Problem Set on Translation and Indirect Interpretation:
Answers and Notes

1. Notes on the Answers

In Section 2, I have copied some illustrative answers from the problem sets submitted to me. In this section, I provide some notes on the answers below as well as on the problems themselves.

(1) Polynomial Operations and Algebras

- I have no comments regarding the answers to (1a)-(1c). Please see section 2 for illustrative answers to each of them.

- Regarding (1d), several folks supplied answers that simply stated something akin to:

  "Because A is closed under G and $F_1, \ldots F_n$, A will also be closed under their composition $G{<}F_1, \ldots F_n{>}$."

  Although this statement is true, and its truth may be patently obvious, the exercise requires you to spell out your reasoning here a bit more. Something along the following lines would be an ideal answer:

  "Let $a_1, \ldots, a_m \in A$. By assumption, A is closed under $F_1, \ldots F_n$. Consequently, $F_1(a_1,\ldots, a_m) \in A, \ldots, F_n(a_1,\ldots, a_m) \in A$. By assumption, A is also closed under G. Therefore, $G(F_1(a_1,\ldots, a_m), \ldots, F_n(a_1,\ldots, a_m)) \in A$. Thus, by definition of function composition, $G{<}F_1, \ldots F_n{>}(a_1, \ldots, a_m) \in A$. Since $a_1, \ldots, a_m$ were arbitrary, A is closed under $G{<}F_1, \ldots F_n{>}$."

(2) Derived Syntactic Rules and Meaningful Expressions

- I have no comments regarding the answers to (2a)-(2c). Please see section 2 for illustrative answers to each of them.

- Regarding (2d), an ideal answer would again be something along the following lines:

  "Let $\varphi_1, \ldots, \varphi_m$ be such that each $\varphi_i \in C_{\delta' i}$. By assumption, $G_1(\varphi_1, \ldots, \varphi_m) \in C_{\delta 1}, \ldots, G_n(\varphi_1, \ldots, \varphi_m) \in C_{\delta n}$. Therefore, by assumption, $F(G_1(\varphi_1, \ldots, \varphi_m), \ldots, G_n(\varphi_1, \ldots, \varphi_m)) \in C_\delta$. Finally, it therefore follows that $F{<}G_1, \ldots G_n{>}(\varphi_1, \ldots \varphi_m) \in C_\delta$."

(3) **An Exercise in Indirect and Direct Interpretation of a Fragment of English**

- For (3a), in order to define the new fragment of English, each of the following three changes must be made to our earlier fragment:

  (i) A new syntactic operation K_{Neither} must be added to the set of syntactic operations.

  (ii) **The set E in the syntactic algebra must be redefined so as to be closed under the new operation K_{Neither}**

  (iii) A new syntactic rule referencing K_{Neither} must be added to the set of syntactic rules.

  Several folks neglected to explicitly do step (ii); however, strictly speaking it is necessary for the redefinition; otherwise E won't contain any expressions formed by K_{Neither}.

- For (3b), everyone hit upon the correct polynomial operation to correspond in the translation base to K_{Neither} : $F_{\text{And}}<F_{\text{Not}}<\text{Id}_{1,2}>, F_{\text{Not}}<\text{Id}_{2,2}>>$. **However, no one actually showed that the following is a derived rule of Politics-NoQ:**

  $< F_{\text{And}}<F_{\text{Not}}<\text{Id}_{1,2}>, F_{\text{Not}}<\text{Id}_{2,2}>>, <t, t>, t>$

  Rather, everyone simply asserted that this is a derived rule. What I wanted you to do, though, was provide a derivation/proof along the following lines:

  - By (33a) on "The Notion of a Translation Base", $<F_{\text{Not}}, <t>, t> \in K$, the derived rules of Politics-NoQ.
  - By (33b), $<\text{Id}_{1,2}, <t,t>, t>, <\text{Id}_{2,2}, <t,t>, t> \in K$
  - Therefore, by (33d), $< F_{\text{Not}}<\text{Id}_{1,2}>, <t,t>, t>, < F_{\text{Not}}<\text{Id}_{2,2}>, <t,t>, t> \in K$
  - By (33a), $<F_{\text{And}}, <t, t>, t> \in K$
  - Therefore, by (33d), $< F_{\text{And}}<F_{\text{Not}}<\text{Id}_{1,2}>, F_{\text{Not}}<\text{Id}_{2,2}>>, <t, t>, t> \in K$

- For (3c,d), I was ideally wanting to see computations where the behavior of the polynomial operations ($F_{\text{And}}<F_{\text{Not}}<\text{Id}_{1,2}>, F_{\text{Not}}<\text{Id}_{2,2}>>$ and $G_{\text{And}}<G_{\text{Not}}<\text{Id}_{1,2}>, G_{\text{Not}}<\text{Id}_{2,2}>>$) are broken down step-by-step. Please see section 2 for an illustration.

