

**Remko Scha (1981/1984):
“Distributive, Collective and Cumulative Quantification”**

1. Introduction

(1) The Importance of Scha (1981/1984)

- The first ‘modern’ work on plurals (Landman 2000)
- There are many ideas in this work that later works offer critical reaction to.
- *We won’t cover the whole paper in much depth, what we are mainly interested in is the essentials of Scha’s treatment of the phenomena in (2)-(5).*
- (For much more extensive discussion and exposition, see Landman (2000: Chapter 4)

Our discussion of Scha will focus on his treatment of the following phenomena, which can possibly be grouped under the label of ‘**readings that are neither collective nor distributive**’

(2) Cumulative Readings

We’ve already discussed that the sentence in (a) is T in a situation like (b), and how this is a problem for our current semantics of plurals.

- Sentence: Three TAs graded thirty exams.
- Situation: Paul graded 10 exams, Tony graded 15, and John graded 5.
- The Key Proposal of Scha (1981/1984)

The truth of sentence (2a) in a situation like (2b) is due to a special *reading* the sentence has, one where it has the T-conditions informally stated below.

- Scha’s T-Conditions for ‘Cumulative Readings’ (see his (4b))

The **total number** of TAs that graded exams is **3**, and the **total number** of exams graded by TAs is **30**.

(3) 'Cover Readings'

Scha (1981/84) famously observes that a sentence like (3a) is true in a situation like (3b).

- a. Sentence: Six boys met in the park.
- b. Situation: Tom and Frank met in the park. So did Bill and Tony. So did Dave and Roger.
- c. The Key Proposal of Scha (1981/1984)

The truth of sentence (3a) in a situation like (3b) is due to a special *reading* of the numeral *six*, one that results in the T-conditions stated below.

(Note; although Scha doesn't himself give a name to this reading, Landman (2000: Ch. 4) introduces the term 'cover reading' to describe it.)

- d. Scha's T-Conditions for 'Cover Readings' (Landman 2000: Ch. 4)

$|\{x : \text{ATOM}(x) \ \& \ \exists y : x \leq y \ \& \ y \in *[[\text{boy}]] \ \& \ y \in [[\text{met in the park}]]\}| = 6$

'the set of atomic entities x such that
 x is a part of a plurality of boys that met in the park has 6 elements'

('if you add up all the boys that met in the park, you get six individual boys')

(4) Side-Note on 'Cover Readings'

- The fact that (3a) is true in (3b) is actually not a problem for the semantic theory we developed in class.
- Note that the LF in (a) is true in situation (3b), as it is assigned the T-conditions in (b)

- a. An LF That is True in Situation (3b)

[Six boys [* [met in the park]]

- b. T-Conditions Assigned to (3a)

$\exists x. [[\text{boys}]](x) = T \ \& \ |\text{ATOMS}(x)| = 6 \ \& \ x \in * [\lambda z : z \text{ met in the park }]$

'There is a group of six boys that you can form from the set of things (pluralities) that met in the park.'

(5) **Co-Distributive Readings**

Scha (1981/84) notes that many sentences containing **definite plurals** seem to admit of readings that are neither wholly cumulative nor wholly distributive.

a. **First Example: Circles and Squares**

(i) Sentence: The squares contain the circles.

(ii) True in Following Situation:



(iii) Inadequate T-Conditions

1. Every square contains every circle.
2. Every square contains a circle.

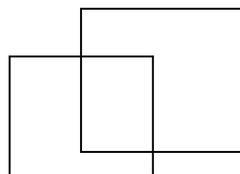
(iv) Scha's (1981/84) Postulated T-Conditions

Every circle is contained within some square.

b. **Second Example**

(i) Sentence: The sides of square 1 are parallel to the sides of square 2

(ii) True in the Following Situation



(iii) Inadequate T-Conditions:

Every side of S1 is parallel to every side of S2

(iv) Scha's (1981/1984) Postulated T-Conditions

Every side of square 1 is parallel to some side of square 2, and every side of square 2 is parallel to some side of square 1.

Note: The T-conditions in (5biv) are kinda similar to those for the 'cumulative reading' in (2d)...

c. **Third Example**

- (i) Sentence: The sides of square 1 cross the sides of square 2.
- (ii) True in the Following Situation: (5bii)
- (iii) Inadequate T-conditions
Every side of square 1 crosses every side of square 2.
- (iii) Scha's (1981/1984) Postulated T-Conditions
Some sides of square 1 crosses some sides of square 2

Note: Scha (1981/84) doesn't himself have a special term for these 'readings', but subsequent authors have used the term 'co-distributive' to refer to them (Winter 2000).

Generally speaking, the term 'co-distributive reading' is typically used to refer to readings that

- (i) are not 'doubly distributive', and
- (ii) are associated with sentences containing multiple definite plurals...

