, then it isn’t closed under $G_{\text{Concat}}, G_{\text{Not}}, G_{\text{And}}$
 - $G_{\text{Not}}, G_{\text{And}}$ are only defined for elements of B that are in $D_{t, E}$
 - G_{Concat} is only defined for pairs α, β such that $\alpha \in D_{\langle \sigma, \tau \rangle, E}$ and $\beta \in D_{\sigma, E}$,

(10) **A Related Problem**

- If $\langle B, G_{\gamma}, f \rangle_{\gamma \in \{\text{Concat}, \text{Not}, \text{And}\}}$ is to be an interpretation, then the meaning assignment based on it must be homomorphism from A in (5) to $B = \bigcup_{\tau \in T} D_{\tau, E}$
- **But remember that A contains a whole bunch of ‘syntactic garbage’.**
- Thus, there has to be ‘meanings’ in B that this syntactic garbage gets mapped to
- **For example, since G_{Not} and F_{Not} would ‘correspond’ in the homomorphism, we need to have it that for an individual constant ‘ a ’ where $f(a) = \alpha \in D_{e, E}$**

$$g(\sim a) =$$

$$g(F_{\text{Not}}(a)) =$$

$$G_{\text{Not}}(g(a)) =$$

$$G_{\text{Not}}(f(a)) =$$

$$G_{\text{Not}}(\alpha) = \quad ??? \quad = \quad \text{some kind of ‘semantic garbage’ corresponding to the ‘syntactic garbage’ } \sim a$$

(11) **Incorporating ‘Semantic Garbage’ Into Our System**

What is ‘semantic garbage’, however? And how do we incorporate it into our system?

- Montague is conspicuously silent on the matter in UG and PTQ. To my knowledge, there’s no indication in his extant writings on how he thought the issue in (9)/(10) could/should be concretely handled (*probably it was too trivial to discuss...*)
- Halvorsen & Ladusaw (1979) have some more focused discussion of the matter, but they also leave the nature of the ‘semantic garbage’ entirely open (and cryptic).
- For better or worse, the following is my own proposal: **we are going to slightly redefine D_e so that it contains a special element ‘garbage’**
 - **We are then going to redefine the operations G_γ in light of this...**

(12) **The Set of ‘Possible Denotations’ (To be Revised)**

Let T be the set of types and E be some non-empty set (of entities). The set $D_{\tau, E}$ of denotations of type τ based on E is defined as follows:

- (i) $D_{e,E} = E \cup \{ \mathbf{garbage} \}$
- (ii) $D_{t,E} = \{ 0, 1 \}$
- (iii) If $\sigma, \tau \in T$, then $D_{\langle \sigma, \tau \rangle, E} =$ the set of functions from $D_{\sigma, E}$ to $D_{\tau, E}$
 $= D_{\tau, E}^{D_{\sigma, E}}$

(13) **The Semantic Operations**

a. The Operation ‘ G_{Not} ’

- (i) If $\alpha \notin D_{t,E}$, then $G_{\text{Not}}(\alpha) = \mathbf{garbage}$
- (ii) If $\alpha \in D_{t,E}$, then $G_{\text{Not}}(\alpha) =$ 1, if $\alpha = 0$
0, otherwise

b. The Operation ‘ G_{And} ’

- (i) If $\alpha \notin D_{t,E}$ or $\beta \notin D_{t,E}$, then $G_{\text{And}}(\alpha, \beta) = \mathbf{garbage}$
- (ii) If $\alpha, \beta \in D_{t,E}$, then $G_{\text{And}}(\alpha, \beta) =$ 1, if $\alpha = \beta = 1$
0, otherwise

c. The Operation ‘ G_{Concat} ’

- (i) If $\alpha = \mathbf{garbage}$ or $\beta = \mathbf{garbage}$, then $G_{\text{Concat}}(\alpha, \beta) = \mathbf{garbage}$
- (ii) If not $\alpha \in D_{\langle \sigma, \tau \rangle, E}$ and $\beta \in D_{\sigma, E}$, then $G_{\text{Concat}}(\alpha, \beta) = \mathbf{garbage}$
- (iii) Otherwise, if $\alpha \in D_{\langle \sigma, \tau \rangle, E}$ and $\beta \in D_{\sigma, E}$, then $G_{\text{Concat}}(\alpha, \beta) = \alpha(\beta)$