2. Illustrative Answers from Submitted Problem Sets

(1) **Polynomial Operations and Algebras**

Let **A** be an algebra $\langle A, F_\gamma \rangle_{\gamma \in \Gamma}$, and let H be a member of the polynomial operations K over **A**. Show that A is closed under H.

a. Step One: Show that A is closed under F_γ for all $\gamma \in \Gamma$

Since A is an algebra, by definition, it follows that A is closed under F_γ for all $\gamma \in \Gamma$.

b. Step Two:
Let $Id_{n,m}$ be any identity function (projection function). Show that A is closed under $Id_{n,m}$.

By the definition of $Id_{n,m}$, $\forall n, m \in \mathbb{N}$, where $n \leq m$ and $\alpha = \langle \alpha_1, \ldots, \alpha_m \rangle \in A^m$, $Id_{n,m}(\alpha) = \alpha_n \in A$. Therefore, A is closed under H.

c. Step Three: Let $a \in A$. Show that A is closed under $C_{a,m}$.

- Let $a \in A$. The constant function $C_{a,m}$ takes as argument any m-tuple β and returns a.
- For any m-tuple member β of A $C_{a,m}$ takes, $C_{a,m}$ will return a.
- Since $a \in A$, we can conclude that A is closed under $C_{a,m}$.

d. Step Four:
Let G be an n-ary function that A is closed under. Let $F_1, \ldots F_n$ be n m-ary functions that A is closed under. Show that A is closed under $G\langle F_1, \ldots F_n \rangle$.

Let $a_1, \ldots, a_m \in A$. By definition of function composition,

$$G\langle F_1, \ldots, F_n \rangle (a_1, \ldots a_m) = G(F_1(a_1, \ldots, a_m), \ldots, F_n(a_1, \ldots, a_m)).$$

Because A is closed under F_1 through F_n by assumption, the outputs of $F_1(a_1, \ldots, a_m)$ through $F_n(a_1, \ldots, a_m)$ will be in A. Call these elements $\alpha_1, \ldots, \alpha_n$, respectively. The above is hence equivalent to $G(\alpha_1, \ldots, \alpha_n)$ with $\alpha_1, \ldots, \alpha_m \in A$. Because A is closed under G by assumption, the output will be in A.

(2) **Derived Syntactic Rules and Meaningful Expressions**

Let **L** be a language $<<A, F_\gamma, X_\delta, S, \delta_0>_{\gamma \in \Gamma, \delta \in \Delta}, R>$, and let $<H, <\delta_1, \ldots \delta_n>, \delta>$ be a derived syntactic rule of **L**. Show that if $\varphi_1, \ldots \varphi_n$ are such that each $\varphi_i \in C_{\delta i}$, then $H(\varphi_1, \ldots \varphi_n) \in C_\delta$.

a. <u>Step One:</u>
Let $<H, <\delta_1, \ldots \delta_n>, \delta> \in S$. Show that if $\varphi_1, \ldots \varphi_n$ are such that each $\varphi_i \in C_{\delta i}$, then $H(\varphi_1, \ldots \varphi_n) \in C_\delta$.

By the definition of syntactic category, since $\mathscr{H}$ is a syntactic rule of $\mathscr{L}$, $H(\varphi_1, \ldots, \varphi_m) \in C_\delta$.

b. <u>Step Two:</u>
Let $<H, <\delta_1, \ldots \delta_n>, \delta>$ be a rule of the form $< Id_{n,m}, <\delta_1, \ldots, \delta_n, \ldots, \delta_m>, \delta_n>$.
Show that if $\varphi_1, \ldots \varphi_m$ are such that each $\varphi_i \in C_{\delta i}$, then $Id_{n,m}(\varphi_1, \ldots \varphi_m) \in C_{\delta n}$.