(6) **A Key Property of Scha (1981/1984) that Many View as Problematic**

As we will see, Scha (1981/1984) proposes *three different* explanations for the phenomena above:

- 'Co-distributive readings' (5) are captured via *meaning postulates*.
- 'Cover readings' (3) are captured via *an ambiguity in the meaning of numerals*
- 'Cumulative readings' (2) are captured via *a complex process of forming binary generalized quantifiers...*

However, as has long been noted, the exact boundaries between these phenomena is not *a priori* so clear...

... note in particular the similarity between certain 'co-distributive' readings (5biv) and the cumulative readings (2d)

For this and other such reasons, folks tend to prefer systems that can provide largely unified accounts of (2)-(6)...

We will, though, follow Scha (1981/1984) in discussing each of these phenomena, and his analysis of them, in turn...

2. The ‘Co-Distributive Readings’ of Sentences Containing Definite Plurals

(7) The Problem to be Solved

We need our semantics to predict that sentences like (5ai), (5bi) and (5ci) true in situations (5a_{ii}), (5b_{ii}) and (5c_{ii}), respectively.

To build towards Scha’s solution, let us recall the facts below...

(8) The Existence of ‘Stubbornly Distributive Predicates’

Certain verbs (e.g. *run, sleep, smile*) seem to validate the inference below:

X ran / slept / smiled → All atoms in X ran / slept / smiled .

(9) The World Before Scha (1981 / 1984)

- In work preceding Scha’s, it was proposed that a plural definite like “the boys” was ambiguous, and allowed an interpretation akin to that below.

a. [[the boys]] = [λP : for all x such that x is a boy, $P(x) = T$]

- In these systems, it was possible to capture the facts in (8) by assuming that the verbs in question *subcategorized* for DPs possessing the ‘distributive’ interpretation in (9a).
- Thus, in these systems, a sentence like (b) would necessarily be assigned the T-conditions in (c), and so the inference in (8) would thereby be validated.

b. The boys ran / slept / smiled

c. for all x such that x is a boy, x ran / slept / smiled

(10) Scha’s (1981 / 1984) Key Observation and Critique

- Sentences like those in (9b) don’t have *exactly* the T-conditions in (9b)....
Relatedly, the inference in (8) isn’t *exactly* valid for these Vs...
- Rather, the T-conditions of (9b) seem to be vaguer than (9c)...
*...it seems like (9b) would be T even in situations where **not every boy** ran/slept/smiled...*
*... just so long as **some significant** number of the boys ran/slept/smiled...*
- Key Observation:
This vagueness of (9b) seems quite similar to the general vagueness surrounding collective readings of sentences...

... consider that the collective reading of “the boys built a canoe” is T even when one or two boys didn’t really do anything (Link 1983, Lasersohn 1999, Brisson 2003)

(11) **The Proposal: Distributivity Through Collectivity and Meaning Postulates**

- A sentence like “the boys ran / slept / smiled” can be mapped to the LF in (a), which is assigned the T-conditions in (b)
 - a. [The boys [ran / slept / smiled]]
 - b. $\text{boy-1+boy-2+...boy-}n \in [[\text{ran / slept / smiled}]]$
- Given the ‘primacy of the collective reading’, such LFs and T-conditions correspond to a ‘collective reading’
- However, *just as in all collective readings*, a collective reading of “the boys ran / slept / smiled” requires that some significant subpart of the boys *participate in the event in question...*
- Given the nature of the Vs in question (perhaps the fact that they are activities), the only kind of ‘participation’ this significant subpart of the boys could be doing is, in fact, *running, sleeping, smiling, etc....*
- Thus, our world-knowledge / lexical-knowledge of run entails that the following holds:
 - c. Meaning Postulate for ‘Stubbornly Distributive Predicates’
 $X \in [[\text{ran / slept / smiled}]]$ *only if*
some ‘significant’ subpart Y of X is such that
for all atoms y in Y, y ‘participated’ in the act of running, sleeping, smiling
 - We can formalize this notion by including the generalization in (c) as a ‘meaning postulate’ for the verb in question...
... such a meaning postulate is similar to one like that in (d), which we might need for collective readings of Vs like ‘build’
 - d. Meaning Postulate for *Build*
 $\langle X, z \rangle \in [[\text{build}]]$ *only if*
some ‘significant’ subpart Y of X is such that
for all atoms y in Y, y ‘participated’ in the act of building z

This much should be familiar to you from our earlier discussion of verbs like ‘run’...

...but what does any of this have to do with the problem in (7)?...