(14) **Illustration of These Definitions**

Let f be the $\langle \text{et} \rangle$ function $\{ \langle \text{Barack}, 0 \rangle, \langle \text{Michelle}, 1 \rangle \}^3$

- | | | | |
|----|--|----|---|
| a. | $G_{\text{Not}}(1) = 0$ | b. | $G_{\text{Not}}(\text{Barack}) = \mathbf{garbage}$ |
| c. | $G_{\text{And}}(1,1) = 1$ | d. | $G_{\text{And}}(1,f) = \mathbf{garbage}$ |
| e. | $G_{\text{Concat}}(f, \text{Barack}) = f(\text{Barack}) = 0$ | f. | $G_{\text{Concat}}(f, \mathbf{garbage}) = \mathbf{garbage}$ |
| g. | $G_{\text{Concat}}(\text{Michelle}, \text{Barack}) = \mathbf{garbage}$ | h. | $G_{\text{Concat}}(f, f) = \mathbf{garbage}$ |

With the formalization of 'semantic garbage' in (12) and (13) at hand, we can now properly define a 'fregean interpretation' for FOL-NoQ...

(15) **Fregean Interpretation of FOL-NoQ (To Be *Substantially* Revised)**

Let E be a non-empty set (of entities). A *Fregean Interpretation of FOL-NoQ based on E* is an interpretation of FOL-NoQ $\langle B, G_{\gamma}, f \rangle_{\gamma \in \{\text{Concat}, \text{Not}, \text{And}\}}$ such that

- a. $B = \bigcup_{\tau \in T} D_{\tau, E}$

Note: Again, the set B of semantic values is the union of *all* the denotations of type τ based on E .

Note: This definition now makes use of the revised definition in (12), which adds the special element **garbage** to $D_{e, E}$

- b. The operations $G_{\text{Concat}}, G_{\text{Not}}, G_{\text{And}}$ are as defined in (13).

Note: Under the new definitions in (13), B is closed under $G_{\text{Concat}}, G_{\text{Not}}, G_{\text{And}}$

- c. The function f is such that
- (i) For all $\tau \in T$, if $\alpha \in X_{\tau}$ then $f(\alpha) \in D_{\tau, E}$
 - (ii) For all $\alpha \in X_e, f(\alpha) \neq \mathbf{garbage}$

Note: The conditions in (15c) entail that:

- Every individual constant will be mapped to some entity in D_e
- No individual constant will be mapped to (interpreted as) **garbage** (in D_e)
- A n -ary predicate letter of type $\langle e, \dots, t \rangle$ will be mapped to a function in $D_{\langle e, \dots, t \rangle, E}$
 - Thus, an n -ary predicate letter is mapped to the curried characteristic function of an n -ary relation! (Just as it should be).

³ Technically speaking f must also now be defined for **garbage**. We can simply assume that $f(\mathbf{garbage}) = 0$ for all $\langle \text{et} \rangle$ functions f . Similar such assumptions can be made for functions of the other types. From this point on, we'll simply ignore **garbage** in our specifications of the denotations of the basic lexical items.

(16) **A Concrete Instance of a Fregean Interpretation of FOL-NoQ**

Let the set $S = \{ \text{Michelle, Barack, Mitt} \}$. Let $\mathbf{B} = \langle B, G_\gamma, f \rangle_{\gamma \in \{\text{Concat, Not, And}\}}$ be the Fregean interpretation based on S , such that f contains the following mappings:

- a. $f(a) = \text{Michelle}$
- b. $f(b) = \text{Barack}$
- c. $f(c) = \text{Mitt}$
- d. $f(P) = h = \{ \langle \text{Michelle}, 0 \rangle, \langle \text{Barack}, 1 \rangle, \langle \text{Mitt}, 0 \rangle \}$
- e. $f(Q) = j = \left(\begin{array}{l} \text{Michelle} \rightarrow \left\{ \begin{array}{l} \text{Michelle} \rightarrow 1 \\ \text{Barack} \rightarrow 1 \\ \text{Mitt} \rightarrow 0 \end{array} \right\} \\ \\ \text{Barack} \rightarrow \left\{ \begin{array}{l} \text{Michelle} \rightarrow 1 \\ \text{Barack} \rightarrow 1 \\ \text{Mitt} \rightarrow 0 \end{array} \right\} \\ \\ \text{Mitt} \rightarrow \left\{ \begin{array}{l} \text{Michelle} \rightarrow 0 \\ \text{Barack} \rightarrow 0 \\ \text{Mitt} \rightarrow 1 \end{array} \right\} \end{array} \right)$

Note: We've basically interpreted 'P' as the property 'is president', and we've interpreted 'Q' as the relation 'x is loved by y'.