- Let $\langle H, \langle \delta_1, \ldots, \delta_n \rangle, \delta \rangle$ be a rule of the form $\langle Id_{n,m}, \langle \delta_1, \ldots, \delta_n, \ldots, \delta_m \rangle, \delta_n \rangle$. If $\varphi_1, \ldots, \varphi_m$ are such that each $\varphi_i \in C_{\delta i}$, then according to the definition of the identity function, $Id_{n,m}(\varphi_1, \ldots, \varphi_m)$ will return the n^{th} member φ_n of $\langle \varphi_1, \ldots, \varphi_m \rangle$, which is of category $C_{\delta n}$.
- Therefore, $Id_{n,m}(\varphi_1, \ldots, \varphi_m) \in C_{\delta n}$.

c. <u>Step Three:</u>
Let $<H, <\delta_1, \ldots \delta_n>, \delta>$ be of the form $< C_{a,n}, <\delta_1, \ldots, \delta_n>, \delta >$, where $a \in C_\delta$.
Show that if $\varphi_1, \ldots \varphi_n$ are such that each $\varphi_i \in C_{\delta i}$, then $C_{a,n}(\varphi_1, \ldots \varphi_n) \in C_\delta$.

c. Take $\langle H, \langle \delta_1, \ldots, \delta_n \rangle, \delta \rangle$ to be a rule of the form $\langle C_{a,n}, \langle \delta_1, \ldots, \delta_n \rangle, \delta \rangle$, where $a \in C_\delta$.
By the definition of a constant function, $C_{a,n}(\phi_1, \ldots, \phi_n) = a$,
so $C_{a,n}(\phi_1, \ldots, \phi_n) \in C_\delta$.

d. <u>Step Four:</u>
Let the rule $<F, <\delta_1, \ldots \delta_n>, \delta>$ have the property that if $\varphi_1, \ldots \varphi_n$ are such that each $\varphi_i \in C_{\delta i}$, then $F(\varphi_1, \ldots \varphi_n) \in C_\delta$. In addition, for each $G_1, \ldots G_n$, let the rule $<G_j, <\delta'_1, \ldots \delta'_m>, \delta_j>$ have the property that if $\varphi_1, \ldots \varphi_m$ are such that each $\varphi_i \in C_{\delta' i}$, then $G_j(\varphi_1, \ldots \varphi_m) \in C_{\delta j}$.

Show that the rule $<F<G_1, \ldots G_n>, <\delta'_1, \ldots \delta'_m>, \delta>$ has the property that if $\varphi_1, \ldots, \varphi_m$ are such that each $\varphi_i \in C_{\delta' i}$, then $F<G_1, \ldots G_n>(\varphi_1, \ldots \varphi_m) \in C_\delta$.

d. *Step 4:*
$F(G_1, \ldots, G_n)(\phi_1, \ldots, \phi_m)$ is equivalent to $F(G_1(\phi_1, \ldots, \phi_m), \ldots, G_n(\phi_1, \ldots, \phi_m))$ by the definition of function composition. By the syntactic rules for $G_1, \ldots, G_n$ and the category labels of $\phi_1, \ldots, \phi_m$, $G_j(\phi_1, \ldots, \phi_m) \in C_{\delta_j}$. By the syntactic rule for F, the result of applying F to these outputs will be of category C_δ.

> <u>Note:</u> *To maintain consistency across the posted answers to Problem (3), I will excerpt from only one submitted assignment. It should be noted, however, that several other students also submitted comparable answers.*

(3) **An Exercise in Indirect and Direct Interpretation of a Fragment of English**

a. Minimally alter our language Mini-English so that its expressions now include strings like *Neither Mitt smokes nor Barack smokes*.

a. Let Mini-English be the structure ⟨L, R⟩, where

I. L is Disambiguated Mini-English (DME) defined as follows:
$\langle E, K_\gamma, X_\delta, S_E, S\rangle_{\gamma \in \{\text{Merge-S, Merge-IV, Not, And, If, Neither}\},\ \delta \in \Delta\ (\Delta = \{\text{NP, IV, TV, S}\})}$

- **The syntactic operations:**
  In the definitions below, α and β are trees whose root nodes are ordered pairs. In addition α' and β' are the first members of the root nodes of α and β (respectively).