(12) **The Proposal: Co-Distributive Readings Through Meaning Postulates**

- Why not suppose that the truth of (5ai), (5bi) and (5ci) in situations (5a_{ii}), (5b_{ii}) and (5c_{ii}) simply reflects those sentences' *collective interpretations*.
- That is, why not simply suppose it's a feature of the *lexical* meaning of those predicates that they hold of plural arguments in those kinds of situations.
- That is, why not suppose that our grammar of English contains the following meaning postulates.

a. Meaning Postulate for *Contain*

$\langle X, Y \rangle \in [[\text{contain}]]$ *only if*
every atomic member y of Y is contained in some atomic member x of X

b. Meaning Postulate for *Parallel To*

$\langle X, Y \rangle \in [[\text{parallel to}]]$ *only if*
for every atomic member x of X, there is some atomic member y of Y such that x is parallel to y, *and*
for every atomic member y of Y, there is some atomic member x of X such that y is parallel to x.

c. Meaning Postulate for *Cross*

$\langle X, Y \rangle \in [[\text{parallel to}]]$ *only if*
there is a set of atomic member x of X such that each member of that set crosses some atomic member y of Y.

(13) **The Idea Simply Put**

- We've seen that 'collective readings' of sentences always bring with them some kind of entailments regarding the individual members of the pluralities involved...
- We've also seen that the exact nature of these entailments depends upon the lexical semantics of the predicates involved...
(... and so we've modeled them using 'meaning postulates'....)
- So, when we see the facts in (5), it's reasonable to suppose that they simply reflect the special entailments associated with the collective readings of those verbs...
- Thus, following our general program for such entailments, we will model them by pairing those predicates with meaning postulates of the appropriate form!...

(14) **Some Support for This Line of Thought (Landman 2000)**

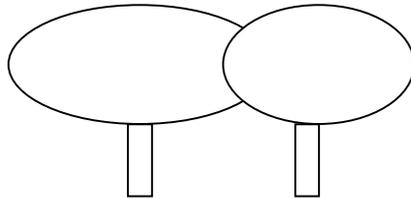
There are cases superficially similar to those in (5), where an approach like that in (12) seems like its got to be the right answer.

a. Case Study, Part 1

Sentence (i) is true in a situation like that in (ii). Thus, we might associate with (i) the T-conditions in (iii).

(i) The leaves of tree A are touching the leaves of tree B.

(ii)



*Note, not **all** the leaves are touching.*

(iii) Some of the leaves of tree A are touching some of the leaves of tree B.

b. Case Study, Part 2

It seems, though, that for any two entities X and Y, the following generalization holds:

Meaning Postulate for 'Touch'

$\langle X, Y \rangle \in [[\text{parallel to}]]$ *only if*

there is some part x of X and some part y of Y such that x is in physical contact with y.

Consider, for example, the situations where a sentence like *Dave touched Sue* is true...

c. Case Study, Part 3

Thus, if we are right that the 'meaning postulate' in (b) governs collective readings of *touch*, we easily predict that (14ai) will be true in situation (14aii)...

(15) **A Key Problem for This Line of Thought (Roberts 1987)**

There are cases superficially similar to those in (5), where an approach like that in (12) *can't* be the right answer.

a. Case Study, Part 1

Sentence (i) is true in a situation like that in (ii). Thus, we might associate with (i) the T-conditions in (iii).

(i) The boys lifted the pianos.

(ii) Boy-1 lifted Piano-1, Boy-2 lifted Piano-2, Boy-3 lifted Piano-3.

(iii) Every boy lifted some piano, and every piano was lifted by some boy.

b. Case Study, Part 2

Given the meaning postulate in (12b), by parity of reasoning, we might be lead to suppose that *lift* should be associated with the meaning postulate below.

$\langle X, Y \rangle \in [[\text{lifted}]]$ *only if*

for every atomic member x of X , there is some atomic member y of Y such that x lifted y , *and*

for every atomic member y of Y , there is some atomic member x of X such that y lifted x .

c. Case Study, Part 3

However, this would wrongly predict that sentence (15ai) is *only* true in situations like (15aii)...

(16) **Conclusion (Roberts 1987)**

- There must be some means *other than meaning postulates (lexical semantics)* for capturing the truth of (15ai) in the situation in (15aii)
- This mechanism may well also be responsible for those 'co-distributive' readings that are similar in form...
- THUS...
 - While some of those co-distributive readings in (5) may indeed just be the 'collective readings' of those predicates...
 - Others are likely the result of mechanisms not found in Scha's system...

3. The ‘Cover Readings’

(17) The Problem to be Solved

A sentence like (a) can be understood as T in a situation like (b).

- a. Sentence: Six boys met in the park.
- b. Situation: Tom and Frank met in the park. So did Bill and Tony. So did Dave and Roger.

(18) The Solution Offered by Scha (1981/1984)

The sentence in (17a) has a *reading* where its T-conditions are as follows; this reading is due to an ambiguity in the meaning of the numeral modifier.

