Problem Set on Propositional Logic and First Order Logic:
Translation and Natural Deduction [1]

(1) Translation from English to Propositional Logic

Please encode the following English statements as formulae in PL. Be sure to include a 'key' indicating what English statements each propositional letter 'stands for'.

a. God willing, peace will come.
b. If it isn't summer, then it is damp and cold, if it is evening or night.
c. If you do not help me if I need you, I will not help you if you need me.
d. John comes only if Peter does not come.
e. We are going, unless it is raining.
f. If Johnny is nice, then he will get a bicycle from Santa Clause, whether he wants one or not.

(2) Derivations in Our Natural Deduction System for Propositional Logic

Please provide derivations in our natural deduction system for PL establishing each of the following.

a. $(p \vee q) \vdash ((p \rightarrow q) \rightarrow q)$
b. $(p \,\&\, {\sim}q) \vdash {\sim}(p \rightarrow q)$
c. $(p \rightarrow {\sim}q) \vdash (q \rightarrow {\sim}p)$
d. $(p \,\&\, (q \vee r)) \vdash ((p \,\&\, q) \vee (p \,\&\, r))$
e. $((p \,\&\, q) \vee (p \,\&\, r)) \vdash (p \,\&\, (q \vee r))$

(3) Translations from English to First Order Logic

Please encode the following English statements as formulae in FOL. Be sure to include a 'key' indicating what the predicate letters and individual constants 'stand for'.

a. Although John and Mary love each other deeply, they make each other unhappy.
b. It is not the case that all ambitious people are not honest.
c. Lyn got a present from John, but she didn't get anything from Peter.
d. Nobody lives in Hadley who wasn't born there.
e. People who live in Amherst or close buy own a car.
f. If somebody is noisy, then everybody is annoyed at him.

[1] Most of these exercises are taken from Gamut (1991), Volume 1: Chapters 2, 3 and 4

(4) **Identifying Sentences of FOL**
Please identify whether the following formulae of FOL are sentences or not.

a. (∃xPxa & Bx)
b. ∃x(∀yPxy → Bx)
c. ∀y~∃xPxy
d. ∃x(~Ba → (~∀y(~Pxy ∨ Qb) → Cy))
e. ∀x∀y((Pxy & By) → ∃wCxw)
f. (∀x∀yPyy → Bx)
g. ∀x(∀yPyy → Bx)

(5) **Proof by Induction on the Complexity of Formula**
Please show the following by constructing a proof on the complexity of formula:

<u>Claim:</u> No formula of FOL begins with a variable.

(6) **Derivations in Our Natural Deduction System for First Order Logic**

Please provide derivations in our natural deduction system for FOL establishing each of the following.

a. ∀x(Px & Bx) ⊢ (∀xPx & ∀xBx)
b. ∀xPx & ∀xBx ⊢∀x(Px & Bx)
c. ∃x(Px & Bx) ⊢ (∃xPx & ∃xBx)
d. ∃x~Px ⊢~∀xPx
e. ~∀xPx ⊢∃x~Px