Seth Cable | *Proseminar on Semantic Theory*
Fall 2013 | *Ling 720*

Problem Set on the 'Formal Preliminaries' [1]

(1) **Basic Comprehension Questions on Relations and Functions**

a. Let A = {b,c} and B = {2,3}. State whether the following are true or false:

(i) $(A \times B) \cap (B \times A) = \varnothing$
(ii) $\langle c,c \rangle \subseteq A^2$
(iii) $\{\langle b,3 \rangle, \langle 2,a \rangle\} \subseteq (A \times B) \cup (B \times A)$
(iv) $\varnothing \subseteq (A \times A)$

b. Let A = {b,c} and B = {2,3}. Let $R = \{\langle b,b \rangle, \langle b,2 \rangle, \langle c,2 \rangle, \langle c,3 \rangle\}$

(i) What is the range and domain of R?
(ii) What is R^{-1}?

c. Let f: A→B and g: B→C both be *bijections*. Show that $(g \circ f)^{-1} = (f^{-1}) \circ (g^{-1})$

d. Let $R = \{ \langle x,y \rangle : x, y \in \{ 1, 2 \} \text{ and } x < y \}$.

(i) Represent R as a set of pairs.
(ii) Represent the characteristic function of R as a set of pairs [2]
(iii) Represent the characteristic function of R as a ternary relation
(iv) Represent the curried characteristic function of R as a matrix

Reminder: By 'matrix' I mean a diagram like the following:

$$\left(\begin{array}{ll} a \rightarrow & \left(\begin{array}{l} a \rightarrow a \\ b \rightarrow c \\ c \rightarrow b \end{array} \right) \\ b \rightarrow & \left(\begin{array}{l} a \rightarrow b \\ b \rightarrow c \\ c \rightarrow a \end{array} \right) \\ c \rightarrow & \left(\begin{array}{l} a \rightarrow c \\ b \rightarrow c \\ c \rightarrow c \end{array} \right) \end{array} \right)$$

[1] Most of these exercises are taken from Partee *et al.* (1993), Chapters 2 and 4.
[2] Assume that the domain of the characteristic function of R is $\{1,2\} \times \{1,2\}$.

(2) **Proving that Sets are Countable**

a. Show that the set of integer powers of ten {10, 100, 1000, 10000, … } is countable.

b. Suppose that the following is true of English:

(i) There is a finite alphabet for writing sentences, consisting of 26 letters and a space (forget punctuation marks for now)

(ii) Every sentence of English is a finite string in the alphabet in (i)

(iii) There is no upper bound on the length of sentences of English. That is, for any sentence S of English, there is always a longer sentence S'.

Show that the set of English sentences is countably infinite.

<u>Some Hints:</u>

- To solve this, you simply have to show that there is a defined way of ordering the sentences of English into an infinite 'list' (as we did for the rationals greater than 0).
- One way of solving this makes use of 'alphabetical order'.
- Another, more round-about way of solving this makes use of the following key consequence of the 'fundamental theorem of arithmetic'

  <u>Consequence of Fundamental Theorem of Arithmetic:</u>
  For every positive integer n > 1, there is *exactly one* way of representing n as a product of powers of primes:

  $n = p_1^{a1} \times p_2^{a2} \times \ldots \times p_k^{ak}$ where each p_i is prime, each $a_i \in \mathbb{N}$

(3) **Proofs by Induction**

Construct a proof by induction for the following general equivalence:

$(X_1 \cup \ldots \cup X_n)' = X_1' \cap \ldots \cap X_n'$

<u>Hint:</u>
You can appeal to the following general equivalences:

(i) $X'' = X$

(ii) $X_1 \cap \ldots \cap X_n = (X_1 \cap \ldots \cap X_{(n-1)}) \cap X_n$

(iii) $X_1 \cup \ldots \cup X_n = (X_1 \cup \ldots \cup X_{(n-1)}) \cup X_n$