The ‘Cumulative Reading’

$| \{ x : \text{ATOM}(x) \ \& \ \exists y : x \leq y \ \& \ y \in *[[\text{boy}]] \ \& \ y \in [[\text{met in the park}]] \} | = 6$

(19) Some Commentary

- We’ve already seen that the fact in (17) is not a problem for a theory of plurals admitting of the *-operator
- Moreover, this kind of approach has the advantage that it predicts similar facts for sentences *not* containing numeral expressions. For example, note that (a) can be true in situation (17b), which seems quite similar to the phenomenon in (17).

- a. Sentence: The boys met in the park.
- *However*, we’ll spend a little bit of time unpacking Scha’s analysis, since it will also introduce us to his syntax/semantics for numerical modification, which is key to understanding his approach to ‘cumulative readings’...

(20) First Caveat

- A significant of Scha’s (1981/1984) system (which we have thus far been ignoring) is that he assumes that plural and singular NPs are semantically equivalent...
- In Scha’s actual system, both *boy* and *boys* denotes (more or less) $*\{x : x \text{ is a boy}\}$
- This feature of his system, however is not central to the claims that are of interest to us, and so I will be ‘abstracting away from it’ in my presentation of his system.
- This will have the result, though, that the actual logical expressions I write here differ rather markedly in their appearance from those in Scha (1981/1984)...
(...please verify for yourself that they *are* equivalent...)

(21) **Second Caveat**

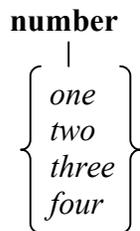
- Scha's proposals are formalized within the syntactic-semantic framework of Montague Grammar.
- I'm also going to be abstracting away from that, and will use syntactic representations and semantic 'rules' that will be more familiar to those of raised on Heim & Kratzer (1998).
(...my guide in this has been Landman 2000....)
- However, since they are based within the syntax of Scha (1981/1984), they will still look a bit 'funny' to our eyes...

With those caveats in mind, let's see what Scha (1981) does...

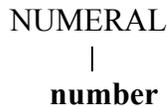
(22) **Scha's (1981 / 1984) Treatment of Numerals, Part 1**

A key component of Scha's theory is the syntactic and semantic distinction between (i) numbers, (ii) numerals, and (iii) numerical determiners.

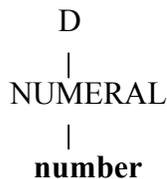
- a. Numbers:
Numbers are (more or less) syntactic heads, and can be filled by the following lexical items.



- b. Numerals
A number can project a 'NUMERAL' node, as follows.

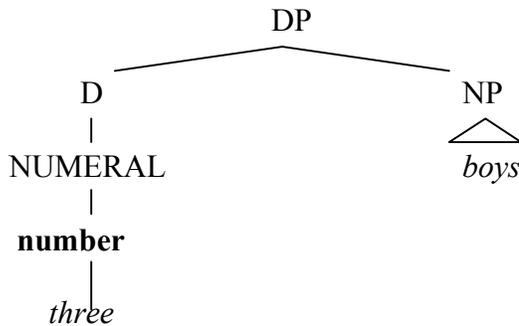


- c. Numerical Determiners
Any 'NUMERAL' node can project a 'D' node, as follows, creating a numerical determiner.



(23) **Illustration**

A DP like “three boys” will have (basically) the structure below.



Why this funky structure?...

*Basically, it's to allow one to define the meaning of numerical determiners in terms of the meanings of NUMERALS, and (ultimately) **numbers**...*

This will be done through a set of special semantic rules...

(24) **The Semantics of Numbers**

Numbers simply denote numbers. Easy peasy!

- a. [[one]] = 1
- b. [[two]] = 2
- c. [[three]] = 3 *et cetera...*

(25) **The Semantics of Numerals**

NUMERALS are $\langle et \rangle$ predicates based upon numbers; their interpretation is based upon the following general rule.

$$[[\text{[NUMERAL number] }]] = [\lambda x : | \text{ATOMS}(x) | = [[\text{number}]]]$$

With these ingredients in place, we will now introduce a variety of rules for interpreting numerical determiners....

*The existence of these multiple rules implements the key notion that numerical determiners are **ambiguous** between ‘collective’, ‘distributive’ and ‘cover’ interpretations...*

(26) **Preliminary: A New and Useful Notation**

Let S be a set. ‘+S’ = MAX(*S)
the entity you get by making the sum of all the entities in S

(26) **First Rule for Numerical Determiners: The Distributive Reading**

The following rule corresponds to rule C1 on page 151 of Scha (1984).

$$\begin{aligned} &[[\text{D [NUMERAL number] }]] = \\ &[\lambda P : \lambda Q : [[\text{[NUMERAL number] }]](+\{ y : y \in P \ \& \ AT(y) \ \& \ Q(y) = T \})] \end{aligned}$$

(27) **Illustration: Using Rule (26) To Derive the ‘Distributive Reading’ of Six**