(i) $K_{\text{Merge-S}}(\alpha, \beta)$ = ⟨α'β', Merge-S⟩ with daughters α β

(ii) $K_{\text{Merge-IV}}(\alpha, \beta)$ = ⟨α'β', Merge-IV⟩ with daughters α β

(iii) $K_{\text{Not}}(\alpha)$ = ⟨It is not the case that α', Not⟩ with daughter α

(iv) $K_{\text{And}}(\alpha, \beta)$ = ⟨α' and β', And⟩ with daughters α β

(v) $K_{\text{If}}(\alpha, \beta)$ = ⟨If α' then β', If⟩ with daughters α β

(vi) $K_{\text{Neither}}(\alpha, \beta)$ = ⟨Neither α' nor β', Neither⟩ with daughters α β

- E is the smallest set that:
  (i) For all $\delta \in \Delta$, $X_\delta \subseteq E$
  (ii) E is closed under the operations $K_{\text{Merge-S}}$, $K_{\text{Merge-IV}}$, K_{Not}, K_{And}, K_{If}, and K_{Neither}
- The set of syntactic rules S_E is defined as follows:
  (i) $\langle K_{\text{Merge-IV}}, \langle \text{TV, NP}\rangle, \text{IV}\rangle$
  (ii) $\langle K_{\text{Merge-S}}, \langle \text{NP, IV}\rangle, \text{S}\rangle$
  (iii) $\langle K_{\text{And}}, \langle \text{S, S}\rangle, \text{S}\rangle$
  (iv) $\langle K_{\text{If}}, \langle \text{S, S}\rangle, \text{S}\rangle$

  (v) $\langle K_{\text{Neither}}, \langle \text{S, S}\rangle, \text{S}\rangle$
  (vi) $\langle K_{\text{Not}}, \langle \text{S}\rangle, \text{S}\rangle$

II. R is a function which takes as input a tree T in E, and returns as output the first member of the root node of T.

b. Take our translation base in (46)-(50) on the handout "The Notion of a Translation Base", and minimally alter it so that strings like *Neither Mitt smokes nor Barack smokes* receive appropriate translations in Politics-NoQ.

b. Let **T** be the structure $\langle g, H_\gamma, j\rangle_{\gamma \in \{\text{Merge-S, Merge-IV, Not, And, If, Neither}\}}$, where g, H_γ, and j are defined as follows. T is a translation base from Mini-English to Politics-NoQ.

- The function g: $\Delta \rightarrow T$ (T = the set of all types) is defined as follows:
  $g(\text{NP}) = e \quad g(\text{TV}) = \langle e, \langle e, t\rangle\rangle \quad g(\text{IV}) = \langle e, t\rangle \quad g(\text{S}) = t$
- **The polynomial operations:**
  (over the algebra $\langle A, F_\gamma\rangle_{\gamma \in \{\text{Concat, Not, And}\}}$)
  (i) H_{Not} and H_{And} = F_{Not} and F_{And}, respectively
  (ii) $H_{\text{Merge-IV}}$ = F_{Concat}
  (iii) H_{If} = $F_{\text{Not}}\langle F_{\text{And}}\langle \text{Id}_{1,2}, F_{\text{Not}}\langle \text{Id}_{2,2}\rangle\rangle\rangle$
  (iv) $H_{\text{Merge-S}}$ = $F_{\text{Concat}}\langle \text{Id}_{2,2}, \text{Id}_{1,2}\rangle$
  (v) H_{Neither} = $F_{\text{And}}\langle F_{\text{Not}}\langle \text{Id}_{1,2}\rangle, F_{\text{Not}}\langle \text{Id}_{2,2}\rangle\rangle$
- (i) $\langle H_{\text{Not}}, \langle t\rangle, t\rangle$ is a derived rule of Politics-NoQ
  (ii) $\langle H_{\text{And}}, \langle t, t\rangle, t\rangle$ is a derived rule of Politics-NoQ
  (iii) $\langle H_{\text{Merge-IV}}, \langle\langle e, \langle e, t\rangle\rangle, e\rangle, \langle e, t\rangle\rangle$ is a derived rule of Politics-NoQ

(iv) $\langle H_{\text{If}}, \langle \text{t}, \text{t}\rangle, \text{t}\rangle$ is a derived rule of Politics-NoQ

(v) $\langle H_{\text{Merge-S}}, \langle\langle \text{e}, \text{t}\rangle, \text{e}\rangle, \text{t}\rangle$ is a derived rule of Politics-NoQ

