The Basics of Intensional Semantics, Part 1:
The Motivation for Intensions and How to Formalize Them

1. The Inadequacies of a Purely Extensional Semantics

(1) Extensional Semantics

a. The interpretation function “[[ ]]” is (always) a function from natural language expressions to their extensions in the (actual) world.

b. The extension of a complex phrase is (always) derived by computing the extensions (and only the extensions) of its component parts.

In LING 610, you saw that – amazingly – an extensional semantics is sufficient for a semantic theory of many natural language structures...
...However, it won’t be enough for all natural language...

(2) Negative Consequence: No ‘Counterfactual’ Language

a. The Consequence:
In an extensional semantics, all predicates are of type <et>, <eet>, etc.. Thus, all sentences will be T or F depending only upon properties and relations that hold in the actual world.

- [[smokes]] = [λx : x smokes] = yields T iff x smokes (in the actual world)
- [[Barack smokes]] = T iff Barack smokes (in the actual world)

b. The Empirical Problem:
There are sentences of natural language whose truth or falsity depends on properties or relations holding in (purely) hypothetical worlds.

(i) Barack might smoke.
(ii) Michelle believes that Barack smokes.
(iii) If Joe smoked, then Barack wouldn’t be alone.

Nothing in our purely extensional ‘semantic toolkit’ seems like it will provide a decent analysis of these kinds of structures.

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1 These notes are based upon Heim & Kratzer (1998; Chapter 12) and von Fintel & Heim (2011; Chapter 1).
Negative Consequence: Arguments are Always Extensions

a. The Consequence:
If some phrase XP is argument to a head H, then the extension of H is a function that takes the extension of XP as its argument.

b. The Empirical Problem:
Consider the verb “believe”; from sentences like the following, it seems to have a meaning that takes as argument the ‘meaning’ of a sentence (its complement CP).

(i) Orly believes [ that Barack smokes ].
In an extensional semantics, the ‘meaning’ (i.e., semantic value) of a sentence is its truth value (since that’s what the extension of a sentence is).

(ii) \[ \llbracket \text{Barack smokes} \rrbracket = T \]
So, if we were to analyze the verb “believes” in an extensional semantics, we would have to view it as a function of type \(<t, e t\>>. But, now consider the fact that the extensions of sentences (i) and (ii) are both \(T\).

(iii) \[ \llbracket \text{Orly believes [that Barack smokes]} \rrbracket = T \]
Thus, the extension of “believes” must (qua function) contain \(<T, <\text{Orly}, T>>\). But, now consider that the extension of the following sentence is also \(T\):

(iv) \[ \llbracket \text{Barack is a natural-born citizen} \rrbracket = T \]
Thus, our extensional semantics for “believe” would entail/predict that:

(v) \[ \llbracket \text{Orly believes [that Barack is a natural-born citizen]} \rrbracket = T \]

Epic Fail: Our extensional semantics for “believe” makes the obviously false prediction that if X believes one true/false sentence, then X believes all true/false sentences!

But this obviously false prediction is a necessary consequence of two core assumptions of our purely extensional semantic system:

(i) The semantic value of a structure is (always) its extension

(ii) The extension of a sentence is its truth value.
(4) **Conclusion: An Extensional Semantics is Not Enough**

For words like “believe”, their extension does *not* take as argument the *extension* of their sentential complement (unlike purely extensional logical operators like “and”).

a. Consequently, in this structural context, our ‘semantic valuation’ function “[[ ]]]” has to provide something *other* than the extension of the complement clause.

b. Consequently, for sentences containing the verb “believes”, their extension is not determined purely by computing the *extensions* of their component parts.

Thus the core assumptions of our (purely) extensional semantics in (1) are wrong!
(Extensions just aren’t enough for a complete semantics of human language…)

2. **Towards a Solution: Intensions?**

(5) **A Recap: What Do We Need?**

a. From examining the intuitive content of words like “believe”, “might”, *etc.*, it appears that we must extend our theory of semantic/*types* to include objects that would permit ‘talk about purely hypothetical worlds/situations’.

b. From examining how the compositional semantics for “believe” must operate, it appears that we must extend our semantic theory so that the interpretation function “[[ ]]” sometimes yields objects other than extensions.

So, how do we get what we need? Well, let’s take a break and recall the notion of an ‘intension’:

(6) **The Intension of a Structure X**

A kind of ‘concept’ / ‘formula’ / ‘function’ which – given how things stand in the actual world – determines the extension of X in the actual world.

a. “The president”
   (i) \( \text{Extension} = \text{referent} \) Barack Obama
   (ii) \( \text{Intension} = \text{concept} \) ‘The person who holds the presidency’

b. “The president smokes”
   (i) \( \text{Extension} = \text{truth value} \) \( T \)
   (ii) \( \text{Intension} \neq \text{truth conditions/proposition} \) The president smokes.