We can now use the 'meaning assignment g determined by B' to interpret formulae in our FOL-NoQ language!...

(17) **Using the Fregean Interpretation in (16) to Interpret Sentences of FOL-NoQ**

Let g be the meaning assignment for FOL-NoQ that is determined by \mathbf{B} in (16).

- (i) $g(\sim(Pa)) =$ (by definition of FOL-NoQ)
- (ii) $g(F_{\text{Not}}(F_{\text{Concat}}(P, a))) =$ (by homomorphism property of g)
- (iii) $G_{\text{Not}}(g(F_{\text{Concat}}(P, a))) =$ (by homomorphism property of g)
- (iv) $G_{\text{Not}}(G_{\text{Concat}}(g(P), g(a))) =$ (by definition of g)⁴
- (v) $G_{\text{Not}}(G_{\text{Concat}}(f(P), f(a))) =$ (by definition of f in (16))
- (vi) $G_{\text{Not}}(G_{\text{Concat}}(h, \text{Michelle})) =$ (by definition of G_{Concat})
- (vii) $G_{\text{Not}}(h(\text{Michelle})) =$ (by definition of h in (16d))
- (viii) $G_{\text{Not}}(0) =$ (by definition of G_{Not})
- (ix) 1

⁴ Note that by the definition in (3), $f \subseteq g$, and so if $\alpha \in X$, then $g(\alpha) = f(\alpha)$.

(18) **Using the Fregean Interpretation in (16) to Interpret Sentences of FOL-NoQ**

Let g be the meaning assignment for FOL-NoQ that is determined by \mathbf{B} in (16).

- (i) $g(((Qa)b) \& (Pb)) =$ (by definition of FOL-NoQ)
- (ii) $g(F_{\text{And}}(F_{\text{Concat}}(F_{\text{Concat}}(Q,a),b), F_{\text{Concat}}(P,b))) =$ (by homomorphism property of g)
- (iii) $G_{\text{And}}(g(F_{\text{Concat}}(F_{\text{Concat}}(Q,a),b)), g(F_{\text{Concat}}(P,b))) =$ (by homomorphism property of g)
- (iv) $G_{\text{And}}(G_{\text{Concat}}(g(F_{\text{Concat}}(Q,a), g(b))), G_{\text{Concat}}(g(P),g(b))) =$
(by homomorphism property of g)
- (v) $G_{\text{And}}(G_{\text{Concat}}(G_{\text{Concat}}(g(Q), g(a)), g(b)), G_{\text{Concat}}(g(P),g(b))) =$ (by definition of g)
- (vi) $G_{\text{And}}(G_{\text{Concat}}(G_{\text{Concat}}(f(Q), f(a)), f(b)), G_{\text{Concat}}(f(P), f(b))) =$ (by definition of f)
- (vii) $G_{\text{And}}(G_{\text{Concat}}(G_{\text{Concat}}(j, Michelle), Barack), G_{\text{Concat}}(h, Barack)) =$
(by definition of G_{Concat})
- (viii) $G_{\text{And}}(G_{\text{Concat}}(G_{\text{Concat}}(j, Michelle), Barack), h(Barack)) =$ (by definition of h)
- (ix) $G_{\text{And}}(G_{\text{Concat}}(G_{\text{Concat}}(j, Michelle), Barack), 1) =$ (by definition of G_{Concat})
- (x) $G_{\text{And}}(G_{\text{Concat}}(j(Michelle), Barack), 1) =$ (by definition of j)
- (xi) $G_{\text{And}}(G_{\text{Concat}}(\{ \langle Michelle, 1 \rangle, \langle Barack, 1 \rangle, \langle Mitt, 0 \rangle \}, Barack), 1) =$
(by definition of G_{Concat})
- (xii) $G_{\text{And}}(1, 1) = 1$ (by definition of G_{And})