(vi) $\langle H_{\text{Neither}}, \langle \text{t}, \text{t}\rangle, \text{t}\rangle$ is a derived rule of Politics-NoQ

- **The (lexical translation) function j:**

  (i) j(⟨Barack, ∅⟩) = **barack'**

  (ii) j(⟨Michelle, ∅⟩) = **michelle'**

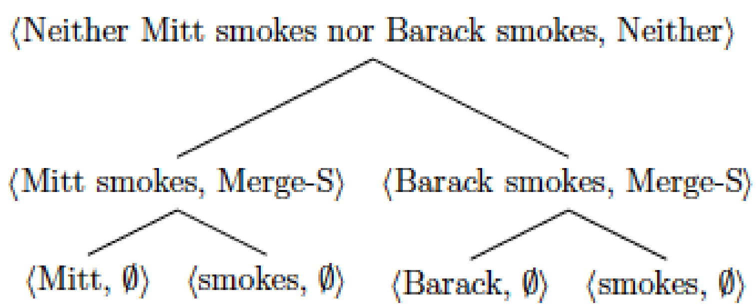
  (iii) j(⟨Mitt, ∅⟩) = **mitt'**

  (iv) j(⟨smokes, ∅⟩) = **smokes'**

  (v) j(⟨loves, ∅⟩) = **loves'**

c. Please show how your new translation base, along with our interpretation for Politics-NoQ, assigns a truth-value to the analysis tree for *Neither Mitt smokes nor Barack smokes.*

c. The sentence *Neither Mitt smokes nor Barack smokes* would be the analysis tree below in DME:



- Let k be the translation function determined by the translation base **T** defined in (3b).
- Let T be the tree above.

  k(T) = (by definition of DME)

  k(K_{Neither}($K_{\text{Merge-S}}$(⟨Mitt, ∅⟩, ⟨smokes, ∅⟩),
  $K_{\text{Merge-S}}$(⟨Barack, ∅⟩, ⟨smokes, ∅⟩))) =

  (by homomorphism property of k)

  H_{Neither}(k($K_{\text{Merge-S}}$(⟨Mitt, ∅⟩, ⟨smokes, ∅⟩)),
  k($K_{\text{Merge-S}}$(⟨Barack, ∅⟩, ⟨smokes, ∅⟩))) =

  (by homomorphism property of k)

$H_{\text{Neither}}(H_{\text{Merge-S}}(\text{k}(\langle\text{Mitt}, \emptyset\rangle), \text{k}(\langle\text{smokes}, \emptyset\rangle)),$
$\quad H_{\text{Merge-S}}(\text{k}(\langle\text{Barack}, \emptyset\rangle), \text{k}(\langle\text{smokes}, \emptyset\rangle))) =$

(by definition of k and j)

$H_{\text{Neither}}(H_{\text{Merge-S}}(\text{j}(\langle\text{Mitt}, \emptyset\rangle), \text{j}(\langle\text{smokes}, \emptyset\rangle)),$
$\quad H_{\text{Merge-S}}(\text{j}(\langle\text{Barack}, \emptyset\rangle), \text{j}(\langle\text{smokes}, \emptyset\rangle))) =$

(by definition of j)

$H_{\text{Neither}}(H_{\text{Merge-S}}(\textbf{mitt'}, \textbf{smokes'}),$
$\quad H_{\text{Merge-S}}(\textbf{barack'}, \textbf{smokes'})) =$ (by the definition of $H_{\text{Merge-S}}$)

$H_{\text{Neither}}(F_{\text{Concat}}\langle\text{Id}_{2,2}, \text{Id}_{1,2}\rangle(\textbf{mitt'}, \textbf{smokes'}),$
$\quad F_{\text{Concat}}\langle\text{Id}_{2,2}, \text{Id}_{1,2}\rangle(\textbf{barack'}, \textbf{smokes'})) =$

(by definition of function composition)

$H_{\text{Neither}}(F_{\text{Concat}}(\text{Id}_{2,2}(\textbf{mitt'}, \textbf{smokes'}), \text{Id}_{1,2}(\textbf{mitt'}, \textbf{smokes'})),$
$\quad F_{\text{Concat}}(\text{Id}_{2,2}(\textbf{barack'}, \textbf{smokes'}), \text{Id}_{1,2}(\textbf{barack'}, \textbf{smokes'}))) =$

