Problem Set on Languages and Interpretations

(1) **Basic Comprehension Questions on Our (Provisional) Definition of a 'Language'**

Let L be a language $< A, F_\gamma, X_\delta, S, \delta_0>_{\gamma \in \Gamma, \delta \in \Delta}$ as defined in (21) on the handout "An Algebraic Perspective on the Syntax of First Order Logic".

a. What are the syntactic rules of *L*?

b. What are the syntactic category labels of *L*?

c. What are the basic expressions ('lexical items') of *L*? (Please represent as an indexed family of sets.)

d. Which elements in *L* form an algebra together?

f. What is the category label for the declarative sentences of *L*?

g. What are the syntactic operations of *L*? (Please represent as an indexed family of sets).

h. What is the difference between A and the meaningful expressions of *L*? Can they ever be the same?

i. Let CAT be the syntactic categories of *L*.
 (i) Which element are the members of CAT subsets of?
 (ii) Which element serves to index the members of CAT?
 (iii) Please represent CAT as an indexed family of sets.
 (iv) Which elements in *L* work together to generate CAT?

(2) **Basic Exercise in Language Design**

Let $< A, F_\gamma, X_\delta, S_E, S >_{\gamma \in \{Merge, And, Not\}\ \delta \in \{NP, IV, TV, S\}}$ be the language 'Mini-English', as defined in (29) on the handout "An Algebraic Perspective on the Syntax of First Order Logic".

a. Please alter this structure minimally, so that the category C_S includes strings like the following:
 If Mitt smokes, then Barack smokes.

b. Please provide an analysis tree showing how the following string is derived.
 If Barack loves Michelle, then it is not the case that Mitt smokes.

c. Are the following meaningful expressions of the language you defined? Why or why not?
 (i) *Then Barack smokes.*
 (ii) *If Barack loves Michelle.*
 (ii) *And Barack smokes*.

(3) **More Advanced Exercise on Language Design**
Let First Order Logic (FOL) be the language defined in (16) on the handout "An Algebraic Perspective on Propositional Logic."

a. Represent FOL as a language $< A , F_\gamma , X_\delta , S, \delta_0>_{\gamma \in \Gamma , \delta \in \Delta}$, following the definition in (21) on the handout "An Algebraic Perspective on the Syntax of First Order Logic". To do this, you will need to do the following:

(i) Identify a set of syntactic operations $\{ F_\gamma \}_{\gamma \in \Gamma}$ that will generate the WFFs of FOL (as defined in (16)).

(ii) Use these operations to define an algebra $<A, F_\gamma>_{\gamma \in \Gamma}$ such that the WFFs of FOL are a subset of A.

(iii) **Special Hint:** Let the syntactic category labels Δ be T ∪ {var}, where T is the set of types, and 'var' is the category label for variables.

(iv) Organize the basic expressions of FOL into sets $\{ X_\delta \}_{\delta \in \Delta}$
Special Hint: Make sure that X_e contains both the individual constants and the variables.

(v) Write out a set of syntactic rules S for FOL.
Special Hint: You *might* find it helpful to consult (17) on "An Algebraic Perspective on Propositional Logic"

b. Given your representation of FOL as a language, please provide an analysis tree showing how the following formula is generated by your system (where P is a unary predicate letter and Q is a binary predicate letter):

∀x(Px & ~∃y((Qa)y))

c. Let R be a ternary predicate letter, a and c be individual constants, and x be a variable.

(i) Please state whether the following are or are not meaningful expressions of the language you defined.
1. ((Ra)x)
2. (((Ra)x)c)
3. ∃x((Ra)x)

(ii) If the string is a meaningful expression, please provide a calculation showing what category it is a member of (follow the format in (17) on the handout "An Algebraic Perspective on the Syntax of First Order Logic".

(iii) If the string is not a meaningful expression, please provide a brief explanation of why it isn't.

(4) **Another Exercise on Language Design**
Let Propositional Logic (PL) be the language defined in (4) on the handout "An Algebraic Perspective on Propositional Logic." Please represent PL as a language, following the definition in (21) on the handout "An Algebraic Perspective on the Syntax of First Order Logic".

HINT: Adapt what you did in exercise (3).

(5) **An Exercise on Models and (Montagovian) Interpretations**
Let FOL-NoQ be the language $< A , F_\gamma , X_\tau , S , t >_{\gamma \in \{\text{Concat, Not, And}\}, \tau \in T}$ defined in (5) on the handout "Montague's General Theory of Semantics" (MGTS).

a. Let E be a set of entities, and let $\mathbf{B} = <B, G_\gamma , f>_{\gamma \in \{\text{Concat, Not, And}\}}$ be a Fregean interpretation for FOL-NoQ based on E (as defined in (15) on MGTS). Let $\mathcal{M}$ be a model <E, I> (as defined in (21) on MGTS), where I = *f*.

Please show via induction on structural complexity that every $\varphi \in C_t$ is such that $[[\varphi]]^M = g(\varphi)$, where g is the meaning assignment based on **B**.

Some Hints:

1. First, show that if φ is an atomic formula of FOL-NoQ, then $[[\varphi]]^M = g(\varphi)$.
To show this, first use our definition of a model $\mathcal{M}$ to show:
$[[(\ldots(\Phi\alpha_1)\ldots\alpha_n)]]^M = g(\Phi)(g(\alpha_1))\ldots(g(\alpha_n))$
Then, use the homomorphism property of g to show:
$g(\Phi)(g(\alpha_1))\ldots(g(\alpha_n)) = g((\ldots(\Phi\alpha_1)\ldots\alpha_n))$

2. Next, assume that φ is a conjunction (ψ & χ), and that ψ and χ are both such that $[[\psi]]^M = g(\psi)$ and $[[\chi]]^M = g(\chi)$. Show that $[[\varphi]]^M = g(\varphi)$.
To show this, use the definition of a model $\mathcal{M}$ and the induction assumption to show:
$[[(\psi \,\&\, \chi)]]^M = G_{\text{And}}(g(\psi), g(\chi))$
Then use the homomorphism property of g to show:
$G_{\text{And}}(g(\psi), g(\chi)) = g((\psi \,\&\, \chi))$

3. Next, assume that φ is a negation ~ψ, and that ψ is such that $[[\psi]]^M = g(\psi)$. Show that $[[\varphi]]^M = g(\varphi)$.
To show this, follow the same general strategy laid out in 2.

b. Let $\mathcal{M}$ be a model <D, I>. Let $\mathbf{B} = <B, G_\gamma , f>_{\gamma \in \{\text{Concat, Not, And}\}}$ be a Fregean interpretation for FOL-NoQ based on D, where *f* = I.

Please show via induction on structural complexity that every $\varphi \in C_t$ is such that $[[\varphi]]^M = g(\varphi)$, where g is the meaning assignment based on **B**.