(19) **Using the Fregean Interpretation in (16) Interpret Syntactic Garbage**

- (i) $g((P \sim a)) =$ (by definition of FOL-NoQ)
- (ii) $g(F_{\text{Concat}}(P, F_{\text{Not}}(a))) =$ (by homomorphism property of g)
- (iii) $G_{\text{Concat}}(g(P), G_{\text{Not}}(g(a))) =$ (by definition of g)
- (iv) $G_{\text{Concat}}(f(P), G_{\text{Not}}(f(a))) =$ (by definition of f)
- (v) $G_{\text{Concat}}(h, G_{\text{Not}}(Michelle)) =$ (by definition of G_{Not} in (13a))
- (vi) $G_{\text{Concat}}(h, \mathbf{garbage}) =$ (by definition of G_{Concat} in (13c))
- (vii) **garbage**

3. Fregean Interpretations and Models of FOL-NoQ

(20) Taking Stock of What We've Done So Far

- In the last set of notes, we developed the general definition of 'language' in (1), according to which any language is built upon a 'syntactic algebra' $\langle A, F_\gamma \rangle_{\gamma \in \Gamma}$
- We developed a notion of 'meaning', according to which a semantics for a given language is a homomorphism from the syntactic algebra $\langle A, F_\gamma \rangle_{\gamma \in \Gamma}$ to a semantic algebra $\langle B, G_\gamma \rangle_{\gamma \in \Gamma}$
- We developed a notion of 'Fregean interpretation', a special *kind* of interpretation for our FOL-NoQ language.
- We saw how one specific such 'Fregean interpretation' can be used to map sentences of FOL-NoQ to truth-values (just like a classical model!)

There's an important relation between our concept of 'Fregean Interpretation' in (15) and our concept of a 'model' for FOL, repeated below:

(21) Definition of a 'Model' for First Order Logic

A model \mathcal{M} is a pair $\langle D, I \rangle$ consisting of:

- a. A *non-empty* set D , called the 'domain of \mathcal{M} '
- b. A function I , whose domain is the individual constants and predicate letters, and whose range satisfies the following conditions:
 - (i) If α is an individual constant, then $I(\alpha) \in D$
 - (ii) If Φ is an n -ary predicate letter, then $I(\Phi)$ is **the curried characteristic function of an n -ary relation $R \subseteq D^n$**

(22) The Correspondence Between (Fregean) Interpretations and Models, Part 1

Let $\mathbf{B} = \langle B, G_\gamma, f \rangle_{\gamma \in \{\text{Concat, Not, And}\}}$ be a Fregean interpretation (for FOL-NoQ); let g be meaning assignment determined by \mathbf{B} . Let $\mathcal{M} = \langle D, I \rangle$ be such that $D = D_{e,E}$ and $I = f$.

- a. Claim: $\langle D, I \rangle$ is a model (for FOL-NoQ)
 - $D = D_{e,E}$ is by definition a non-empty set.
 - By definition in (15c), f is a function satisfying the condition in (21b).

- b. Claim: For any sentence φ (of FOL-NoQ), $g(\varphi) = [[\varphi]]^{\mathcal{M}}$

(proof left as exercise for the student)

(23) **The Correspondence Between (Fregean) Interpretations and Models, Part 2**

Let $\mathcal{M} = \langle D, I \rangle$ be a model (for FOL-NoQ). Let $\mathbf{B} = \langle B, G_\gamma, f \rangle_{\gamma \in \{\text{Concat, Not, And}\}}$ be such that $B = \bigcup_{\tau \in T} D_{\tau, D}$ and $f = I$.

a. Claim: $\langle B, G_\gamma, f \rangle_{\gamma \in \{\text{Concat, Not, And}\}}$ is a Fregean interpretation (for FOL-NoQ)

- By definition, $B = \bigcup_{\tau \in T} D_{\tau, E}$, for a (non-empty) set $E = D$
- By definition, $G_{\text{Concat}}, G_{\text{Not}}, G_{\text{And}}$ are as defined in (13).
- By definition (21b), function I satisfies the conditions in (15c); it is such that
 - (i) For all $\tau \in T$, if $\alpha \in X_\tau$ then $f(\alpha) \in D_{\tau, E}$
 - (ii) For all $\alpha \in X_e$, $f(\alpha) \neq \text{garbage}$

b. Claim: Let g be the meaning assignment determined by \mathbf{B} . For any sentence φ (of FOL-NoQ), $g(\varphi) = [[\varphi]]^M$

(proof left as exercise for the student)