- a. $[[\text{D [NUMERAL six] }]] =$ (by (26))
- b. $[\lambda P : \lambda Q : [[\text{[NUMERAL six] }]](+\{ y : y \in P \ \& \ AT(y) \ \& \ Q(y) = T \})] =$ (by (25))
- c. $[\lambda P : \lambda Q : [\lambda y : | \text{ATOMS}(y) | = [[\text{six}]]](+\{ y : y \in P \ \& \ AT(y) \ \& \ Q(y) = T \})] =$
- d. $[\lambda P : \lambda Q : [\lambda y : | \text{ATOMS}(y) | = 6](+\{ y : y \in P \ \& \ AT(y) \ \& \ Q(y) = T \})] =$
- e. $[\lambda P : \lambda Q : | \text{ATOMS}(+\{ y : y \in P \ \& \ AT(y) \ \& \ Q(y) = T \}) | = 6] =$
‘there are six atoms in the entity you get by summing together all the atoms y in P that are such that Q(y)’
- e. $[\lambda P : \lambda Q : | \{ y : y \in P \ \& \ AT(y) \ \& \ Q(y) = T \} | = 6]$
‘the number of atoms in P that are such that Q(y) = T is six’

(28) **Illustration of the Derived ‘Distributive Reading’**

$$\begin{aligned} &[[\text{six boys lifted a piano}]] = \\ &| \{ y : y \in *[[\text{boy}]] \ \& \ AT(y) \ \& \ y \in [[\text{lift a piano}]] \} | = 6 \\ &\text{‘the set of boy-atoms } y \text{ such that } y \text{ lifted piano has a cardinality of 6’} \\ &\text{‘there are six individual boys } y \text{ such that } y \text{ lifted a piano’} \end{aligned}$$

(29) **Second Rule for Numerical Determiners: The Collective Reading**

The following rule corresponds to rule C2 on page 151 of Scha (1984)

$$\begin{aligned} &[[\text{D [NUMERAL number] }]] = \\ &[\lambda P : \lambda Q : \exists x . x \in P \ \& \ [[\text{[NUMERAL number] }]](x) \ \& \ Q(x)] \end{aligned}$$

(30) **Illustration: Using Rule (29) To Derive the ‘Collective Reading’ of Six**

- a. $[[[D \text{ NUMERAL six}]]] =$ (by (29))
- b. $[\lambda P : \lambda Q : \exists x . x \in P \ \& \ [[\text{NUMERAL six}]](x) \ \& \ Q(x)] =$ (by (25))
- c. $[\lambda P : \lambda Q : \exists x . x \in P \ \& \ [\lambda y : | \text{ATOMS}(y) | = [[\text{six}]]](x) \ \& \ Q(x)] =$
- d. $[\lambda P : \lambda Q : \exists x . x \in P \ \& \ [\lambda y : | \text{ATOMS}(y) | = 6](x) \ \& \ Q(x)] =$
- e. $[\lambda P : \lambda Q : \exists x . x \in P \ \& \ | \text{ATOMS}(x) | = 6 \ \& \ Q(x)]$

(31) **Observation:**

The meaning derived by (29) is exactly that we’ve been assuming for a numerals...
... thus, this will clearly derive for us the collective reading of a sentence like “six boys lifted a piano”...

(32) **Third Rule for Numerical Determiners: The Cover Reading**

The following rule corresponds to rule C3 on page 151 of Scha (1984)

$$[[[D \text{ NUMERAL number}]]] =$$

$$[\lambda P : \lambda Q : [[\text{NUMERAL number}]](+\{ y : y \in P \ \& \ Q(y) = T \})]$$

(33) **Illustration: Using Rule (32) To Derive the ‘Cover Reading’ of Six**

- a. $[[[D \text{ NUMERAL six}]]] =$ (by (29))
- b. $[\lambda P : \lambda Q : [[\text{NUMERAL six}]](+\{ y : y \in P \ \& \ Q(y) = T \})] =$ (by (25))
- c. $[\lambda P : \lambda Q : [\lambda y : | \text{ATOMS}(y) | = [[\text{six}]]](+\{ y : y \in P \ \& \ Q(y) = T \})] =$
- d. $[\lambda P : \lambda Q : [\lambda y : | \text{ATOMS}(y) | = 6](+\{ y : y \in P \ \& \ Q(y) = T \})] =$
- e. $[\lambda P : \lambda Q : | \text{ATOMS}(+\{ y : y \in P \ \& \ Q(y) = T \}) | = 6] =$
‘take all those y such that they are pluralities of P and satisfy Q...
sum them together...
the resulting entity has six atoms in it...

(34) **Illustration of the Derived ‘Cover Reading’**

[[six boys met in the park]] =
ATOMS(+{ y : y ∈ *[[boy]] & [[met in the park]](y) = T }) | = 6]

‘take all those groups of boys that met in the park...
sum them together....
the resulting entity has six atoms in it...’

