Problem Set on Propositional Logic and First Order Logic:
Formal Semantics [1]

(1) **Computing Entailment in Propositional Logic**
Please use truth-tables to establish whether the following claims are accurate.

a. $\{ (\sim p \vee \sim q) \} \vDash \sim(p \vee q)$

b. $\{ (p \vee q), (\sim p \rightarrow \sim q) \} \vDash q$

c. $\{ ((p \rightarrow q) \rightarrow r) \} \vDash (p \rightarrow (q \rightarrow r))$

(2) **Computing Logical Equivalence in Propositional Logic**
Please use truth-tables to show that the following pairs are logically equivalent.

a.	(i)	$(\varphi \,\&\, \psi)$	(ii)	$\sim(\sim\varphi \vee \sim\psi)$
b.	(i)	$(\varphi \rightarrow \psi)$	(ii)	$(\sim\psi \rightarrow \sim\varphi)$
c.	(i)	$(\varphi \,\&\, (\psi \vee \chi))$	(ii)	$((\varphi \,\&\, \psi) \vee (\varphi \,\&\, \chi))$

(3) **Defining Operators in Propositional Logic**
Please show how the operators '~' and '∨' can be defined using the 'nor'-operator (↓) defined below.

φ	ψ	$(\varphi \downarrow \psi)$
1	1	0
1	0	0
0	1	0
0	0	1

(4) **Computing Truth of Formulas Relative to a Model**
Let $\mathcal{M}$ be the model < L, I>, where L is the set of English letters { a, b, c, d, e, f, g, h, I, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z } and I consists of at least the following mappings:

I(A) = $\{ x : x \in L$ and x is a vowel $\}$
I(R) = $\{ \langle x,y \rangle : x, y \in L$ and x precedes y in alphabetical order $\}$

Please calculate the truth-values of the following sentences relative to $\mathcal{M}$:

a. $\exists x \exists y \exists z(Rxy \,\&\, (Ay \,\&\, (Rxz \,\&\, \sim Az)))$
b. $\forall x(Rxx \rightarrow \sim Ax)$
c. $\forall x(\sim Ax \rightarrow \exists y Rxy)$

[1] Most of these exercises are taken from Gamut (1991), Volume 1: Chapter 2, Chapter 3

(5) **Proving Logical Equivalence for First Order Logic Sentences**
Please show that the following pairs of formulae are logically equivalent.

a. (i) ∀x∀yPxy (ii) ∀y∀xPxy
b. (i) ∃x∃yPxy (ii) ∃y∃xPxy

(6) **Proving Entailment for First Order Logic Sentences**
Please show that the following entailment relations hold.

a. ∀xPx ⊨ ∃xPx [2]

b. ∃x(Px & Qx) ⊨ (∃xPx & ∃xQx)

c. ∃y∀xPxy ⊨ ∀x∃yPxy

[2] Hint: Recall that the domain D of a model has to be a *non-empty* set.