(by definition of $\text{Id}_{2,2}$ and $\text{Id}_{1,2}$)

$H_{\text{Neither}}(F_{\text{Concat}}(\textbf{smokes'}, \textbf{mitt'}), F_{\text{Concat}}(\textbf{smokes'}, \textbf{barack'})) =$

(by definition of H_{Neither})

$F_{\text{And}}\langle F_{\text{Not}}\langle\text{Id}_{1,2}\rangle, F_{\text{Not}}\langle\text{Id}_{2,2}\rangle\rangle(F_{\text{Concat}}(\textbf{smokes'}, \textbf{mitt'}),$
$\quad F_{\text{Concat}}(\textbf{smokes'}, \textbf{barack'})) =$

(by definition of function composition)

$F_{\text{And}}(F_{\text{Not}}\langle\text{Id}_{1,2}\rangle(F_{\text{Concat}}(\textbf{smokes'}, \textbf{mitt'}), F_{\text{Concat}}(\textbf{smokes'}, \textbf{barack'})),$
$\quad F_{\text{Not}}\langle\text{Id}_{2,2}\rangle(F_{\text{Concat}}(\textbf{smokes'}, \textbf{mitt'}), F_{\text{Concat}}(\textbf{smokes'}, \textbf{barack'}))) =$

(by definition of function composition)

$F_{\text{And}}(F_{\text{Not}}(\text{Id}_{1,2}(F_{\text{Concat}}(\textbf{smokes'}, \textbf{mitt'}), F_{\text{Concat}}(\textbf{smokes'}, \textbf{barack'}))),$
$\quad F_{\text{Not}}(\text{Id}_{2,2}(F_{\text{Concat}}(\textbf{smokes'}, \textbf{mitt'}), F_{\text{Concat}}(\textbf{smokes'}, \textbf{barack'})))) =$

(by definition of F_{Concat})

$F_{\text{And}}(F_{\text{Not}}(\text{Id}_{1,2}((\textbf{smokes' mitt'}), (\textbf{smokes' barack'}))),$
$\quad F_{\text{Not}}(\text{Id}_{2,2}((\textbf{smokes' mitt'}), (\textbf{smokes' barack'})))) =$

(by definition of $\text{Id}_{1,2}$ and $\text{Id}_{2,2}$)

$F_{\text{And}}(F_{\text{Not}}(\textbf{smokes' mitt'}), F_{\text{Not}}(\textbf{smokes' barack'})) =$

(by definition of F_{Not})

$F_{\text{And}}(\sim(\textbf{smokes' mitt'}), \sim(\textbf{smokes' barack'})) =$

(by definition of F_{And})

$(\sim(\textbf{smokes' mitt'})\ \&\ \sim(\textbf{smokes' barack'}))$

- Now that we have mapped the tree in DME to the meaningful expression in Politics-NoQ, (~(**smokes' mitt'**) & ~(**smokes' barack'**)), we can use the interpretation **B** for Politics-NoQ to assign a truth-value to the meaningful expression.
- Let $\mathbf{B} = \langle B, G_\gamma, f \rangle_{\gamma \in \{\text{Concat, Not, And}\}}$ be the Fregean interpretation for Politics-NoQ, as defined in (16) on the handout "Montague's Theory of Translation: Laying the Groundwork".
- Let the meaning assignment function g be a homomorphism from Politics-NoQ to **B**.

g((~(**smokes' mitt'**) & ~(**smokes' barack'**))) =
(by definition of Politics-NoQ)

$g(F_{\text{And}}(F_{\text{Not}}(F_{\text{Concat}}(\textbf{smokes'}, \textbf{mitt'})), F_{\text{Not}}(F_{\text{Concat}}(\textbf{smokes'}, \textbf{barack'})))) =$
(by homomorphism property of g)

$G_{\text{And}}(g(F_{\text{Not}}(F_{\text{Concat}}(\textbf{smokes'}, \textbf{mitt'})), g(F_{\text{Not}}(F_{\text{Concat}}(\textbf{smokes'}, \textbf{barack'})))) =$
(by homomorphism property of g)

$G_{\text{And}}(G_{\text{Not}}(g(F_{\text{Concat}}(\textbf{smokes'}, \textbf{mitt'}))), G_{\text{Not}}(g(F_{\text{Concat}}(\textbf{smokes'}, \textbf{barack'})))) =$
(by homomorphism property of g)