*Now, consider the following line of thought…*
(7) **One Line of Thought…**

a. Sentences with the same *extension* (truth value) can nevertheless have two different *intensions* (truth conditions)
   - “Barack smokes” is $T \iff$ *Barack smokes*
   - “Barack is a natural-born citizen” is $T \iff$ *Barack is a natural-born citizen.*

b. Thus, if a word like “believe” took as argument the *intension* of its complement clause, we could avoid the ‘epic fail’ in (3).
   - Since the intension of “Barack smokes” is distinct from the intension of “Barack is a natural-born citizen”, Orly can stand in the ‘believes’-relation to the former, but not the latter!

c. *Consequently*, if we (somehow) allow that the semantic valuation function “[[ ]]” yields the *intension* of the complement of “believes”, we can avoid the empirical problem in (3)!!

(8) **Some Independent, Conceptual Motivation**

*Question:* What kind of relation does the verb “believes” represent?

a. Not a Relation Between an Entity and a *Sentence* $<e, <e, t>>$
The following seems true: “Qin Shi Huang thought terracotta was nice.”
   - But, what kind of possible relation could Qin Shi Huang have to the *English sentence* “terracotta was nice”.
     - He predated the possibility of the sentence by at least a millennium!

b. Not a Relation Between an Entity and *Truth Value* $<t <e, t>>$
(see reasoning above in (3))

c. A Relation Between an Entity and a *Proposition / Truth Conditions*?
   *Seems to match to our informal pre-theoretic notions…*
   - Even though Qin Shi Huang never uttered or assented to the *English sentence* “terracotta was nice”…
     - He *did* utter/assent to an *Archaic Chinese* sentence that expressed the same ‘proposition’ (had the same truth-conditions).

So, even at a very pre-theoretic, informal level, it seems profitable to analyze “believe” as a relation between an individual and the *intension* of a sentence (i.e., truth conditions or ‘proposition’).
The Plan

Let’s flesh-out and formalize this idea that semantic values can sometimes be *intensions*.

a. The resulting theory will avoid the empirical problem in (3).

b. **Moreover**, we will see over subsequent weeks that it also resolves the more general empirical issue in (2).

*The resulting ‘intensional semantics’ will provide the tools necessary to analyze those sentences that seem to describe purely ‘hypothetical’ situations and relations...*

3. Formalizing the Notion of an Intension

So, following the plan in (9), we want to have a fully fleshed-out, formalized semantic system that manipulates *intensions*...

(10) **Fundamental Problem**

a. Thus far, our concept of an ‘intension’ has been a rather informal one (*viz.* (6)).

b. If we want a formal system that ‘manipulates’ intensions, we need some kind of a formal model of what an ‘intension’ is.

c. Thus, we need to have a way of modeling the notion of an ‘intension’ in our formal machinery (*i.e.* using set-theoretic concepts and our lambda notation).

(11) **Towards A Formal Model of ‘Intensions’: The Basic Idea**

a. **The Core Property of an ‘Intension’ (*cf.* (6))**

   For any structure X, the intension of X – combined with how things stand in the actual world – determines the extension of X (in the actual world).

   *If you ‘combine’ the intension of X with the actual world, that yields the extension of X.*

b. **The Formal Insight:** We can think of the ‘intension of X’ as a kind of *function*!

   It’s a function which, if you give it a reality / universe, it gives you back some object – the object that is the extension of X (in that reality / universe)

   *So, let’s try to flesh out in more detail the basic idea in (11b)!*
3.1 Step 1: The Ontology of ‘Possible Worlds’

The ‘basic idea’ in (11b) is that intensions are functions from ‘realities / universes’ to things (extensions)…

Thus, the semantic system we are aiming to create is one where we have functions whose domain is a set of ‘realities / universes’…

Thus, we must somehow add a set of ‘realities / universes’ to the overall inventory of ‘things’ that the meanings of natural language structures make use of...

(12) Terminology

Instead of the term ‘realities / universes’, we’ll adopt the (centuries-old, but initially confusing) term ‘possible worlds’

• Thus, intensions will be functions from ‘possible worlds’ to things.
• The actual world is considered one of an infinite set of ‘possible worlds’

(13) A Picture of the ‘Metaphysics’ this Semantics Assumes

a. The Actual World

The ‘actual world’ is the sum total of all facts - past, present and future - until the end of time. It encompasses such facts as:

• Barack Obama is president in 2013
• Seth Cable lived at 67 Harvard Avenue in 2003
• Julius Caesar was stabbed exactly 23 times.
• 5,981 years ago, there were exactly 239,653 California condors.

b. The Contingency of the Actual World

Many of these ‘facts’ making up the actual world could have been otherwise.