‘**six boys were involved in events of meeting in the park.**’

Observation: *These T-conditions indeed hold in situation (17b)!!*

(35) **Summary**

- For sentence containing numerically modified plurals, Scha (1981 / 1984) posits a systematic ambiguity in the meaning of the numeral determiner.
- The *systematicity* of this ambiguity is captured via the postulation of the three general interpretation rules in (26), (29) and (32).
(i.e., he doesn’t just stipulate each reading separately in the lexicon...)
- One of the meanings he generates – the ‘cover reading’ of the numeral – is able to capture the facts in (17)...
- ... however, this might be criticized for its lack of coverage, since similar interpretation seems to surround sentences containing plural definites....

4. The ‘Cumulative Readings’

(36) **The Problem to be Solved**

Sentence (a) can be read as T in a situation like (b). Following Scha (1981/1984), then, it seems that (a) can receive the T-conditions in (c).

- Sentence: Three TAs graded thirty exams.
- Situation: Paul graded 10 exams, Tony graded 15, and John graded 5.
- Scha’s T-Conditions for ‘Cumulative Readings’ (see his (4b))
The **total number** of TAs that graded exams is **3**, and the **total number** of exams graded by TAs is **30**.

(37) **Some Preliminary Remarks**

- Much of the complex technology that Scha introduces to derive the T-conditions in (36c) is tied to his *particular* way of representing those T-conditions in a formal meta-language...
- So, before we can start spelling out the semantics, we need to introduce some of Scha's meta-language formalisms...
- Important Note
 - Much of the complexity of the system in Scha (1981 / 1984) is due to its *generality*...
... Scha set out to create a system that would derive cumulative readings for predicates of *any* arity (transitives, di-transitives, etc...)
 - To simplify our presentation, we will focus only on cumulative readings with binary predicates, such as the verb *grade* in our example (36)...
 - For more information on how the system here can be generalized to other cases, see Scha (1984 / 1981)...

(38) **Preliminary 1: Projection Functions**

Let the set S be a set of pairs $\langle x,y \rangle$.

- a. $\text{proj}_1(S)$ = $\{ x : \text{there is some } y \text{ such that } \langle x,y \rangle \in S \}$
'the set of all the first members of ordered pairs in S'
- b. $\text{proj}_2(S)$ = $\{ y : \text{there is some } x \text{ such that } \langle x,y \rangle \in S \}$
'the set of all the second members of ordered pairs in S'

(39) **Preliminary 2: Cartesian Products**

Let S1 and S2 be two sets. $S1 \times S2$ = $\{ \langle x,y \rangle : x \in S1 \text{ and } y \in S2 \}$

(40) **Illustration**

If $A = \{ a, b \}$ and $B = \{ c, d \}$, then...

$$A \times B = \{ \langle a,c \rangle, \langle a,d \rangle, \langle b,c \rangle, \langle b,d \rangle \}$$

$$\text{proj}_1(\{ \langle a,c \rangle, \langle a,d \rangle, \langle b,c \rangle, \langle b,d \rangle \}) = \{ a, b \} = A$$
$$\text{proj}_2(\{ \langle a,c \rangle, \langle a,d \rangle, \langle b,c \rangle, \langle b,d \rangle \}) = \{ c, d \} = B$$

(41) **Scha's Formal Statement of the 'Cumulative Reading' of (36a)**

a. Sentence: Three TAs graded 30 exams

b. Truth-Conditions:

$| \text{ATOMS}(+\text{proj}_1(\{ \langle x,y \rangle \in *[[\text{TA}]] \times *[[\text{exam}]] : \langle x,y \rangle \in [[\text{graded}]] \})) | = 3 \ \&$
 $| \text{ATOMS}(+\text{proj}_2(\{ \langle x,y \rangle \in *[[\text{TA}]] \times *[[\text{exam}]] : \langle x,y \rangle \in [[\text{graded}]] \})) | = 30$

c. Taking the Formula Above Apart

$| \text{ATOMS}(+\text{proj}_1(\{ \langle x,y \rangle \in *[[\text{TA}]] \times *[[\text{exam}]] : \langle x,y \rangle \in [[\text{graded}]] \})) | = 3 \ \&$

Take all those pairs $\langle x,y \rangle$ such that x is a plurality of TAs and y is a plurality of exams, and x graded y ...

Take all the first members of those pairs (all the TAs that graded some exam)...

Sum all those TAs together...

The plurality you get has three atoms in it...

$| \text{ATOMS}(+\text{proj}_2(\{ \langle x,y \rangle \in *[[\text{TA}]] \times *[[\text{exam}]] : \langle x,y \rangle \in [[\text{graded}]] \})) | = 30$

Take all those pairs $\langle x,y \rangle$ such that x is a plurality of TAs and y is a plurality of exams, and x graded y ...

Take all the second members of those pairs (all the exams graded by some TA)...

Sum all those exams together...

The plurality you get has thirty atoms in it.

... Thus, all the TAs 'involved' in grading exams was three, and all the exams graded by some TA (or group of TAs) was thirty...