$G_{\text{And}}(G_{\text{Not}}(G_{\text{Concat}}(g(\textbf{smokes'}), g(\textbf{mitt'}))), G_{\text{Not}}(G_{\text{Concat}}(g(\textbf{smokes'}), g(\textbf{barack'})))) =$ (by definition of g)

$G_{\text{And}}(G_{\text{Not}}(G_{\text{Concat}}(f(\textbf{smokes'}), f(\textbf{mitt'}))), G_{\text{Not}}(G_{\text{Concat}}(f(\textbf{smokes'}), f(\textbf{barack'})))) =$
(by definition of f in (16))

$G_{\text{And}}(G_{\text{Not}}(G_{\text{Concat}}(h, \text{Mitt})), G_{\text{Not}}(G_{\text{Concat}}(h, \text{Barack}))) =$
(by definition of G_{Concat})

$G_{\text{And}}(G_{\text{Not}}(h(\text{Mitt})), G_{\text{Not}}(h(\text{Barack}))) =$ (by definition of h in (16))

$G_{\text{And}}(G_{\text{Not}}(0), G_{\text{Not}}(1)) =$ (by definition of G_{Not})

$G_{\text{And}}(1, 0) =$ (by definition of G_{And})

0

d. Given your proposed translation base, construct a direct interpretation of Disambiguated Mini-English, and show how it interprets *Neither Mitt smokes nor Michelle smokes.*

d. Let $\mathbf{B} = \langle \text{B}, J_\gamma, l \rangle_{\gamma \in \{\text{Merge-S, Merge-IV, Not, And, If, Neither}\}}$ be the Fregean interpretation of DME, defined as follows:

- **The definition of the set B:**
  The set B is the same set as the set B in $\mathbf{B} = \langle \text{B}, G_\gamma, f \rangle_{\gamma \in \{\text{Concat, Not, And}\}}$, the interpretation of Politics-NoQ previously defined.

  $\Rightarrow \text{B} = \bigcup_{\tau \in \text{T}} D_\tau, \{\text{Michelle, Barack, Mitt}\}$

- **The definition of semantic operations:**
  The polynomial operations over $\langle \text{B}, G_\gamma \rangle_{\gamma \in \{\text{Concat, Not, And}\}}$, $\{J_\gamma\}_{\gamma \in \{\text{Merge-S, Merge-IV, Not, And, If, Neither}\}}$, are defined as follows:

  (i) $J_{\text{Not}} = G_{\text{Not}}$

  (ii) $J_{\text{And}} = G_{\text{And}}$

  (iii) $J_{\text{If}} = G_{\text{Not}}\langle G_{\text{And}}\langle \text{Id}_{1,2},\ G_{\text{Not}}\langle \text{Id}_{2,2}\rangle\rangle\rangle$

  (iv) $J_{\text{Merge-IV}} = G_{\text{Concat}}$

  (v) $J_{\text{Merge-S}} = G_{\text{Concat}}\langle \text{Id}_{2,2}, \text{Id}_{1,2}\rangle$

  (vi) $J_{\text{Neither}} = G_{\text{And}}\langle G_{\text{Not}}\langle \text{Id}_{1,2}\rangle, G_{\text{Not}}\langle \text{Id}_{2,2}\rangle\rangle$

  $\Rightarrow \langle \text{B}, J_\gamma \rangle_{\gamma \in \{\text{Merge-S, Merge-IV, Not, And, If, Neither}\}}$ is an algebra.

- **The definition of the Lexical Interpretation Function:**
  The lexical interpretation function l is defined as follows:

  (i) $l(\langle \text{Barack}, \emptyset \rangle) = \text{Barack}$

  (ii) $l(\langle \text{Michelle}, \emptyset \rangle) = \text{Michelle}$

  (iii) $l(\langle \text{Mitt}, \emptyset \rangle) = \text{Mitt}$

  (iv) $l(\langle \text{smokes}, \emptyset \rangle) = h = f(\textbf{smokes'})$

  (v) $l(\langle \text{loves}, \emptyset \rangle) = j = f(\textbf{loves'})$

- Let the meaning assignment function g be the unique homomorphism determined by $\mathbf{B}$: $l \subseteq$ g.