• Obama could have decided not to run for president.
• I could have stayed in the grad dorms until 2007.
• One of the Roman senators could have missed.
• 5,982 years ago, one of those condor eggs might not have hatched.

c. The HUGE Metaphysical Leap

For every fact about the actual world that could have been otherwise, there actually exists an ‘alternative reality’ where that fact is otherwise.

• Our actual universe is one of a set of (very real) alternative universes.
• The set of these alternate univeses will be represented by \( W \) (the set of all possible worlds.)
• The actual world (just one member of \( W \)) can be represented as ‘\( w_0 \)’
3.2 Step 2: Formalizing the ‘World-Dependency’ of Extensions

With this ontology of ‘possible worlds’ in place, let us now reflect upon the following key property of our *extensional* semantic valuation function “[[ . ]]”

(14) **The Value of “[[ ]]” Depends Upon the State of the Actual World**

- If the (extensional) semantic valuation function “[[ ]]” takes a sentence as argument, it gives back T or F depending upon the state of the (actual) world.

  a. $[[ \text{Barack smokes}]]= \text{T}$ iff Barack smokes.

- Note, though, that in our extensional semantics, these equations are always implicitly about *truth in the actual world*.

- Consequently, the equation in (b) would be a more explicit restatement of (a).

  b. $[[ \text{Barack smokes}]]= \text{T in } w_0$ iff Barack smokes in $w_0$

Thus, our extensional semantics derives generalizations of the form in (14b)…

*However, when we consider the truth of sentences across possible worlds, it’s apparent that (14b) is simply a limiting case of a much broader generalization.*

(15) **Truth of Sentences Across Possible Worlds**

- Note that there are possible worlds other than $w_0$ where Barack smokes.
  - In these other possible worlds, some other facts are different (maybe water is purple) but the fact remains that Barack smokes.

- Intuitively, in these *other* possible worlds, the sentence “Barack smokes” is T.

- Next, note that there are possible worlds other than $w_0$ where Barack *doesn’t* smoke. Intuitively, in *these* possible worlds, the sentence “Barack smokes” is F.

- Thus, we arrive at this generalization (of which (14b) is a limiting case):

  Let $w$ be any possible world in $W$:
  
  $[[ \text{Barack smokes} ]] = \text{T in } w$ iff Barack smokes in $w$.

**New Sub-Goal:**

Let us augment our semantic theory so that it derives not simply the (limited) set of statements in (14b), but the more general, trans-world statements in (15).
Sub-Step 1: New Notation

Let’s first introduce the following, more compact notation for the “left hand” part of the targeted generalization in (15).

a. \[[ X ]\]^w = the extension of X at world w.

With this, we can rewrite our targeted generalization as the following:

b. Let w be any possible world in W:
\[[ Barack \text{ smokes } ]\]^w = \text{T iff Barack smokes in w.}

Terminology

In the notation \[[ X ]\]^w, the possible world w paired with “[[ ]]” is the evaluation world.

Sub-Step 2: New Lexical Entries

Using the new notation in (16a), let us re-write our lexical entries for “smokes” and “Barack” accordingly.

a. \[[ \text{smokes} ]\]^w = \[\lambda x : x \text{smokes in w }\]

What This Equation Says:
For any possible world w, the extension of “smokes” at w is:
the function from entities x to truth-values which yields T iff x smokes in w.

Some Further Notes:
According to this equation, the extension of “smokes” will be different at different possible worlds.
• In a world w’ where Barack smokes: [[smokes]]^(w)\text{ (Barack) } = T
• In a world w’’ where Barack doesn’t smoke: [[smokes]]^(w’’)\text{ (Barack) } = T

b. \[[ \text{Barack} ]\]^w = Barack

What This Equation Says
For any possible world w, the extension of “Barack” at w is Barack

Some Further Notes:
According to this equation, the extension of “Barack” is the same at all possible worlds.
• The term for such an expression is a rigid designator.
• In this class, the claim that proper names are rigid designators is simply a stipulated (simplifying) assumptions.
  o But, there are serious philosophical arguments for it (Saul Kripke)
With lexical entries like those in (18), our semantic theory can now derive statements of the more general form in (15)/(16b)!