(24) **Some Remarks**

- a. Thus, for every Fregean interpretation \mathbf{B} based on E , there is a corresponding model \mathcal{M} whose domain is E , and which makes exactly the same sentences true.
- b. Similarly, for every model $\mathcal{M} = \langle D, I \rangle$, there is a Fregean interpretation \mathbf{B} based on D , and which makes exactly the same sentences true.
- c. Given this relationship between models and Fregean interpretations, we can freely shift between the two (*cf.* sets and their characteristic functions).
- d. Similarly, we can view Fregean interpretations and models as being *in essence* ‘the same thing’ (even though they are different set-theoretic objects) (*cf.* sets and their characteristic functions)
- e. **Thus, if we can provide a Fregean interpretation for a given language, we’ve also thereby provided that language with a model-theoretic semantics.**
- f. Finally, given this correspondence between models and *Fregean* interpretations, we can see that our general notion of ‘interpretation’ in (2) is truly a more general concept of ‘interpretation’ than is found in our model-theoretic semantics of FOL.

4. Extending The Framework to Natural Languages

(25) **Burning Question:**

Can we provide a (Fregean) interpretation to (a fragment of) a natural language, like English?

A second fundamental contribution of Montague's was the discovery that there are, in principle, two different ways of providing an interpretation for a language.

(26) **Method 1: Direct Interpretation of a (Natural) Language L**

a. Step 1:

Provide an analysis of language L as a structure $\langle A, F_\gamma, X_\delta, S, \delta_0 \rangle_{\gamma \in \Gamma, \delta \in \Delta}$ satisfying the core definition in (1).

b. Step 2: Specify a structure $\langle B, G_\gamma, f \rangle_{\gamma \in \Gamma}$ satisfying the core definition in (2) of an 'interpretation' for $\langle A, F_\gamma, X_\delta, S, \delta_0 \rangle_{\gamma \in \Gamma, \delta \in \Delta}$

- This is the method pursued by Montague in his "English as a Formal Language"

(27) **Method 2: Indirect Interpretation (To be Revised)**

a. Step 1:

Provide an analysis of language L as a structure $\langle A, F_\gamma, X_\delta, S, \delta_0 \rangle_{\gamma \in \Gamma, \delta \in \Delta}$ satisfying the core definition in (1).

b. Step 2:

Take an (artificial) language $L' = \langle A', F'_{\gamma'}, X'_{\delta'}, S', \delta'_0 \rangle_{\gamma' \in \Gamma', \delta' \in \Delta'}$ such that *there is already a known interpretation $\mathbf{B} = \langle B, G_{\gamma'}, f \rangle_{\gamma' \in \Gamma'}$ for L' .*

c. Step 3: (To be Revised)

Give a homomorphism h from A (the structures of L) to A' (the structures of L')
○ *This homomorphism h is effectively a 'translation' from L into L' .*

- This is the method pursued by Montague in his most seminal papers, UG and PTQ

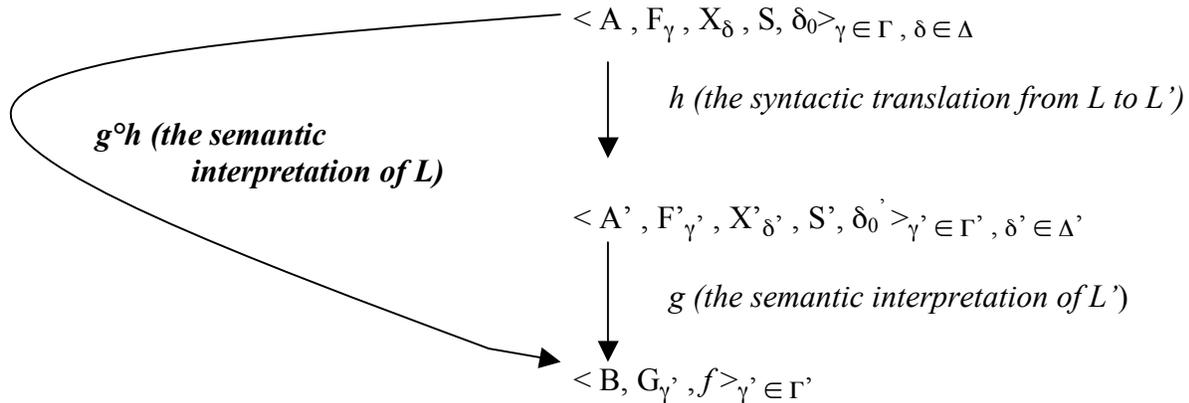
(28) **Key Skeptical Question**

With this 'indirect interpretation', all we really do is supply a 'translation' h from one language L into another L' . *Have we really provided L with a semantics when we do this?*