OK... so this formal statement in (41b) does seem to amount to the T-conditions informally stated in (36c)...

But, how in the name of god do we derive it from an LF for (36a)?...

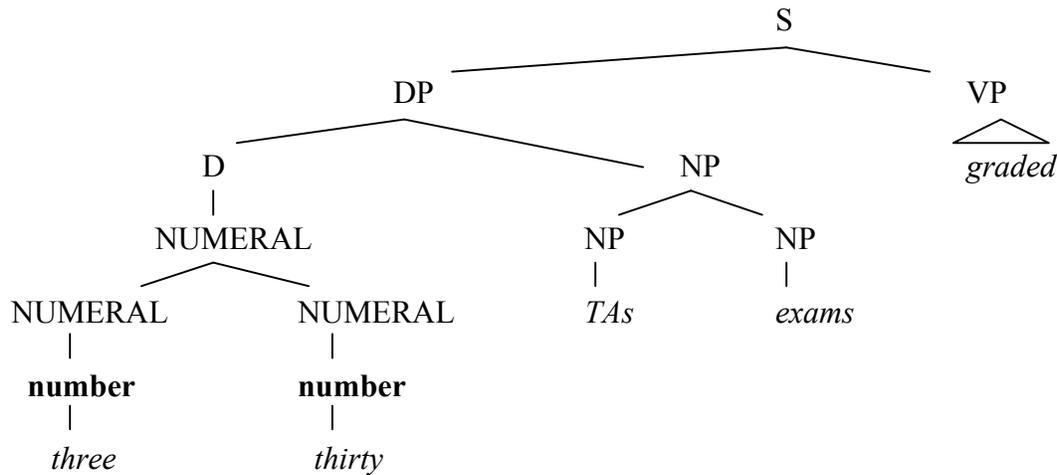
(42) **First Question**

What is the LF which we should assume for (36a) under its proposed 'cumulative' interpretation?...

(43) **Unsatisfying Answer**

- Scha (1981/1984) isn't working within GB syntax, but rather Montague Grammar, and so technically speaking, there isn't an 'LF' *per se*...
- However, if we were to import his overall story into a GB syntax, the LF would look something like (44) [see Landman (2000: Ch. 4)]

(44) The LF for (36a) Under its Cumulative Reading



- In this structure, the two numerals *three* and *thirty* have joined together to form a single constituent, a *binary numeral*.
- In this structure, the two NPs *TAs* and *exams* have joined together to form a single constituent, a *binary NP*.
- **Don't worry yourself with how this kind of a structure can be derived... no one seems to think there's really a natural way of doing this...**
(Even in their original context, Scha's syntactic derivations were considered rather baroque...)

(45) **Problem!**

The semantic rules we spelled out for numerical determiners in Section 3 will not be sufficient to interpret this structure:

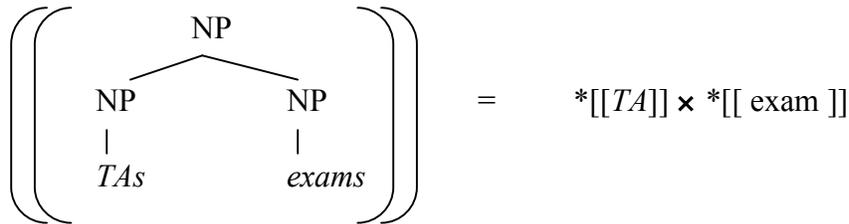
- We have no rule for interpreting the branching, binary 'NP' node.
- We have no rule for interpreting the branching, binary 'NUMERAL' node.
- We have no rule for interpreting D when dominating a binary 'NUMERAL' node
(Notice the exact statement of rules (26), (29) and (32)...))

(46) **Interpreting the Binary NP (Scha 1984: 152, Rule D)**

Let's interpret the binary branching NP as the *Cartesian product* of its constituent NPs.

$$[[[_{NP} NP_1 NP_2]]] = [[NP_1]] \times [[NP_2]]$$

(47) **Illustration**

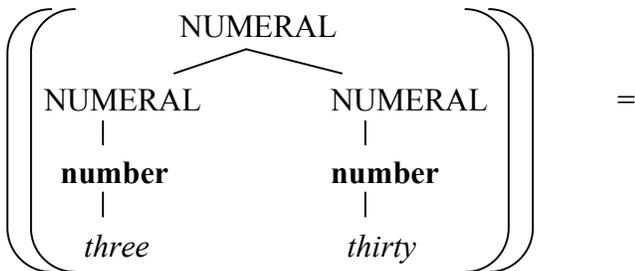


That's the easy part... what should we do about the binary numerals?

