Problem Set on Translation and Indirect Interpretation

(1) **Polynomial Operations and Algebras**

Let **A** be an algebra $\langle A, F_\gamma \rangle_{\gamma \in \Gamma}$, and let H be a member of the polynomial operations K over **A**. Show that A is closed under H.

There are four steps to showing that A is closed under H, corresponding to the four 'ways' by which H could be a member of K.

a. Step One: Show that A is closed under F_γ for all $\gamma \in \Gamma$

b. Step Two:
Let $Id_{n,m}$ be any identity function (projection function). Show that A is closed under $Id_{n,m}$.

c. Step Three: Let $a \in A$. Show that A is closed under $C_{a,m}$.

d. Step Four:
Let G be an n-ary function that A is closed under. Let $F_1, \ldots F_n$ be n m-ary functions that A is closed under. Show that A is closed under $G\langle F_1, \ldots F_n \rangle$.

(2) **Derived Syntactic Rules and Meaningful Expressions**

Let **L** be a language $\langle\langle A , F_\gamma , X_\delta , S, \delta_0 \rangle_{\gamma \in \Gamma , \delta \in \Delta}, R\rangle$, and let $\langle H, \langle \delta_1 , \ldots \delta_n \rangle, \delta \rangle$ be a derived syntactic rule of **L**. Show that if $\varphi_1, \ldots \varphi_n$ are such that each $\varphi_i \in C_{\delta i}$, then $H(\varphi_1, \ldots \varphi_n) \in C_\delta$.

There are four steps to showing that $H(\varphi_1, \ldots \varphi_n) \in C_\delta$, *corresponding to the four 'ways' by which* $\langle H, \langle \delta_1 , \ldots \delta_n \rangle, \delta \rangle$ *could be a derived syntactic rule of* ***L****.*

a. Step One:
Let $\langle H, \langle \delta_1 , \ldots \delta_n \rangle, \delta \rangle \in S$. Show that if $\varphi_1, \ldots \varphi_n$ are such that each $\varphi_i \in C_{\delta i}$, then $H(\varphi_1, \ldots \varphi_n) \in C_\delta$.

b. Step Two:
Let $\langle H, \langle \delta_1 , \ldots \delta_n \rangle, \delta \rangle$ be a rule of the form $\langle Id_{n,m} , \langle \delta_1, \ldots , \delta_n, \ldots , \delta_m \rangle , \delta_n \rangle$. Show that if $\varphi_1, \ldots \varphi_m$ are such that each $\varphi_i \in C_{\delta i}$, then $Id_{n,m}(\varphi_1, \ldots \varphi_m) \in C_{\delta n}$.

c. Step Three:
Let $\langle H, \langle \delta_1 , \ldots \delta_n \rangle, \delta \rangle$ be of the form $\langle C_{a,n}, \langle \delta_1, \ldots, \delta_n \rangle, \delta \rangle$, where $a \in C_\delta$. Show that if $\varphi_1, \ldots \varphi_n$ are such that each $\varphi_i \in C_{\delta i}$, then $C_{a,n}(\varphi_1, \ldots \varphi_n) \in C_\delta$.

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d. Step Four:
Let the rule $<F , <\delta_1 , \ldots \delta_n>, \delta>$ have the property that if $\varphi_1, \ldots \varphi_n$ are such that each $\varphi_i \in C_{\delta i}$, then $F(\varphi_1, \ldots \varphi_n) \in C_\delta$. In addition, for each $G_1, \ldots G_n$, let the rule $<G_j , <\delta'_1 , \ldots \delta'_m>, \delta_j >$ have the property that if $\varphi_1, \ldots \varphi_m$ are such that each $\varphi_i \in C_{\delta' i}$, then $G_j(\varphi_1, \ldots \varphi_m) \in C_{\delta j}$.

Show that the rule $<F<G_1, \ldots G_n>, <\delta'_1 , \ldots \delta'_m>, \delta>$ has the property that if $\varphi_1, \ldots, \varphi_m$ are such that each $\varphi_i \in C_{\delta' i}$, then $F<G_1, \ldots G_n>(\varphi_1, \ldots \varphi_m) \in C_\delta$.

(3) **An Exercise in Indirect and Direct Interpretation of a Fragment of English**

a. Minimally alter our language Mini-English so that its expressions now include strings like *Neither Mitt smokes nor Barack smokes*.

Note:
Don't worry if your system also produces such marginal strings as *Neither it is not the case that Mitt smokes nor Barack loves Michelle.*

b. Take our translation base in (46)-(50) on the handout "The Notion of a Translation Base", and minimally alter it so that strings like *Neither Mitt smokes nor Barack smokes* receive appropriate translations in Politics-NoQ.

Note:
Be sure to show that any *new* polynomial operations in your translation base have the property in (40c) on handout "The Notion of a Translation Base."

c. Please show how your new translation base, along with our interpretation for Politics-NoQ, assigns a truth-value to the analysis tree for *Neither Mitt smokes nor Barack smokes.*

d. Given your proposed translation base, construct a direct interpretation of Disambiguated Mini-English, and show how it interprets *Neither Mitt smokes nor Michelle smokes.*