(29) **Answer to the Skeptical Question**

- Recall that the composition of two homomorphisms is itself a homomorphism (Assignment 4)
- Therefore, if \mathbf{B} is the interpretation of L' , and g is the (homomorphic) meaning assignment determined by \mathbf{B} , and h is the (homomorphic) translation from L to L' , **then $g \circ h$ is a homomorphism from A (the expressions of L) to B (the meanings assigned to L)**.
- Therefore, by providing an indirect interpretation for a language L , **we've also provided a (model-theoretic) semantics for L**

Indirect Interpretation in a Picture (Oversimplified):



In other words, with ‘indirect interpretation’, we get our semantics for L ‘indirectly’, through the translation language L' . **But we still do get a semantics for L .**

This notion of ‘indirect interpretation’ will become clearer once we work through a concrete example...

There are a few important properties of ‘indirect interpretation’ that we can go ahead and observe, however....

(30) **Indirect Interpretation is ‘Eliminable’**

- Note that if we ‘indirectly interpreted’ language L via language L' , then we have also thereby ‘directly interpreted’ L , via the composition of the two homomorphisms $g \circ h$
- Thus, indirect interpretation is never a necessary part of giving a semantics for a language L ; it’s ‘eliminable’:
 - Once we’ve done ‘indirect interpretation’ we could recast everything we’ve done as ‘direct interpretation’ via the composition $g \circ h$

(31) **Some Choice Quotes**

- From Halvorsen & Ladusaw (1979), p. 210:
“An understanding of [the eliminability of indirect interpretation] is necessary to understand the use of...logics of PTQ and most other analyses within Montague grammar. As has been stated elsewhere, the [logical language] is an *expository device* and is in no way a necessary part of an analysis of any language offered within this theory. By using the easily interpreted [logical language] as a mediator, natural languages can be analyzed syntactically and then provided with a translation...from them to [the logical language] to induce their interpretation...*This method of analysis amounts to direct interpretation of natural language.*” (emphasis mine)
- From Dowty *et al.* (1981), p. 263:
“Translating English into [a logical language] was therefore not essential to interpreting the English phrases we generated; *it was simply a convenient intermediate step in assigning them meanings. This step could have been eliminated had we chosen to describe the interpretation of English directly...* This point is important, because anyone who does not appreciate it may misunderstand the role of [logical languages] in applications of Montague’s descriptive framework to natural languages.” (emphasis mine)

The general point here is that, to give a semantics for (e.g.) English in Montague Grammar, you don’t *have* to translate English into any logical language...

- It’s just that doing so can be a very handy, elegant means of specifying an interpretation for the language.

Again, this will all become clearer when we’ve seen some concrete instances of it...

(32) **Why Do Indirect Interpretation?**

If the logical language is well-designed and familiar to readers, then it can provide a more ‘perspicuous’ representation (statement / name) of the meanings that we wish to be assigned to the (natural) language expressions.

Illustration (From 610):

‘ $\lambda x_e : x \text{ smokes}$ ’ vs. ‘The function f from D_e to D_t such that for all $x \in D_e$, $f(x) = 1$ iff x smokes.’

- From Dowty *et al.* (1981), p. 264:
“[The purpose of indirect interpretation] was to have a convenient, compact notation for giving a briefer statement of semantic rules than we were able to give in earlier chapters of this book, where semantic rules were formulated rather long-windedly in English... [The logical language] could provide us with names for meanings...”

(33) **Partisan Comment: Indirect Interpretation and Semantics in the 21st Century**

- In a certain sense, much of the semantic literature nowadays is rife with ‘indirect interpretation’, *but without an explicit, model-theoretic semantics for the translation language.*
- When in semantics paper we write things like $[[\text{-er}]] = [\lambda P_{\langle dt \rangle}: \lambda Q_{\langle dt \rangle}: \dots]$, we assume that whatever appears to the right of ‘=’ is understood to our readers (and ourselves).
 - *This isn’t always the case though... sometimes formulas and notations creep in that seem to make sense, but turn out not to upon deeper probing...*
 - In the original Montague Grammar framework, there is much lower risk of this happening, *since one must also provide an explicit model-theoretic semantics for their interpretation language.*

So, this is the general gist of how ‘indirect interpretation’ in MG works...

Note, however, that the characterization in (27) has some gross oversimplifications...

In the next set of notes, we’ll set the record straight by delving into Montague’s theory of translation...