(48) **Interpreting the Binary Numeral (Scha 1984: 151, Rule B)**

$$\begin{aligned} &[[[\text{NUMERAL NUMERAL}_1 \text{ NUMERAL}_2]]] = \\ &[\lambda P : [[\text{NUMERAL}_1]] (+\text{proj}_1(P)) \& [[\text{NUMERAL}_2]] (+\text{proj}_2(P))] \end{aligned}$$

(49) **Illustration:**



$$[\lambda P : [[[\text{NUMERAL three}]]] (+\text{proj}_1(P)) \& [[[\text{NUMERAL thirty}]]] (+\text{proj}_2(P))] = \text{(by (25))}$$

$$[\lambda P : [\lambda x : | \text{ATOMS}(x) | = 3] (+\text{proj}_1(P)) \& [\lambda x : | \text{ATOMS}(x) | = 30] (+\text{proj}_2(P))] =$$

$$[\lambda P : | \text{ATOMS}(+\text{proj}_1(P)) | = 3] \& | \text{ATOMS}(+\text{proj}_2(P)) | = 30]]$$

‘if you add up all of the first members of the pairs in P, you get something with 3 atoms...
...and if you add up all the second members of the pairs in P, you get something
with 30 atoms...’

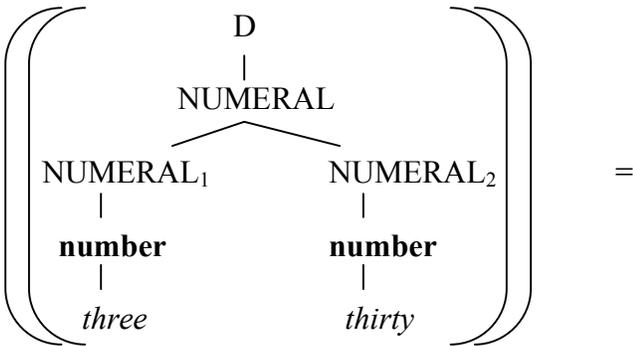
You can see how this is now starting to build towards the T-conditions in (41)...

(50) **Interpreting a Binary Numerical Determiner (Scha 1984: 152, Rule F2)**

$$[[[D [NUMERAL NUMERAL_1 NUMERAL_2]]]] =$$

$$[\lambda P: \lambda Q: [[[NUMERAL NUMERAL_1 NUMERAL_2]]] (\{ \langle x,y \rangle \in P : \langle x,y \rangle \in Q \})]$$

(50) **Illustration**



$$[\lambda P: \lambda Q: [[[_{NUM} NUMERAL_1 NUMERAL_2]]] (\{ \langle x,y \rangle \in P : \langle x,y \rangle \in Q \})] = \text{(by (49))}$$

$$[\lambda P: \lambda Q: [\lambda Z : | ATOMS(+proj_1(Z)) | = 3] \& | ATOMS(+proj_2(Z)) | = 30]$$

$$(\{ \langle x,y \rangle \in P : \langle x,y \rangle \in Q \})] =$$

$$[\lambda P: \lambda Q: | ATOMS(+proj_1(\{ \langle x,y \rangle \in P : \langle x,y \rangle \in Q \})) | = 3]$$

$$\& | ATOMS(+proj_2(\{ \langle x,y \rangle \in P : \langle x,y \rangle \in Q \})) | = 30]]$$

With these ingredients in place, we can now interpret the LF in (44)!

(51) **Putting it all Together: Deriving the Cumulative Reading**

[[(44)]] =

[[DP]] ([[VP]]) =

[[DP]] ([[graded]]) =

[[D]] ([[NP]]) ([[graded]]) = (by (47))

[[D]] (*[[TA]] × *[[exam]]) ([[graded]]) = (by (50))

[λP: λQ: | ATOMS(+proj₁({ <x,y> ∈ P : <x,y> ∈ Q })) | = 3] &
| ATOMS(+proj₂({ <x,y> ∈ P : <x,y> ∈ Q })) | = 30] (*[[TA]] × *[[exam]])
([[graded]]) =

| ATOMS(+proj₁({ <x,y> ∈ *[[TA]] × *[[exam]] : <x,y> ∈ [[graded]] })) | = 3] &
| ATOMS(+proj₂({ <x,y> ∈ *[[TA]] × *[[exams]] : <x,y> ∈ [[graded]] })) | = 30]

Yay! Our system derives the bee-yoo-tiful set of T-conditions in (41)!!!

Q to-the E to-the D!!

(52) **Some Commentary**

- So, this is where the study of cumulative readings begins, with the system sketched out here...
- There are some issues with this system that people usually pick on, which have driven subsequent work towards ‘something better’:
 - Each of the three phenomena discussed above (cumulative readings, cover readings, co-distributive readings) are captured via three completely different mechanisms...
 - The cumulative readings are captured through a rather abstruse syntax and rather stipulative semantics (based upon special rules of interpretation)...
- We’ll next turn to one well-known and popular alternative to Scha (1981/1984), one that has come to influence most subsequent work in the literature...