- Let T be the analysis tree for *Neither Mitt smokes nor Michelle smokes* in DME.

$g(T) =$ (by definition of DME)

$g(K_{\text{Neither}}(K_{\text{Merge-S}}(\langle \text{Mitt}, \emptyset\rangle, \langle \text{smokes}, \emptyset\rangle),$
$K_{\text{Merge-S}}(\langle \text{Michelle}, \emptyset\rangle, \langle \text{smokes}, \emptyset\rangle))) =$
(by homomorphism property of g)

$J_{\text{Neither}}(g(K_{\text{Merge-S}}(\langle \text{Mitt}, \emptyset\rangle, \langle \text{smokes}, \emptyset\rangle)),$
$g(K_{\text{Merge-S}}(\langle \text{Michelle}, \emptyset\rangle, \langle \text{smokes}, \emptyset\rangle))) =$
(by homomorphism property of g)

$J_{\text{Neither}}(J_{\text{Merge-S}}(g(\langle \text{Mitt}, \emptyset\rangle), g(\langle \text{smokes}, \emptyset\rangle)),$
$J_{\text{Merge-S}}(g(\langle \text{Michelle}, \emptyset\rangle), g(\langle \text{smokes}, \emptyset\rangle))) =$ (by definition of g)

$J_{\text{Neither}}(J_{\text{Merge-S}}(l(\langle \text{Mitt}, \emptyset\rangle), l(\langle \text{smokes}, \emptyset\rangle)),$
$J_{\text{Merge-S}}(l(\langle \text{Michelle}, \emptyset\rangle), l(\langle \text{smokes}, \emptyset\rangle))) =$ (by definition of l)

$J_{\text{Neither}}(J_{\text{Merge-S}}(\text{Mitt}, h), J_{\text{Merge-S}}(\text{Michelle}, h)) =$
(by definition of $J_{\text{Merge-S}}$)

$J_{\text{Neither}}(G_{\text{Concat}}\langle \text{Id}_{2,2}, \text{Id}_{1,2}\rangle(\text{Mitt}, h),$
$G_{\text{Concat}}\langle \text{Id}_{2,2}, \text{Id}_{1,2}\rangle(\text{Michelle}, h)) =$
(by definition of function composition)

$J_{\text{Neither}}(G_{\text{Concat}}(\text{Id}_{2,2}(\text{Mitt}, h), \text{Id}_{1,2}(\text{Mitt}, h)),$
$G_{\text{Concat}}(\text{Id}_{2,2}(\text{Michelle}, h), \text{Id}_{1,2}(\text{Michelle}, h))) =$
(by definition of $\text{Id}_{2,2}$ and $\text{Id}_{1,2}$)

$J_{\text{Neither}}(G_{\text{Concat}}(h, \text{Mitt}), G_{\text{Concat}}(h, \text{Michelle})) =$ (by definition of G_{Concat})

$J_{\text{Neither}}(h(\text{Mitt}), h(\text{Michelle})) =$ (be definition of J_{Neither}

$G_{\text{And}}\langle G_{\text{Not}}\langle \text{Id}_{1,2}\rangle, G_{\text{Not}}\langle \text{Id}_{2,2}\rangle\rangle(h(\text{Mitt}), h(\text{Michelle})) =$
(by definition of function composition)

$G_{\text{And}}(G_{\text{Not}}\langle \text{Id}_{1,2}\rangle(h(\text{Mitt}), h(\text{Michelle})),$
$G_{\text{Not}}\langle \text{Id}_{2,2}\rangle(h(\text{Mitt}), h(\text{Michelle}))) =$
(by definition of function composition)

$G_{\text{And}}(G_{\text{Not}}(\text{Id}_{1,2}(h(\text{Mitt}), h(\text{Michelle}))),$
$G_{\text{Not}}(\text{Id}_{2,2}(h(\text{Mitt}), h(\text{Michelle})))) =$ (by definition of $\text{Id}_{1,2}$ and $\text{Id}_{2,2}$)

$G_{\text{And}}(G_{\text{Not}}(h(\text{Mitt})), G_{\text{Not}}(h(\text{Michelle}))) =$ (by definition of h)

$G_{\text{And}}(G_{\text{Not}}(0), G_{\text{Not}}(0)) =$ (by definition of G_{Not})

$G_{\text{And}}(1, 1) =$ (by definition of G_{And})

1