(19) Deriving the Trans-World Generalization in (15)/(16b)

Let \( w \) be any possible world in \( W \)…

a. \( [[ \text{Barack smokes }]^w] = T \quad \text{iff} \quad \text{(by FA)} \)

b. \( [[ \text{smokes }]^w( [[\text{Barack}]^w] ) = T \quad \text{iff} \quad \text{(by (18b))} \)

c. \( [[ \text{smokes }]^w( \text{Barack} ) = T \quad \text{iff} \quad \text{(by (18a))} \)

d. \( [\lambda x : x \text{smokes in } w ]( \text{Barack} ) = T \quad \text{iff} \quad \text{(by LC)} \)

e. Barack smokes in \( w \).

Okay…So What Have We Done, Exactly?

• By a computation parallel to the one in (19), we can also derive the extensional truth-conditional statement in (14b):

\( [[ \text{Barack smokes }]^{w_0}] = T \text{ in } w_0 \quad \text{iff} \quad \text{Barack smokes in } w_0 \)

• Thus, even with these notational additions, we are still derive all the truth-conditional statements of our original extensional semantics…

• But, our notation also allows us to formally model the way in which the extension of an expression will vary depending upon the possible world…

…and with this, we will now be able to provide a formal model of an intension!

3.3 Step 3: Building an ‘Intension’ from These Ingredients

Let us now recall the ‘formal insight’ we wish our system to capture:

(20) Targeted ‘Formal Insight’

We can think of the ‘intension of \( X \)’ as a kind of \textit{function}, which takes a possible world \( w \) and returns \textit{the extension of \( X \) in } \( w \).

\textit{Well, in this context, consider the function in (21), written variously as (21a), (21b), (21c), (21d)
(21) **The Intension of “Barack Smokes”**

a. \[ \lambda w : [[ \text{Barack smokes} ]]^w = T \]  
   \[
   \]

b. \[ \lambda w : [[ \text{Barack smokes} ]]^w \]  
   \[
   \]

c. \[ \lambda w : \text{Barack smokes in } w \]  
   \[
   \]

d. *The function from possible worlds to T values, which when given a possible world \( w \) as an argument, yields T iff \([ [ \text{Barack smokes} ]]^w = T \) (iff Barack smokes in \( w \)).*

(22) **The Importance of the Function in (21)**

Following the notation we developed in Section 3.2, the function in (22) takes a particular possible world \( w \) as argument, and returns the extension of “Barack smokes” at \( w \).

- It takes \( w \) and returns T iff (the extension of) “Barack smokes” is T at \( w \).
- It takes \( w \) and returns F iff (the extension of) “Barack smokes” is F at \( w \).

Thus, following our ‘targeted formal insight’ in (20), the function in (21) could be regarded as the intension of “Barack smokes”.

(23) **The Intension of “X”**

Recall the definition of the following notation from (16a)

a. \([ [ X ]]^w = \) the extension of X at world \( w \).

Thus, the function in (23b) clearly can be characterized by the prose in (23c)

b. \[ \lambda w : [[ X ]]^w \]  
   \[
   \]

c. *The function whose domain is the set of possible worlds, and when given a possible world \( w \) as argument, yields the extension of \( X \) at \( w \) as its value.*

Thus, following our ‘targeted formal insight’, the function in (23b) is identifiable as ‘the intension of \( X \’\)

(24) **General Conclusion**

For any structure \( X \), the function \([ \lambda w : [[X]]^w \) is the intension of \( X \).
3.4 Step 4: Extensions to Our Type Theory

Recall our theory of semantic types (Section 5 of “A Review of the Essentials of Extensional Semantics”).

- That system is intended to provide a notation for all the ‘things’ that can be the extensions of natural language expressions.
- Recall from Section 2 that we are pursuing the view that some verbs (e.g. “believe”) has as their extensions functions that take intensions as arguments.
- Thus, we need to expand our theory of semantic types so that it includes a way of representing intensions, as formalized in Section 3 above…

(26) ‘Intensional-izing’ our Theory of Semantic Types

If $\alpha$ is a semantic type, then $<s, \alpha>$ is also a semantic type (where we informally read ‘$<s, \alpha>$’ as ‘functions from possible worlds to things of type $\alpha$).

<table>
<thead>
<tr>
<th>Illustration</th>
<th>Proposition:</th>
<th>Function of type $&lt;s, t&gt;$</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Property:</td>
<td>Function of type $&lt;s, &lt;e, t&gt;&gt;$</td>
</tr>
<tr>
<td></td>
<td>Individual Concept:</td>
<td>Function of type $&lt;s, e&gt;$</td>
</tr>
</tbody>
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Side-Note:

- Note that (as of yet) we are not actually adding to our semantic type theory a ‘type s’ of possible world.
- This is because (as far as we’ve seen) there are no expressions of natural language that have specific possible worlds as their values…